Measures and Relative Motions of Some Mostly F. G. W. Struve Doubles

E. O. Wiley
Remote Astronomical Society Observatory
Mayhill, NM

Mailing address: 2503 Atchison Ave.
Lawrence KS 66047
edwiley@sunflower.com

Abstract: Measures of 59 pairs of double stars with long observational histories using "lucky imaging" techniques are reported. Relative motions of 59 pairs are investigated using histories of observation, scatter plots of relative motion, ordinary least-squares (OLS) and total proper motion analyses performed in "R," an open source programming language. A scatter plot of the coefficient of determinations derived from the OLS y|epoch and OLS x|epoch clearly separates common proper motion pairs from optical pairs and what are termed "long-period binary candidates." Differences in proper motion separate optical pairs from long-term binary candidates. An Appendix is provided that details how to use known rectilinear pairs as calibration pairs for the program REDUC.

Introduction

In Wiley (2010) I presented a protocol for estimating rectilinear elements of optical doubles and commented on the use of ordinary least-squares (OLS) analysis for exploring the nature of common proper motion and binary pairs. In this paper I expand on that investigation by measuring and analyzing a number of mainly F. G. W. Struve (STF) doubles where one or both of the pairs have a measurable proper motion, defined as a proper motion that exceeds errors. The exercise is largely an inductive data-exploration exercise to see if there are some relatively simple and approachable analyses available to amateur researchers that would separate optical pairs from physical pairs. Fifty-four "knowns" (proper motions reported for both stars) were selected from the WDS catalog. I included four "unknowns" (proper motion high in one star but unknown in the other) to see if the techniques developed during this inductive exercise would successfully discriminate them. To add value to the exercise I picked my "knowns" in the WDS from pairs with no indication in the WDS Notes column that they have been characterized as optical or physical.

Methods

Many of the pairs measured herein have separations of 3" - 6" and this necessitated using a form of "lucky imaging" with a Takahashi Mewlon 0.3 meter telescope at the GRAS observatories in Mayhill, New Mexico. I typically took 20 - 50 short exposures (0.25 - 1 second) and picked only those where there was a clear separation between the pairs.

Theta and rho were measured using REDUC (Losse, 2010 et seq.) which has a distinct advantage when working with short exposures as one does not have to reduce the plate. In some cases individual images were eliminated due to poor quality ("unlucky" images) and in other cases images were stacked to improve signal-to-noise ratio (see Table 1). REDUC requires at least one calibration pair to determine camera orientation and plate scale. I used two relatively wide optical pairs with excellent rectilinear elements (02157+6740 ENG 10 and 23280+2335 STTA246AB). A protocol for reducing

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rectilinear elements to theta and rho for a given date for a calibration pair is detailed in Appendix A.

Astrophysical data were gathered using Aladin in conjunction with various catalogs. Proper motions were taken from the Washington Double Star Catalog (Mason et al., 2001-2011) and checked against the Tycho 2 (I/259; Hog et al., 2000), Hipparchos (I/239; ESA, 1995), the All-sky Compiled (I/280B; Kharchenko and Roeser, 2009) and PPMXL catalogs (I/317; Roeser et al. 2010) available at the CDS (Bonnarel et al., 2000).

The histories of measures for each pair were requested from the U. S. Naval Observatory (Mason, 2006). Since this is a proof-of-concept paper, most STF pairs were chosen based on pairs with moderate to large proper motions where each pair was had either (1) similar proper motions (≤ 10 mas/year difference in both RA and Dec) and presumed to be physically associated or (2) dissimilar proper motions (≥ 11 mas/year difference in RA and Dec) presumed to be optical pairs. Four pairs were included that had one but not both members with proper motion data greater than 25 mas/year; these act as unknowns. Specific steps in the analysis are listed below.

1. Convert theta and rho measures to Cartesian (x,y) coordinates in an Excel® spread sheet (see Wiley, 2010).

2. Using the plotting functions in Excel®, examine the relative motions in Cartesian space. Examination is facilitated by using the line function to connect observations by epoch of observation and a rough idea of the relationship between x- and y-values can be investigated by line fitting using the trend line function. (The trend line function can return an OLS solution, but more formal analyses should be performed.) Of particular interest are indications of no relative motion (relative position of the secondary clustered around one position), linear motion (relative motion follows a straight line, forming a time series of historically ordered measures) and curvilinear motion (some indication of orbital motion). An OLS y|x solution was calculated for each pair using the "lm" functions of the R programming language (Ihaka and Gentleman, 1996). This "lm" function is simply a call to R to perform linear regression on the variables. I note, based on comments by *Dr*. Richard Branham (pers. comm.), that Total Least Squares (TLS) may be the more appropriate technique since both x- and y-values are subject to errors (e.g. Branham, 2001). I plan to explore TLS techniques in future studies as a TLS package is available in R.

3. Perform two ordinary least squares (OLS) analyses, again using "R," on each pair using epoch of observation as the independent variable and x- and y-values as dependent variables. Test the null hypothesis that slopes of each analysis are statistically no different than zero (flat slope) and harvest the coefficients of variation (R^2) from each analysis. Such analyses can also be performed using the more formal "Regression" function in the Excel® data analysis tool pack. Since epoch of observation can be taken as "without error," OLS, rather than TLS, is the appropriate regression analysis in these cases.

4. In Excel®, visualize relationship between the coefficients of determination of each analysis performed in 3 above, with R^2 of the OLS x|epoch as the x-axis and the R^2 of the OLS y|epoch as the y-axis of a scatter plot.

5. The average relative motion in arc seconds per year along the x-axis and y-axis is the slope function of the regression equations derived from the OLS x|epoch and OLS y|Epoch unless there are obvious changes in velocity signaled by concave relative motion. For example, the equations for the optical pair 00546+3910STF 72 (rounded for simplicity) are:

$$X = 0.02225\text{Epoch} - 41.96$$
$$Y = -0.00677\text{Epoch} + 38.82$$

The motions are 0.0223 ± 0.001 arcsecond/year along the x-axis and -0.007 ± 0.001 arcseconds/year along the y-axis. Using the Pythagorean formula, calculate the average total relative motion. This yields total motion in arc seconds. For example, 00546+3910STF 72:

$$RM_{tot} = \sqrt{a^2 + b^2}$$
$$RM_{tot} = \sqrt{(0.02225)^2 + (-0.0067)^2} = 0.02324 \text{ a.s./yr}$$

6. Convert the total motion to average motion/year in milliseconds (mas/year) by multiplying by 1000 to make the value comparable to those in the WDS, which is in mas/year.

0.02324 a.s./yr * 1000 = 23.24 mas/year

7. To check this calculation against catalog proper

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motion values, calculate average relative motion using catalog motions in right ascension and declination of x- and y-values for the pair and use the distance formula to determine total relative motion.

From WDS, Primary: pm RA = -020 mas/year p, Dec = -020 mas/year

From WDS, Secondary: pm RA -001 mas/year p, Dec = 014 mas/year

$$\text{relative motion } = \sqrt{(-20-(-1))^2 + (-20-(-14))^2}$$
$$= 19.92 \text{ mas/yr}$$

In this particular case, relative motions as shown by analysis of theta and rho in Cartesian space agree fairly well with published catalog values. Large differences would indicate that either the relative motion values or the catalog values are not accurate (or both).

8. Compute the ratio of relative motion to the total motion of the primary. To compute total motion of the primate use the Pythagorean formula and the pmRA and pmDec of the primary For example, 00502+1150STF 63AB. pmRA = +43 mas/yr, pm Dec = -54 mas/yr (WDS):

$$PM_{tot} = \sqrt{a^2 + b^2}$$
$$PM_{tot} = \sqrt{(+43)^2 + (-54)^2} = 69.03 \text{ mas/yr}$$
$$\text{Ratio} = \text{RMtot / PMtot} = (1.5612 \text{ mas/yr}) / (69.03 \text{ mas/yr}) = 2.262$$

A system may exhibit a small relative motion simply because both components have small proper motions, so caution is in order when interpreting the result.

Results

Measures for 59 pairs are reported in Table 1, including measures for the calibration pair 02152+6740ENG 10. Table 2 shows the results of three rounds of OLS analysis (OLS y|x, OLS x|epoch and OLS y|epoch). These results are reported as a single value, the coefficient of determination (R^2) annotated by the probability that the slope of the regression model is statistically different from zero is indicated by a series of asterisks associated with the probability of rejecting a true null hypothesis that the slope is zero (* = 0.5; ** = 0.01, *** = 0.001 or less). Additionally the slopes of each model (XA, YA expressed in milliarcseconds/year of movement) and various calculations of relative motion (RM, the calculated relative motion; CatRM, relative motion from published catalog proper motions) and the ratio of relative motion to total motion of the primary (RM/PM) are presented. Figure 1 visualizes the relationship between R^2-values of OLS x|epoch and OLS y|epoch. Figures 2-6 are visualizations of relative motion geometries and further discussed below. Three pairs (02581+6912STF 317, 03085-0335BU 528AB and 03143+2257BU 530BC) were analyzed for relative motion (Table 2) but I was unable to obtain a satisfactory series of images to report a measure in Table 1 One pair reported in Table 1 (03085-0335Fox 9034CD) had insufficient measures (N = 3) to analyze and does not appear in Table 2.

Discussion

The methods discussed here are applicable to pairs with a long history of observation. The distance measure employed herein assumes that motion is linear or close to linear. A few of the pairs suspected of being long period binaries showed some indications of concave motion, but the second-order polynomial fit was insignificant and is not reported. (A second-order polynomial would not indicate Keplerian motion, but might indicate changes in velocity.) Since no pair analyzed here had a significant second-order polynomial fit, linear motion estimates seem reasonable. However, if a significant second-order polynomial fit is obtained in future analyses, the integral calculus should be employed to check for changes in velocity.

The results seem to discriminate three classes of double stars. The three classes are distinguished by a combination of characteristics.

1. Common proper motion pairs separate from the other two classes in (a) showing no significant changes in theta and rho over their histories of observation, grouping on the bottom left of the OLS analysis of the R^2-values derived from OLS x|epoch and OLS y|epoch analyses (blue dots, Fig. 1) and generally scoring low on the OLS y|x analysis (Table 2); (b) show no correlation between the epoch of observation and relative position in Cartesian space, (c) have low average relative motions on the order of three (3) mas/year (Table 2), and (4) have ratios of relative motion to primary total motion of <0.2 with an average of 0.058 ± 0.037 (Table 2). Relative motions of these pairs are similar to those of 00089+3713STF 1 shown in Figure 2.

2. Optical pairs, some of which are identified as

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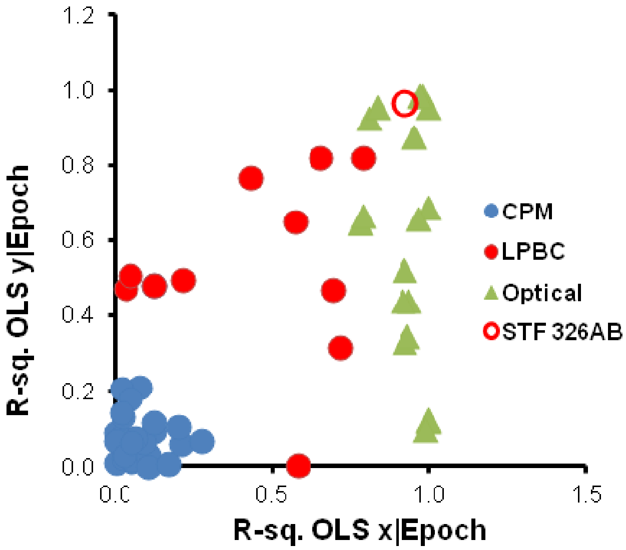

Figure 1. Scatter plot of coefficients of determination (R^2-values) derived from OLS x|Epoch (x-axis) against OLS y|Epoch (y-axis). Blue circles are pairs classified here as common proper motion pairs (CPM). Red circles are pairs classified as candidate long period binaries (LPBC), green triangles are pairs classified as optical pairs (Optical). STF 326AB, a LPBC, is discussed in the text.

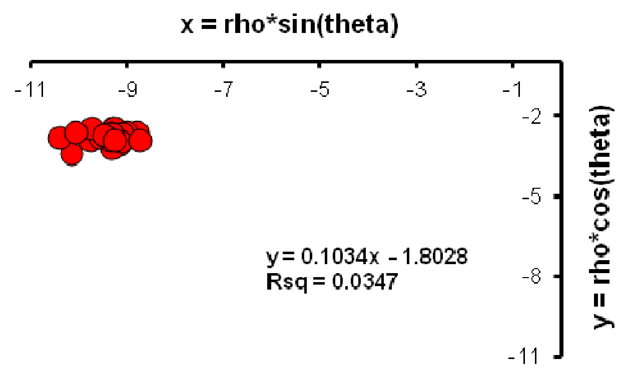

Figure 2. Left: Relative motion of 00089+3713STF 1, an example of a pair with a history of tightly clustered history of measures showing no significant relative motion between primary and secondary.

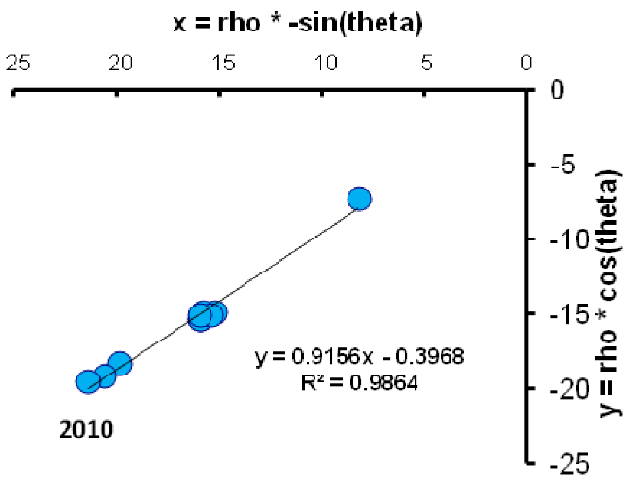

Figure 3. Left: Relative motion of 01035+5019 STF 83. An example of a pair identified as optical.

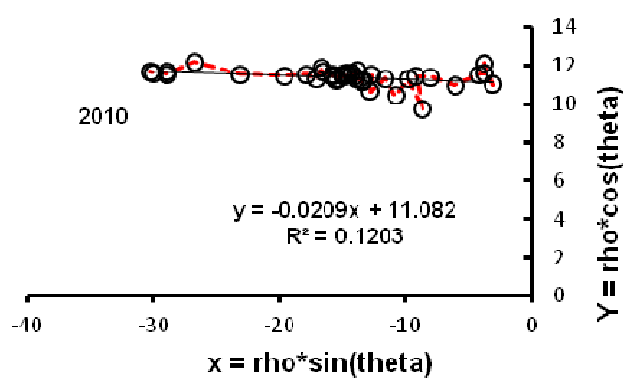

Figure 4. Relative motion of 00502+1150 STF 63AB; typical of an optical pair whose relative motion is along the x-axis, resulting in a low coefficient of determination (R^2).

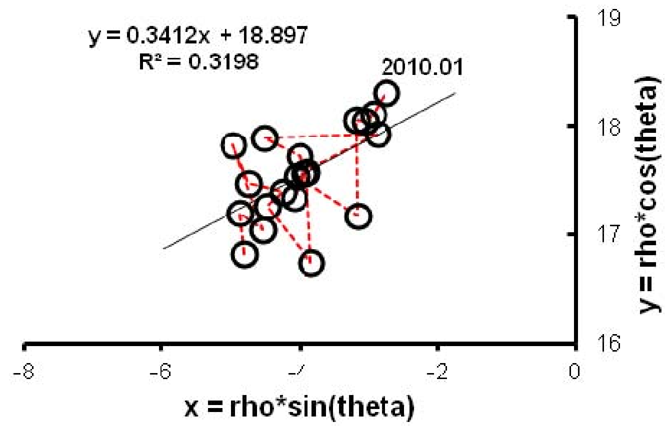

Figure 5. WDS 00444+3332STF 54. Although the observations form a time series correlated with epoch of observation and the relative motion high, as expected for an optical pair, variance of individual measures as visualized in this OLS y|x analysis result in a relatively low coefficient of determination. Dotted lines connect the observations by epoch of observation.

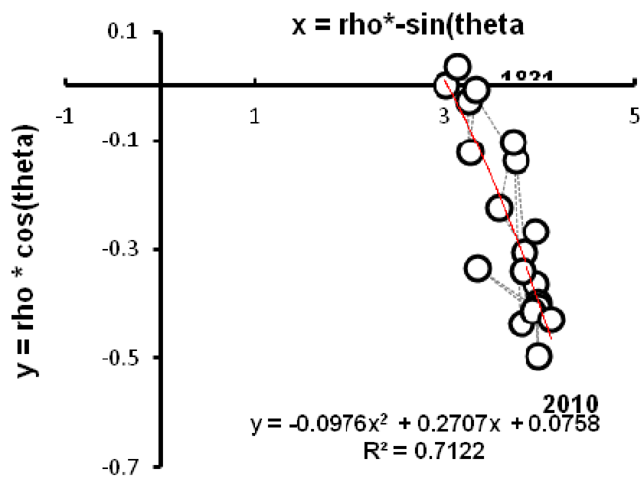

Figure 6. Relative motion of 02581+6912STF 317. Concave motion (albeit not statistically significant) suggests that this pair is binary. The relative motion of this pair is about 5 mas/yr, a value slightly higher than the average of common proper motion pairs. Both stars have the same parallax values. Dotted lines connect the observations by epoch of observation.

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rectilinear (Hartkopf et al., 2008 et seq.), have OLS y|x analysis that are tightly correlated with motion relative to the x and/or y axis, as evidenced by the scatter of green triangles on the right side of Figure 1. They show a correlation between epoch of observation and position in Cartesian space, forming a rectilinear time series. They always have at least one highly significant score in either the OLS y|epoch or x|epoch analysis (Table 2), but may (Fig. 3) or may not (Fig. 4) yield a significant model for OLS y|x. The reason for the difference is simple, if rectilinear movement is along the x-axis, then OLSy|x analysis cannot yield a significant model since x-values cannot predict y-values. Relative motions also serve to discriminate the optical pairs in this study; their relative motions average about78 mas/year with a range of 20.88 - 156.92 mas/year; a range that does not overlap the range of average relative motion of CPM pairs. I note that the scatter in Figure 1 optical pairs may also be caused to simple measurement variation; a pair with highly "noisy" measures will yield R^2-values that are lower than less noisy pairs with the same slope (see Figure 5 and note the slope is similar to Figure 3 but the coefficient of determination is low). Optical pairs, have ratios of relative motion to total motion > 0.4 and usually greater than 1.0 (average = 1.125± 0.539).

3. The third class is what may be long period binary candidate pairs. Their scatter plots and OLS analyses place them above the CPM pairs (relative motion follows the y-axis) or scattered among the optical pairs in Figure 1 where they are plotted as red dots. They are similar to optical pairs in forming a time series. However, their relative motion average motions are more similar to the common proper motions pairs. These long-period binary candidates have relative motions that average 6.16 mas/year, with a range of 1.83 - 8.42 mas/year (excluding STF 326AB, discussed below). The pair 02581+6912STF 317 (Figure 6) is an example; it has one of the higher relative motions (7.08 mas/year) and evidence of a slightly concave relative motion. Finally, they have ratios of total primary total proper motion to relative motion comparable (albeit a bit higher) to common proper motion pairs (average 0.083±0.051). Components of five of these pairs, including STF 317, have similar parallax values (Table 2).

The exception to the generalizations presented above is the pair 02556+2652STF 326AB. This pair is comprised of two very high proper motion stars with an average relative motion of 17.46 mas/yr (placing the system within the optical pairs) but a ratio of primary proper motion to relative motion of 0.075 (placing the system within the long-term binary candidate pool). This pair has both a rectilinear solution

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Table 1. Measures reported in this study. WDS and Disc. Code from the Washington Double Stat Catalog; Epoch; epoch of observation; RA, angle of theta in degrees; SEP, separation of the pair in arcseconds; PAsd, standard deviation of the angle measures; SEPsd, standard deviation of the separation measures; N, number of images from which measures were taken on a single night of observation: a single number indicates that all images taken were used, a fractional number denoted the number of "lucky" images used out of the total indicated by the denominator. In a few cases images were stacked: 1s/24 denoted a single stack of 24 images while 4s/10 indicates that four stacks were compiled from a total of 40 images, 10 images at a time. Notes, in footnotes.

WDS	Disc. Code	Epoch	PA	SEP	PAsd	SEPsd	N	Notes
00089+3713	STF 1	2009.84	287.4	9.73	0.18	0.026	3/5	1, 2
00099+0827	STF 4	2009.84	276.1	5.2	0.73	0.061	13/40	1, 2
00100+4623	STF 3	2009.84	80.44	5.07	0.84	0.082	5/22	1, 2
00148+6250	STF 10AB	2009.85	175.66	17.64	0.16	0.063	8/10	1, 2
00152+7801	STF 11	2010.00	191.84	7.97	0.32	0.094	8/27	1, 2
00174+1631	STF 20	2009.84	233.44	11.884	0.37	0.092	10	1, 2
00214+6700	STF 26AB-C	2010.00	114.4	13.329	0.67	0.119	4	1, 2
00316-0202	STF 35	2010.00	265.7	8.43	0.68	0.401	5	1, 2
00324+0657	STF 36AB	2010.00	82.19	27.133	0.63	0.148	5	1, 2
00324+0657	STF 36AC	2010.00	227.38	167.241	0.07	0.222	5	1, 2
00345-0433	STF 39AB-C	2010.00	44.4	16.63	0.91	0.106	5	1, 2
00345-0433	ALL 1AB-D	2010.00	158.85	202.963	0.04	0.167	5	1, 2
00399+2126	STF 46	2010.00	196.8	6.215	0.31	0.03	13	1, 2
00403+2403	STF 47AB	2010.00	205.45	16.41	0.65	0.186	5	1, 2
00403+2403	BU 1348AC	2010.00	233.06	46.414	0.25	0.124	5	1, 2
00426+7122	STF 48AB	2010.009	332.31	5.51	0.395	0.036	4s/10	1, 2
00426+7122	BAZ 1BC	2010.009	318.93	65.75	0.165	0.061	5s/10	1, 2
00444+3332	STF 54	2010.009	188.58	18.524	0.3	0.104	9	1, 2
00453+1019	STF 58	2010.009	169.47	46.458	0.2	0.115	10	1, 2
00474+7239	STF 57	2010.009	197.67	6.197	0.69	0.11	9	1, 2
00502+1150	STF 63AB	2010.00	248.88	32.417	0.84	0.668	5	1, 2
00502+1150	BU 1351BC	2010.00	318.05	117.898	0.21	0.439	5	1, 2
00503+3548	STF 62	2010.009	302.88	11.789	0.21	0.08	9	1, 2
00538+5242	STF 70AB	2010.009	246.43	7.999	0.2	0.051	20	1, 2
00538+5242	STF 70AC	2010.009	152.94	74.025	0.1	0.119	20	1, 2
00546+3910	STF 72	2010.001	173.23	23.39	0.33	0.173	5	1, 2
01001-0201	STF 81	2010.001	67.24	18.106	0	0	1	1, 2
01035+5019	STF 83	2010.001	312.38	28.949	0.61	0.252	5	1, 2
02157+6740	ENG 10	2010.001	328.27	23.228	0.57	0.154	10	1, 2

Table 1 concludes on next page.

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Table 1 (conclusion). Measures reported in this study. WDS and Disc. Code from the Washington Double Stat Catalog; Epoch; epoch of observation; RA, angle of theta in degrees; SEP, separation of the pair in arcseconds; PAsd, standard deviation of the angle measures; SEPsd, standard deviation of the separation measures; N, number of images from which measures were taken on a single night of observation: a single number indicates that all images taken were used, a fractional number denoted the number of "lucky" images used out of the total indicated by the denominator. In a few cases images were stacked: 1s/24 denoted a single stack of 24 images while 4s/10 indicates that four stacks were compiled from a total of 40 images, 10 images at a time. Notes, in footnotes.

WDS	Disc. Code	Epoch	PA	SEP	PAsd	SEPsd	N	Notes
02216+2338	STF 254	2010.001	14.73	11.904	0.57	0.225	9	1, 2
02521+3718	STF 316	2010.009	134.67	14.464	0.44	0.083	10	1, 2
02556+2652	STF 326	2010.009	221.35	4.798	0.59	0.058	4	1, 2
02558+3429	STF 325AB	2010.009	147.34	22.337	0.34	0.09	5	1, 2
03018+1051	STF 338	2010.009	201.3	19.848	0.37	0.11	10	1, 2
03030-0205	STF 341	2010.009	221	8.741	0.79	0.069	10	1, 2
03066+2038	STF 350	2010.009	118.76	16.502	0.31	0.087	10	1, 2
03067-1319	STF 356	2010.009	13.41	15.316	0.55	0.114	5	1, 2
03083-1236	STF 357	2010.009	294.98	8.679	0.88	0.156	10	1, 2
03085-0335	BU 528AC	2010.009	100.64	51.242	0.178	0.192	10	1, 2
03085-0335	FOX9023CD	2010.009	155.09	17.444	0.47	0.128	10	1, 2
03088-0341	STF 358	2010.009	349.28	15.252	0.39	0.183	10	1, 2
03108+6347	STF 349	2010.009	321.4	5.925	0.99	0.141	18	1, 2
03143+2257	STF 366AB	2010.009	34.33	41.807	0.1	0.039	10	1, 2
03203+1944	STF 376	2010.009	250.83	7.14	0.31	0.037	10	1, 2
03221+6244	STF 373AB	2010.009	118.07	20.323	0.21	0.08	10	1, 2
03221+6244	STTA 33AC	2010.001	111.73	115.547	0.08	0.104	10	1, 2
03221+6244	STU 1AD	2010.009	167.53	179.16	0.04	0.158	8	1, 2
03229+2949	STF 379	2010.009	101.25	10.468	0.75	0.108	10	1, 2
03242+1733	STF 383	2010.041	119.91	5.466	0.3	0.147	7	1, 2
03263-0102	STF 393	2010.041	257.29	15.521	0.7	0.163	10	1, 2
03305+2006	STF 399AB	2010.041	146	19.974	0.31	0.137	10	1, 2
03313+2734	STF 401	2010.041	269.33	11.506	0.62	0.079	18	1, 2
05100-0704	STF 651	2010.001	27.94	46.238	0.39	0.296	5	1, 2
22120+3739	STF2876	2009.85	66.85	11.874	0.36	0.034	5	1, 2
22542+2801	STF2952AB	2009.85	137.8	17.463	0.32	0.05	5	1, 2
22542+2801	STF2952AC	2009.85	244.48	165.28	0.04	0.083	5	1, 2
22567+7830	STF2971	2009.85	3.86	5.44	0	0	1s/24	1, 2
23092-0719	STF2980	2009.85	107.12	4.585	0	0	1s/10	1, 2
23519+3753	STF3042	2009.85	86.41	5.642	0.61	0.088	6	1, 2

Table 1 Notes

1. 0.3 M Dall-Kirkham Cassegrain, f11.5, SBIG ST8E NABG, with resolution of 0.68 arc seconds/pixel.
2. 02157+6740 ENG 10 and 23280+2335 STTA246AB used for calibration, measure of 02157+6740 ENG 10 is control using plate scale and orientation determined from 23280+2335 STTA246AB.

Table 2. Results of OLS and relative motion studies of 59 pairs of doubles stars. WDS, Washington Double Star catalog designation; Disc. Code, discovery code; DF, degrees of freedom of OLS analyses (N-1 observations); next three columns, Rsq, coefficient of determination of OLSy|x, x|epoch, and y|epoch analyses; Asterisk values denote rejection of the null hypothesis in each model that the slope is zero at 0.05 (*), 0.01 (**) and 0.001 (***) probability; Time, time period between first and last observation used in relative motion calculations; XA*1000, slope of the x|Epoch regression model expressed in miliarcseconds/year (mas/yr); YA*1000, slope of the y|Epoch regression model expressed in mas.yr; rel-mas/yr, average relative linear motion over the Time duration as calculated from relative motion of primary and secondary; cat-mas/yr, relative linear motion as calculated from proper motion values in various catalogs; Status, classification of pairs as common proper motion pairs (CMP), long-period binary candidates (LPBC) or optical

WDS	Disc. Code	DF	y|x Rsq	x|epoch R-sq	y|epoch R-sq	Time	XA*1000	YA*1000	RM	Cat RM	PM	RM/PM	Status	note
00089+3713	STF 1	27	0.035	0.002474	0.010	181.004	-0.3211	0.3713	0.49	4.24	19.72	0.025	CPM	
00148+6250	STF 10AB	34	0.144*	0.2115**	0.061	177.79	2.9718	-2.396	3.82	4.47	27.01	0.141	CPM	
00152+7801	STF 11	20	0.179*	0.06261	0.066	178.15	-0.576	1.1823	1.32	0.00	23.09	0.057	CPM	
00174+1631	STF 20	32	0.020	0.08112	0.2096**	184.104	-1.2223	-2.1522	2.48	1.41	43.01	0.058	CPM	
00214+6700	STF 26AB-C	32	0.007	0.02493	0.022	163.26	0.8886	-0.5545	1.05	0.00	25	0.042	CPM	
00316-0202	STF 35	27	0.000	0.002344	0.090	179.84	0.5586	1.945	2.02	0.00	28.18	0.072	CPM	
00324+0657	STF 36AB	62	0.043	0.01002	0.0613*	188.4	-0.9232	1.4854	1.75	6.08	32.35	0.054	CPM	
00345-0433	STF 39AB-C	50	0.035	0.1266**	0.09345*	179.76	-2.6623	-2.786	3.85	8.06	85.15	0.045	CPM	
00399+2126	STF 46	98	0.052*	0.1021**	0.032	178.79	-1.3031	0.9495	1.61	5.10	38.42	0.042	CPM	
00403+2403	STF 47AB	53	0.362***	0.02073	0.07942*	177.56	-0.8644	1.65	1.86	2.24	43.42	0.043	CPM	
00474+7239	STF 57	7	0.310	0.003132	0.06856**	278.18	-0.1993	1.477	1.49	NA	134.41	0.011	CPM	
00503+3548	STF 62	27	0.079	0.2002*	0.105	177.57	-2.563	-1.1876	2.82	3.60	32.02	0.088	CPM	
00538+5242	STF 70AB	25	0.007	0.0533	0.018	178.16	-1.0817	-1.091	1.54	4.24	81.61	0.019	CPM	
02521+3718	STF 316	33	0.113*	0.1081	0.029	179.99	2.082	0.7789	2.22	8.00	40.5	0.055	CPM	
03018+1051	STF 338	33	0.001	0.02004	0.2051**	180.08	-0.6417	-4.517	4.56	0.00	50.45	0.090	CPM	
03066+2038	STF 350	17	0.424**	0.04788	0.179	179.04	-0.804	-3.762	3.85	5.10	52.43	0.073	CPM	
03083-1236	STF 357	19	0.236*	0.2734*	0.067	176.96	-2.3136	-0.9583	2.50	2.24	81.69	0.031	CPM	
03088-0341	STF 358	12	0.717***	0.09373	0.064	177.04	-2.846	-6.969	7.53	11.40	50.33	0.150	CPM	
03143+2257	BU 530BC	24	0.131	0.0676	0.073	127.95	0.716	1.149	1.35	NA	28.79	0.047	CPM	
03203+1944	STF 376	50	0.012	0.1228*	0.1172*	227.03	-2.867	1.5159	3.24	7.21	21.4	0.152	CPM	
03221+6244	STF 373AB	15	0.003	0.106	0.000	134.34	2.074	-0.06942	2.08	17.09	45	0.046	CPM	1
03229+2949	STF 379	84	0.084*	0.028	0.030	179.17	0.4	1.6	1.65	4.01	40.31	0.041	CPM	
03242+1733	STF 383	28	0.431***	0.02579	0.130	179.49	-1.111	-2.028	2.31	6.08	58.82	0.039	CPM	
03313+2734	STF 401	94	0.011	0.02146	0.1413***	179.08	-0.8212	1.4884	1.70	7.00	50	0.034	CPM	
22120+3739	STF2876	27	0.079	0.05133	0.064	180.4	-1.0499	-1.0068	1.45	10.44	53.24	0.027	CPM	1
22567+7830	STF2971AB	52	0.261***	0.1268	0.4799**	176.96	-3.484	3.522	4.95	1.00	134.85	0.037	CPM	
23092-0719	STF2980	22	0.152	0.1689*	0.006	178.76	2.414	0.1984	2.42	18.29	65	0.037	CPM	1
00099+0827	STF 3	65	0.333***	0.2136***	0.4958***	187.59	1.6113	-1.6324	2.29	32.31	56.09	0.041	LPBC	1
00100+4623	STF 4	76	0.106**	0.03484	0.474***	179.14	-1.1819	-2.0475	2.36	1.41	14.04	0.168	LPBC	

Table 2 concludes on next page.

Measures and Relative Motions of Some Mostly F. G. W. Struve Doubles

Table 2 (conclusion). Results of OLS and relative motion studies of 59 pairs of doubles stars. WDS, Washington Double Star catalog designation; Disc. Code, discovery code; DF, degrees of freedom of OLS analyses (N-1 observations); next three columns, Rsq, coefficient of determination of OLSy|x, x|epoch, and y|epoch analyses; Asterisk values denote rejection of the null hypothesis in each model that the slope is zero at 0.05 (*), 0.01 (**) and 0.001 (***) probability; Time, time period between first and last observation used in relative motion calculations; XA*1000, slope of the x|Epoch regression model expressed in miliarcseconds/year (mas/yr); YA*1000, slope of the y|Epoch regression model expressed in mas.yr; rel-mas/yr, average relative linear motion over the Time duration as calculated from relative motion of primary and secondary; cat-mas/yr, relative linear motion as calculated from proper motion values in various catalogs; Status, classification of pairs as common proper motion pairs (CMP), long-period binary candidates (LPBC) or optical pairs (Optical)

WDS	Disc. Code	DF	y|x Rsq	x|epoch R-sq	y|epoch R-sq	Time	XA*1000	YA*1000	RM	Cat RM	PM	RM/PM	Status	note
01001-0201	STF 81	9	0.291	0.6927**	0.468*	118.12	3.3471	-2.99	4.49	6.40	89.04	0.050	LPBC	
02556+2652	STF 326AB	73	0.944***	0.9176***	0.9645***	178.55	13.62	-21.031	25.06	17.46	330.6	0.076	LPBC	2,3
02581+6912	STF 317	18	0.709***	0.6533**	0.8202***	169.11	4.6231	-2.6147	5.31	5.00	132.25	0.040	LPBC	2
03030-0205	STF 341	22	0.137	0.5711***	0.6514***	178.58	4.256	5.6188	7.05	17.26	111.2	0.063	LPBC	1
03067-1319	STF 356	18	0.056	0.04842***	0.508***	178.09	-0.6025	3.019	3.08	6.08	82.08	0.038	LPBC	
03085-0335	BU 528AB	36	0.884***	0.7881***	0.8204***	117.95	2.2586	-5.3782	5.83	NA	34.21	0.171	LPBC	
03263-0102	STF 393	14	0.140	0.582***	0.001	182.54	5.15	0.2468	5.16	3.16	33.01	0.156	LPBC	2
03305+2006	STF 399AB	14	0.672***	0.4316**	0.7677***	130.36	6.12	-5.408	8.17	4.12	173.35	0.047	LPBC	2
22542+2801	STF2952AB	11	0.140	0.1268	0.4799**	130.234	-3.484	3.522	4.95	2.00	75.29	0.066	LPBC	2
23519+3753	STF3042	165	0.313***	0.7148***	0.3135***	186.09	7.7456	-1.6534	7.92	NA	102.54	0.077	LPBC	
00324+0657	STF 36AC	12	0.010	0.9182***	0.010	116.16	-40.051	1.798	40.09	41.98	32.35	1.239	Optical	
00345-0433	ALL 1AB-D	3	0.347	0.9973***	0.332	77.99	-94.829	9.232	95.28	85.15	85.15	1.119	Optical	
00403+2403	BU 1348AC	17	0.036	0.9269**	0.121	177.59	-39.371	3.126	39.49	45.54	43.42	0.910	Optical	
00426+7122	STF 48AB	59	0.112**	0.05274	0.013	181.02	-62.58	0.428	62.58	40.80	43.9	1.426	Optical	1
00426+7122	BAZ 1BC	3	0.612	0.9212**	0.344	78.31	-114.9	-38.2	121.08	86.05	66.19	1.829	Optical	1
00444+3332	STF 54	18	0.32**	0.781***	0.523***	179.71	11.963	5.906	13.34	14.87	10.63	1.255	Optical	
00453+1019	STF 58	11	0.74***	0.8046***	.6472***	112.12	20.401	9.846	22.65	24.52	18.38	1.232	Optical	
00502+1150	BU 1351BC	8	0.901***	0.9953***	0.9247***	110.31	101.5	-16.671	102.86	58.60	130.18	0.790	Optical	1
00502+1150	STF 63AB	37	0.12*	0.9979***	0.1229*	177.6	-156.081	3.308	156.12	152.05	69.03	2.262	Optical	
00538+5242	STF 70AC	9	0.612**	0.9634***	0.6888**	106.06	-87.488	-16.819	89.09	95.13	81.61	1.092	Optical	
00546+3910	STF 72	24	0.695***	0.9503***	0.6565***	178.24	22.251	-6.7017	23.24	19.92	28.28	0.822	Optical	
01035+5019	STF 83	8	0.986***	0.9334***	0.8771***	182	-63.73	-56.949	85.47	70.76	116.4	0.734	Optical	1
02216+2338	STF 254	49	0.372***	0.9759***	0.4392**	178.26	46.492	4.636	46.72	50.24	23.54	1.985	Optical	
02558+3429	STF 325AB	45	0.987***	0.9973***	0.9862***	179.03	130.3	88.76	157.66	158.93	12.6	12.513	Optical	
03085-0335	BU 528AC	3	0.629	0.788*	0.9564**	113.94	5.128	23.908	24.45	22.20	34.21	0.715	Optical	
03143+2257	STF 366AB	23	0.712***	0.9852***	0.6653***	132.11	-69.362	13.661	70.69	72.27	60.67	1.165	Optical	
03221+6244	STTA 33AC	14	0.020	0.8319***	0.095	134.34	-17.305	-6.203	18.38	16.64	45	0.409	Optical	
03221+6244	STU 1AD	5	0.431***	0.9687***	0.9556***	109.1`1	-31.011	-44.512	54.25	46.23	45	1.206	Optical	1
05100-0704	STF 651	55	0.912***	0.9139***	0.9837***	180.33	56.706	-232.996	239.80	242.79	247.39	0.969	Optical	
22542+2801	STF2952AC	15	0.112	0.7074***	0.4384**	55.19	80.42	-24.661	84.12	103.81	75.29	1.117	Optical	

Table 2 Notes

1. Average relative motions differ by more than 9 mas/yr.
2. Both components have similar parallax measures. This is only reported for these pairs, other pairs may have similar or different parallax measures.
3. See text for discussion of this pair

Measures and Relative Motions of Some Mostly F. G. W. Struve Doubles

(Continued from page 85)

and a proposed orbital solution (note "C" in the WDS). In fact, all of the long-term binary candidates will have rectilinear solutions given that their relative motions are not significantly different from a rectilinear motion solution. Is STF 236AB optical or binary? Kharchenko and Roeser (2009: CDS catalog I/280B) list three trigometric parallax measures for this system, one composite (based on reported magnitude and position) with high error and proper motions that appears suspect (parallax -63.09 ± 49.9, pmRA 520.84 mas/yr, pmDec -48.79 mas/yr) and two matching the pair in proper motion and magnitude that yield trigometric parallax measures of 43.7 ± 1.25 mas (A component) and 43.9 ± 1.25 mas (B component), suggesting that this pair is physically associated and belongs to the LPBC category.

I have chosen to classify a number of pairs as long period binary candidates to bring them to the attention of the community. At least some of these pairs seem to be physically associated, as evidence by similar parallax measures. However, none of these data definitively corroborates these pairs as binary. It is quite possible that even those at similar distances are simply common proper motion pairs that are diverging or converging. They may even be optical pairs that just happen to be in the same area with similar proper motions. Only a longer history of observations will reveal their nature, but identifying them as possible binaries may encourage continued observation and measurement.

The contrast between relative motions determined from studying theta and rho and those taken from catalogs is instructive. For example, 00538+5242 STF 70AB has a simple change of sign in the WDS that makes relationship appear to be optical, but the actual data in the WDS individual record is correct. The catalog values for 004240.01+712157.3 STF48AB are at variance with those values obtained through relative motion and the same is true for a number of the pairs included in this study (10 in total, see Table 2).

Conclusions

Relative motion studies are well within the capabilities of amateur researchers who have access to a program that can calculate OLS models. Although I used R-language programming, the regression functions in data analysis package of Excel® return similar models and are probably more than adequate for this level of analysis. The nature of long-period binaries cannot be resolved until their motions are shown conform to Keplerian motion, but these methods may help resolve the nature of some double stars, given a long history of observation and sufficiently high proper motions, and may help mark pairs for additional measures in the future. They also seem useful for checking catalog proper motion values.

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APPENDIX A – Using Optical Pairs to Derive Orientation and Scale in REDUC

REDUC uses two values to reduce theta and rho for a double star, the orientation of the camera relative to the optical train and the scale of the image. These are determined by inputting into the program an image of known theta and rho, from which REDUC calculates the parameters. Amateurs using CCD measures may not have a convenient WDS calibration pair, but there are numerous "high-value" pairs in the Catalog of Rectilinear Elements that can serve this purpose and which are separated enough that relative long exposures can insure adequate determination of the centroids. I simply search for pairs with low errors for all elements and separations of at least 10 seconds of arc. The angle and separation for the pair on any given data can be determined by first computing the x,y positions in Cartesian space and then converting these to the angle and distance (with due regard to quadrant). It is a simple matter to check the result against the published ephemeris values to check against gross error. The formulae for x and y are given in the catalog:

$$x = x_a(t - t_0) + x_0$$
$$y = y_a(t - t_0) + y_0$$

Where t_0 is the time of closest approach, t is the date of observation, x_0 and y_0 are the positions at time t_0 in the Cartesian system, and x_a and y_a are the slope and the normal. For the pair 21144+2905 STF2779AB, the calculations proceed as shown for the observation date 2009.844.

x = 0.37006*(2009.844-2011.018) + 7.60913
= 3.8651

y = -0.022599*(2009.844-2011.018) + 12.459682
= 14.7451

tangent (first quadrant) = y/x = 3.8149

Angle (first quadrant) = 75.3112°

Theta 2009.844 = 165.31° (90°+75.31°)

Rho 2009.844 = 15.244 seconds of arc

The trick is determining the correct quadrant, but this can be easily determined from the Ephemeris of the WDS Rectilinear catalog and each angle will have to be determined separately. This compared very favorably with the WDS Ephemeris values for 2010 of 165.3° and 15.242 second of arc. As an additional check, one can calculate the ephemeris of a second pair, image and measure that pair using the Delta (orientation) and E-values (scale) derived from the first standard pair, as reported for 02157+6740ENG 10 in Table 1. I do this routinely as it is relatively simple to find another pair using the information in the catalog.