Determining the Nature of a Double Star: The Law of Conservation of Energy and the Orbital Velocity

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Abstract: I analyze several astromechanics criteria that determine the binarity of a pair of stars. These criteria determine if the observed velocity is within the allowed limits according to the orbital energy conservation equation. They have been used by the LIADA Double Star Section for many years. Finally, I propose to unify these methods and give some examples of applicability.

1. Introduction

The study of visual binary stars has long been recognized as a basic key to the understanding of stellar structure, formation and evolution. Close binary and multiple stars are prime targets for determining and calibrating basic stellar physics in general. The main importance in its study resides in stellar mass determination and in the mass-luminosity calibration. The stellar mass is determined from orbital information which can be obtained using a set of astrometric measures.

Wide binaries are also important for astrophysics. Because of their small binding energies, the interest in studies of wide binaries is because they are good sensors to detect unknown mass concentrations that they may encounter along their galactic trajectories. So, wide binary star systems have became objects of considerable theoretical and observational interest (e.g. Retterer & King (1982), Dommanget (1984), Lathan et al. (1984), Halbwachs (1988), Close & Richer (1990), Poveda & Allen (2004), Sesar, Ivezić & Jurić (2008)). They are relevant to the understanding of the processes of formation and dynamical evolution of the Galaxy. Thus, the present-day distribution of wide binaries can provide information about the disruption process as well as binary formation. Also, it is important for the determination of the characteristics and the frequency of binary stars in different stellar populations and evolutionary stages.

So, it is essential to use good criteria to differentiate between optical pairs and pairs of stars gravitationally bound. Several years ago, the LIADA Double Star Section used several criteria to determine the nature of the pairs (see Rica Romero 2006; Rica Romero 2007; Benavides et al. 2010). Since John Mitchell used, in 1767, probability theory to determine the physical relationship of identical pairs, many criteria have been used by astronomers to determine the nature of a pair. Aitken obtained a rela-

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tionship between the magnitudes of the stars and the maximum angular separation to be considered as a physical pair (more details were published in Rica Romero (2006)). Often astronomers used probability theory (a detailed historical overview was also published by Rica Romero (2007)) and other critera used the astromechanics. Some of them, used by LIADA, are Dommanget (1955, 1956), van de Kamp (1961) and Sinachopoulos & Mouzourakis (1992).

In this work, we comment, in detail, on the criteria based upon astromechanics, and we explain how we can use the orbital energy conservation equation (also referred to as "*Vis-Viva equation*") to determine the nature of a double star. We start from the total energy equation (kinematic energy plus gravitational potential energy) to derivate the "vis viva" equation, that defines the relative orbital velocity for each moment.

The organization of this paper is as follows: in section two, I start with the definition of the total mechanical energy and finish with the "vis viva" equation that defines the maximum orbital velocity. In section three, I make a historical overview for some works that define the same criteria to determine the nature of the double stars. And finally, in section four I present the same example to determine the nature of the double stars. Appendix 1 describes the process for calculating the relative motion and velocity of the double stars.

2. From the Total Energy to the Orbital Velocity

In the two-body problem, the total orbital energy is the sum of the gravitational potential energy and the kinetic energy for the primary and secondary star:

$$E = \frac{-GMm}{r} + \frac{M(v_M)^2}{2} + \frac{m(v_m)^2}{2} \qquad (1)$$

where G is the gravitational constant, r is the distance between the primary and secondary star, v_M (v_m) is the velocity of the primary (secondary) star relative to the center of mass of the two bodies, and M (m) is the mass of the primary (secondary) star.

The total orbital energy can also be calculated using only relative quantities

$$E = \frac{-GMm}{r} + \mu \frac{v^2}{2} \qquad (2)$$

where v is the relative velocity of the secondary with respect to the primary component and $\mu = Mm/(M+m)$ is the reduced mass.

For elliptical and circular orbits, the total energy is given more concisely by

$$E = \frac{-GMm}{2a} \qquad (3)$$

where a is the semi-major axis.

Dividing the total energy by the reduced mass gives the *vis-viva* energy, commonly known as the specific orbital energy.

$$\varepsilon = \frac{v^2}{2} - \frac{G(M+m)}{r} \qquad (4)$$

For elliptical and circular orbits

$$\varepsilon = \frac{-G(M+m)}{2a} \qquad (5)$$

Equating the two previous expressions and solving for v yields the *vis viva* equation:

$$v^2 = G(M+m)\left(\frac{2}{r} - \frac{1}{a}\right) \qquad (6)$$

Equation (6) allows us to determine the orbital velocity of a binary.

3. Historical Overview

In this section, I comment on the same works published earlier, which used the astromechanics to determine the nature of double stars.

3.1 The work of Sinachopoulos & Mouzourakis

Sinachopoulos & Mouzourakis (1992; hereafter SM92) knew about the important number of optical pairs in the visual double star catalogs and they were interested in determining a formula criteria that would permit a differentiation between physical and optical pairs. They compiled a sample of 123 wide visual double stars with angular separations of between 10 and 20 arcseconds and stellar components brighter than 10 magnitudes. SM92 first rejected those pairs with no agreement on their distances (considering them as optical pairs).

For the remaining double stars, they calculated the relative tangential velocity (i.e. the relative projected velocity) of the secondary component with re-

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spect to the primary (in km s^{-1}):

$$V_{\tan} = 0.0474 \, d \, \Delta u \qquad (7)$$

where d is the distance in parsecs and Δu is the relative proper motion of the secondary with respect to the primary (in arcsec yr^{-1}).

The relative radial velocity would allow us to calculate the 3-D relative velocity, but this kinematic data is only known for a very few double stars.

The orbital velocity for a circular orbit used by SM92 was:

$$V_{orb} = \sqrt{\frac{G\,\Sigma M}{r}} \qquad (8)$$

But the r value is unknown and then SM92 used the projected separation (s) using the angular separation (ρ) to determine the maximum orbital velocity (assuming a circular orbit):

$$V_{orb\_max} = 29.78 \sqrt{\frac{\Sigma M}{s}} \qquad (9)$$

where ΣM is the sum of the masses (in solar units) and s is the projected distance between the primary and the secondary components (in Astronomical Units (AU)).

Because $s \leq r,$ the last expression gives the maximum orbital velocity for a circular orbit. Their criterion for a pair to be optical was :

$$V_{tan} - 2\sigma > V_{orb\_max} \qquad (10)$$

where σ is the uncertainty in V_{tan} (due to the measured error on μ). Does this mean that if V$_{tan}$ − 2 σ < V$_{orb\_max}$, then the pair is a true binary? Not exactly. If V$_{tan}$ − 2 σ < V$_{orb\_max}$, the pair MAY BE a true binary. But we must take into account that SM92 did not use the radial velocity, and so, the observed 3D velocity, V, could be greater than V$_{orb\_max}$, and therefore this could be "no binary".

3.2 van de Kamp' test

Prior to SM92, van de Kamp (1961) gave a different view, starting with the "vis viva" equation (Eq. 6) and determined the critical value for a parabolic orbit. In a parabolic orbit the semi-major axis, a, is infinite and the "vis viva" equation can be written as :

$$v^2 = \frac{2G(M + m)}{r} \qquad (12)$$

In AU units for distances, the solar units for masses and the sideral year for time yields G = $4\pi^2$. The condition for a parabolic orbit is the critical value

$$v^2 r = 8\pi^2 (M + m) \qquad (13)$$

Here, as usual, we have the problem of the v and r calculations. If the distance, the relative proper motion, and relative radial velocity are known, then we can calculate a 3-D v value. But for most of the binaries, the radial velocity is unknown and we can only calculate a projection of v. In addition to this, r is always unknown although a projection distance can be determined. Therefore, at best, we can obtain a projected value for $v^2 r$. It will always be smaller than or equal to the true value. If $v^2r > 8\pi^2 (M + m)$ then the orbit must be hyperbolic. If $v^2r < 8\pi^2 (M + m)$ then the orbit MAY BE elliptical.

3.3 The Work of Close et al.

Close, Richer, and Crabtree (1990) determined the binarity of a sample of wide pairs by comparing the relative orbital velocity with the escape velocity for the system. The escape velocity is the minimum speed of an object without propulsion needs, in order to be able to "escape" from the gravity of the companion star. The escape velocity for most wide systems is considerably lower than 5 km s^{-1} so they rejected, as non binary, all of the systems with tangential velocities greater than 5 km s^{-1} + 5σ$_{Vtan}$.

To be considered as bound, the system must satisfy

$$\Delta v_{total} < v_{esc} \qquad (14)$$

where

$$\Delta v_{total} = \sqrt{\Delta v_r^2 + \Delta v_{\tan}^2} \qquad (15a)$$

$$v_{esc} = \sqrt{\frac{2GM_{total}}{r}} \qquad (15b)$$

and Δv_r is the relative radial velocity, M_{total} is the sum of the component masses, and r is the separation between the components.

To derive the formula for the escape velocity, we must start from conservation of energy equation:

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$$E_{total} = E_{kinetic} + E_{potential} \qquad (16)$$

where

$$E_{kinetic} = \frac{1}{2}mv^2 \qquad (17)$$

$$E_{potential} = -G\frac{Mm}{r} \qquad (18)$$

The escape velocity is the speed at which the kinetic energy plus the gravitational potential energy (i.e., the total mechanic energy) of an object is zero, so:

$$E_{total} = \frac{1}{2}\mu v^2 - G\frac{Mm}{r} = 0 \qquad (19)$$

and solving for v:

$$v_{esc} = \sqrt{\frac{2GM_{total}}{r}} \qquad (20)$$

As we commented in other sections, the value of r is unknown and we only can use a projected value, s. So the value obtained for v_{esc} is a maximum limit. That is, we can only calculate a maximum value of v_{esc}.

4. Discussion

The criteria of van de Kamp (1961), Laird et al. (1990), Sinachopoulos & Mouzourakis (1992) and that of Dommanget (1955, 1956) have been used by the LIADA Double Star Section for several years. But, in the last section of this article, we can see that all of them are based upon the Total Mechanic Energy. Therefore, we must unify these "methods":

- SM92, in order to determine the maximum orbital velocity, used the formula (8) which assumed a circular orbit. For circular and elliptical orbits, we must use equation (6).

- van de Kamp (1961) started with equation (6) to determine the parabolic criteria (assuming that the semi major axis = infinity). This is the same as the escape velocity.

Therefore, to determine the maximum orbital velocity and the escape velocity, we must only use equaitons (6) and (15b) and using s instead of r. As the true semi-major axis is unknown, we can determine the expected semi-major axis, E(a), as calculated

by Fischer & Marcy (1992):

$$E(a) = 1.26\,\rho \qquad (21)$$

To calculate the tangential and 3-D velocity of a double star, we must use equations (7) and (15a).

5. Some Practical Examples

In this section we are going to present some practical examples.

5.1 BU 787 AB, an Orbital Pair with an Optical Nature

BU 787 AB (= WDS 03342+4837) is a double star composed of stars of 7.35 (B9V spectral type) and 11.0 magnitudes, separated by 5.3" in 2007-2008. See Figure 1. The Hipparcos satellite (ESA 1997) measured the distance of the primary member ($261^{+60}/_{-40}$ pc). This pair was discovered by Burnham in 1881 (Burnham 1882) and it has 19 measures. Erceg (1984) published its orbital parameters. Surprisingly, a preliminary study made me suspicious of the optical nature of this pair of stars. A detailed study was published by Rica (2011a).

The B component has a large relative motion (see Appendix) with respect to the main component (36.0 ± 2.0 mas yr^{-1}). If I assume that the secondary is located at the same distance as the primary component, then this motion corresponds to a relative velocity of $44.6 \pm$
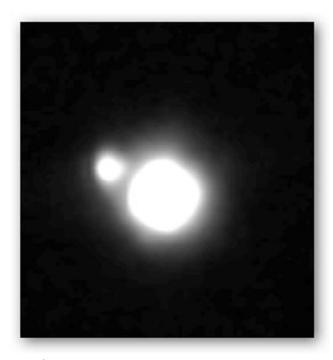

Figure 1: Double star BU 787 = WDS 03342+4837.

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2.5 km s^{-1}. From the stellar masses for the components (3.8 and 1.0 M$_\odot$), the projected separation (957 AU) and the expected semi-major axis (1,206 AU), I calculated a maximum orbital velocity of 2.34 km s^{-1} using equation (6). Using equation (15b), I calculated an escape velocity of 2.98 km s^{-1}. It is very evident that the observed velocity is much greater than the allowed velocity in order to consider this pair as a gravitationally bound binary (even if I take into account the errors in the observed velocity). In addition to this, the large relative motion allows me to reject the possibility of a common origin binary with components not gravitationally bound. Therefore, BU 787 AB is an optical pair.

Surely this is not the first case, and not will be the last one, of an optical double star with orbital parameters. Therefore, my advice to orbit calculators is to confirm the nature of the pairs (especially if the relative motion is rectilinear) before calculating their orbital parameters.

5.2. The case of FMR 19

In number 2 of volume 7 of this journal (Rica 2011b), I studied a new pair of stars called FMR 19 (= WDS 20502-0640) located in the constellation Aquarius and composed of two M1.5V and M2.5V red dwarfs of V magnitudes 17.02 and 18.24. See figure 2. They are separated by 4.2" in the direction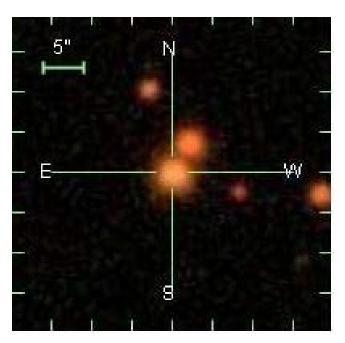 of 333 degrees. Both components have a high and common proper motion. The primary component has a proper motion μ(α) =   -120 mas yr^{-1} and μ(δ) = -119 mas yr^{-1} from UCAC3. For the secondary component μ(α) = -136 ± 6 mas yr^{-1} and μ(δ) = -129 ± 6 mas yr^{-1}. They are located at a photometric distance of 217 ± 14 and 234 ± 25 pc. So, both proper motions and distances are very similar, suggesting binarity.

The estimated relative motion (Δx = -16.0 ± 0.8 mas yr^{-1} and Δy = -10.0 ± 1.8 mas yr^{-1}) corresponds to a relative velocity of 20.2 ± 2.1 km s^{-1}. From the stellar masses for the components (0.37 and 0.29 M$_\odot$), the projected separation (930 AU) and the expected semi-major axis (1,172 AU), I calculated a maximum orbital velocity of 0.87 km s^{-1} using equation (6). Using equation (15b), I calculated an escape velocity of 1.12 km s^{-1}. As in the last example, this pair clearly is not a gravitationally bound binary.

I analyzed the probability that FMR 19 is a gravitationally bound system. I used a Monte Carlo simulation with 25,000 iterations. Monte Carlo methods are a class of computational procedures that rely on repeated random sampling to compute their results. The inputs used in the simulation were: the relative proper motion (in mas yr^{-1}), the distance (in pc), the stellar masses (in solar units), the angular separation, and the expected semi-major axis (in arcsec), E (a). We assumed that the errors for the inputs had Gaussian distributions. For E(a) we used the relation between E(a) and a of 1.16 ± 0.39. The relation E(a)/a was obtained by the author by using several hundred orbital parameters and their ephemerides from the Sixth Catalog of Orbits and Ephemerides of Visual Double Stars.

The distribution of E(a)/a is not exactly a normal distribution, but the effect in the vis viva equation was small. The conclusions obtained in this analysis were: a mean distance for the stellar system of 222 ± 12 pc, with a probability of 83.5 % for having a common distance. The simulation also determined that FMR 19 has no possibility (probability 0%) of being a gravitationally bound binary. However, the high probabilities of a common distance and common proper motions are evidence that allow us to conclude that this pair is likely of common origin and binary in nature although their stellar components don't orbit around the center of mass.

Figure 2: The binary FMR 19 = WDS 20502-0640

Appendix: How to Calculate the Relative Motion and Velocity.

To determine the nature of a double star, we must

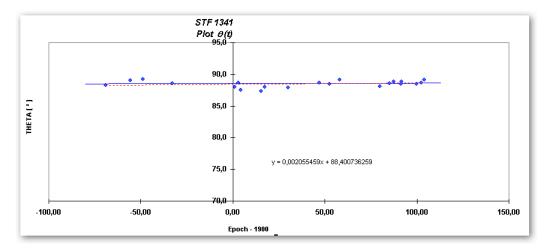**Determining the Nature of a Double Star: The Law of Conservation of Energy ...**

Figure 3: Plot of position angle as function of time for STF 1341.

calculate the relative velocity of B with respect to A. In order to do this, we must perform the following steps:

1.- Make a request to the USNO for a historical report of all WDS astrometries and complete it with our own measures.

2.- Theta values must be corrected for precession effects.

3.- Assign weights to all astrometric data. The weights are assigned from observational techniques, aperture of the telescope, observer, etc. More information can be found in Rica Romero (2010).

4.- Create plots of theta vs time (Figure 3) and rho vs time, fitting a linear or a parabolic function and viewing the mathematical expression of the fitted curve

5.- Calculate the relative motion of B with respect to A using the following formula:

$$\Delta\mu = \sqrt{\left(\rho\frac{d\theta}{dt}\right)^2 + \left(\frac{d\rho}{dt}\right)^2}$$

where ρ and θ are the angular separation and position angle. If the fits are linear, then the derivative of ρ and θ is the slope.

References

Benavides R., Rica F., Reina E., Castellano J., Naves R., Lahuerta L., Lahuerta S., 2010, JDSO, **6**, 30.

Burnham S. W., 1882, Publ. Washburn Obs. **1**.

Close L. M., Richer H. B., 1990, AJ, **100**, 1968.

Close L. M., Richer H. B., Crabtree De. R., 1990, AJ, 100, 1968

Dommanget J., 1955, BAORB, **20**, 1.

Dommanget J., 1956, BAORB, **20**, 183.

Dommanget J., 1984, Ap&SS, **99**, 23.

Erceg, V., 1984, Bull. Obs. Astron. Belgrade #134, **54**.

ESA SP-1200, ESA Noordwijk, 1997, catalogs HIP-PARCOS and TYCHO.

Fischer D. A., Marcy G. W., 1992, AJ, **396**, 178.

Halbwachs J. L., 1988, Ap & SS, **142**, 237.

Lathan D. W., Schechter P, Tonry J., Bahcall J. N., Soniera R. M., 1984, ApJ, **281**, 41.

Poveda A., Allen C., 2004, RMxAC, **21**, 49.

Retterer J. M., King I. R., 1982, ApJ, **254**, 214.

Rica Romero, F. M., 2006, JDSO, 2, 36.

Rica Romero F. M., 2007, JDSO, 3, 21

Rica Romero F. M., 2010, RevMexAA, 45, 263

Rica Romero F. M., 2011a, JDSO, 7, 137

Rica Romero F. M., 2011b, JDSO, 7, 112

Sesar B., Ivezić Ž., Jurić M., 2008, ApJ, **689**, 1244.

Sinachopoulos D., Mouzourakis P., 1992, Complementary Approaches to Double and Multiple Star Research in the IAU Colloquium 135, ASP Conferences Series, Vol. 32.

van de Kamp P., 1961, PASP, **73**, 389.