Relative Motion of the WDS 05110+3203 STF 648 System, With a Protocol for Calculating Relative Motion

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Abstract: Relative motion studies of visual double stars can be investigated using least squares regression techniques and readily accessible programs such as Microsoft Excel® and a calculator. Optical pairs differ from physical pairs under most geometries in both their simple scatter plots and their regression models. A step-by-step protocol for estimating the rectilinear elements of an optical pair is presented. The characteristics of physical pairs using these techniques are discussed.

Introduction

I was inspired to look into studies of relative motion by papers published in the JDSO by Schlimmer (2009a, b) who used least squares analysis to determine the rectilinear elements of several optical pairs. A similar approach was used by Rico Romero (2000). This lead to an incomplete literature review (e.g., Torres, 1985; Clark et al., 2005; Kiyaeva et al., 2008) in the primary literature and a discussion of the uses of regression analysis in astronomy by Isobe et al. (1990) and Feigelson and Babu (1992). I concluded that studies of relative motion were within the skills of amateurs wishing to expand their research program, but that the protocols available in the literature are a bit obscure. This paper is an attempt to outline protocols that can be used by amateurs with modest math skills (such as myself) to study relative motion of double stars with relatively high proper motions (at least one of the pair with a total pm 50 mas/yr).

The system studied, WDS 05110+3203STF 648, consists of four stars. STF 648AB has been recognized as either a common proper motion pair or a binary at least since Burnham (1906) and Lewis (1906). The pair has a similar proper motion (UCAC2.0), parallax and velocity (Kharchenko et al., 2007). There are 62 measures in the WDS and the pair is moving toward the west (ca. 260º) at about -139 mas/yr. The relationships between the AC and AD components are almost certainly optical as evidenced by very different proper motions. The system offers the opportunity to explore the use of least squares techniques and some simple algebra and analytic geometry to study relative motion of both a physically associated and two optical pairs of the same system while making it possible to duplicate the calculations with a hand calculator since neither AC nor AD have many observational measures.

In a nutshell, linear regression is a least-square technique that fits a straight line describing the linear relationships of two variables, usually labeled x and y. These variables are functions of the angle and separation between the primary and secondary that have been converted from polar to Cartesian coordinates. In most applications, x is the independent variable (and without error) while y is the dependent or response variable. Informally, how well can you predict y if you know x by fitting a straight line to the measures? In terms of Isobe et al. (1990), the models employed here are ordinary least squares (OLS) either regressing Y on X (OLS Y|X) or each on Epoch (e.g., OLS Y|Epoch), with a check of the residuals regressed

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against Epoch. Using the regression functions in Excel, I was able to duplicate the results of Schlimmer (2009a), which piqued my interest in seeing if I could estimate the rectilinear elements that appear in the Catalog of Rectilinear Elements (Hartkopf et al, 2008 et seq., hereafter referred to as CRE).

Below I present a protocol for estimating the rectilinear elements of an optical pair, using STF 648AC as an example. I have used what I think is a relatively intuitive approach, not the most efficient one. However, and in the spirit of Tanguay (1998), learning to work these things out manually helps one to understand exactly what is happening in the analysis. After presenting the protocol, I compare the results of a comparative study of calculations of known rectilinear pairs using this protocol with the values that appear in the CRE. I will then discuss the results of the analysis of STF 648AB and suggest that least squares techniques, used with caution, may be one approach to studying physical pairs.

Relative Rectilinear Motion of the Optical Pairs 05110+3203STF 648AC and AD

My measures of both STF 648AC and STF 648AD are shown in Table 1. The goal is to calculate the parameters of closest approach including the angle (THETA0), separation (RHO0) and time of approach (T0). The protocol consists of several steps illustrated using STF 648AC. Results of the relative motion study for both pairs are shown in Tables 2 and 3.

1. Gather all WDS catalog measures of the pair through request from the USNO.

Observation requests forms are on the WDS Web pages (see Mason, 2006). All measures for STF648AC are shown in Table 2.

2. Correct angle for changes due to precession.

Theta can change as a function of precession. Formulae for correcting theta are provided by Aitken (1935: 73) and Greaney (2004) and can be easily programmed in Excel. I have used both an Excel sheet kindly provided by Francisco Rica Romero of LIADA. Below, I work an example the calculation based on the original observation of STF 648AC (Epoch 1906.73). See Table 2 for all corrected measures.

$$\Delta \theta_p = -0.0056^o \sin\alpha \sec\delta \left(t - t_o \right)$$

Table 1: New measures of 05110+3203STF 686AC and 05110+3203STF 686AD.

WDS	Disc. Code	Epoch	PA	SEP	PAsd	SEPsd	N	Notes
05110+3203	STF 648AC	2009.152	113.2	48.1	0.26	0.229	4	1, 2
05110+3203	STF 648AD	2009.152	73	45.9	0.03	0.063	4	1, 2

[1] 0.25m cassegrain, 1.65 arcsec/pixel resolution
[2] N = number of images, single night

Table 2: All measurements of 05110+3203STF 686AC, 1906-2009.152. Coordinates (x, y) are corrected for precession and referred to the pole of 2000.00.

Epoch	PA	RA corrected	Sep	x	y
1906.73	115.7	116.3	36.65	32.856	16.24
1927.2	114.3	114.77	40.25	36.547	16.864
1999.9	113.3	113.3	47.69	43.8	18.864
2009.152	113.14	113.14	48.1	44.23	19.903

Where $\Delta\theta p$ is the angle correction for precession, α is the right ascension of the primary in decimal degrees (= 77.746625º), -0.0056º is the constant for precession, t is the common epoch to which all measures will be referred and to is the epoch of observation.

$$\Delta\theta_p = -0.0056^o \sin(77.746625^o)\left[\sec(32.03875^o) \right](2000.00 - 1906.73)$$

$$\Delta\theta_p = 0.60212^o$$

$$\Delta\theta(corrected) = 115.70^o + .0.60212^o = 116.30^o$$

If one also corrects for the proper motion of the primary, then the resulting calculations will not measure the relative motion of the pair but the motion of the secondary relative to a stationary primary. That calculation would assume that the proper motion of the primary is correct, which does not allow for checking errors in that proper motion (Dr. William I. Hartkopf, personal communication).

3. Place theta and rho in the Cartesian coordinate system and inspect a scatter plot of the converted measures.

Use the formulae:

$$x = -\rho \sin(\theta)$$
$$y = \rho \sin(\theta)$$

These values are also found in Table 2. Several Internet utilities exist that can make the conversion or they can be easily calculated. Conversion from polar to

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Table 3: Rectilinear elements and informal errors for 05110+3203, STF 686AC.

WDS	Disc. Code	X_0	X_A	Y_0	Y_A	θ_1	ρ_0	T_0
05110+3203	STF 648AC	−1.8678	0.10796	7.6435	0.02661	193.7	7.87	1584.1
	Error	0.77	0.008	0.102	0.001	4.6	0.55	1.9
05110+3203	STF 648AD	5.98906	0.11482	−23.161	0.0288	14.5	23.92	1666
	Error	0.49594	0.00654	0.07365	0.00349	1.2	0.058	3.81

Cartesian coordinates allows use of least squares modeling and simple analytic geometry to study relative motion. Signs should be checked to insure that the converted coordinates are in the correct quadrant. The units of the axes are, conveniently, arcseconds. A simple inspection of a scatter plot of an optical pair such as STF 648AC reveals a linear series of plots with observations ordered by Epoch (Figure 1). This is typical of a pair where there is no significant measurement error. The greater the number of measures with error, the greater the scatter and in some cases, measures will have to be eliminated in order to estimate the rectilinear elements (for example, see Schlimmer, 2009a).
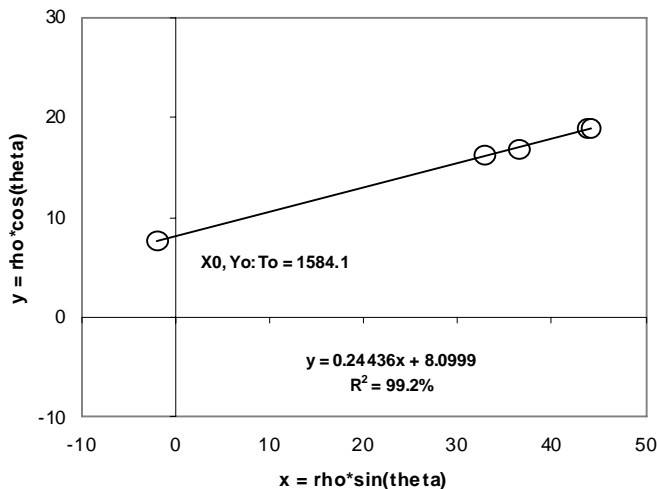
Figure 1: Relative motion of 05110+3203STF 648AC. X0 and Y0 were not included in the analysis and are included to show relative position of the primary and secondary at estimated closest approach. Rho0 is the distance from X0,Y0 to the origin. North is down, east is right.

4. Model the relationship between x and y using linear regression and save the residuals.

This takes the form of the familiar regression model:

$$y = mx + b$$

where y is the dependent variable, b is the intercept of the line through the y-axis, m is the slope of the line,

and x is any value of the x variable. Excel has an easy to use regression analysis routine, but it does not calculate the regression model to the satisfaction of statisticians. For results that will be as close as possible to the CRE either a statistical package should be employed or the exact functions used in Excel. I used Minitab (V. 14; see Meyer and Krueger, 2004). Values of x are treated as the "independent" variable and those of y as the dependent variable. Regression results in the OLS (Y|X) regression were:

$$y = 0.24436x + 8.0999$$

The error of the slope is ±0.01532. The model is highly significant (p = 0.006) with a coefficient of determination (R^2) of 99.2%. A high value for the coefficient of determination is an indication that there is little scatter in the measures relative to the slope of the model. This would be typical of a pair where measurement error is low.

5. Inspect the behavior of residuals by regressing the residual measures against epoch of observation.

Even if you do not prune what seem to be erroneous measures, a plot of residuals against epoch of observation should show a random scatter of residuals
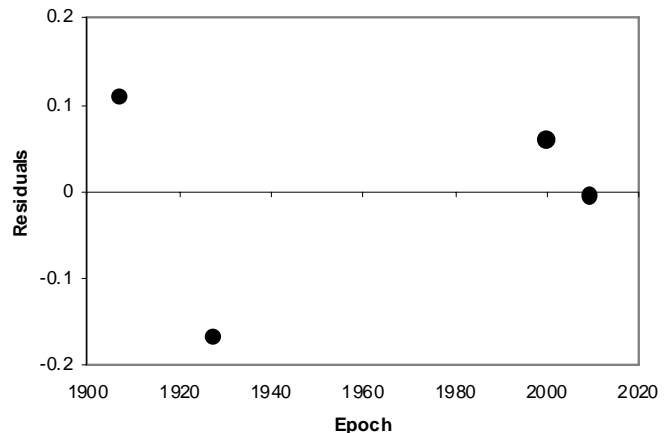
Figure 2: Regression of residuals derived from y on x regression of 05110+3203STF 648AC coordinates suggests no systematic deviation over time.

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with an insignificant regression model (Figure 2).

6. Determine the (x,y) coordinates of the closest approach.

This is accomplished in two steps. First, determine a formula for a regression equation that is perpendicular to the calculated equation and that goes through the origin (the position of the primary). The slope is the "normal" of the calculated slope and is defined as $(-1/m)$. Further calculations take advantage of the alternate form of the regression equation where (X_0, Y_0) are the coordinates at closest approach and (x_2, y_2) are the coordinates of the primary:

$$Y_0 - y_2 = (-1/m)(X_0 - x_2)$$

Since (x_2, y_2) is the position of the primary at the origin (0,0), then the formula simplifies to:

$$Y_0 = (-1/m)X_0$$

Simultaneously solve for X0:

$$mX_0 + b = (-1/m)X_0$$

Rearranged:

$$X_0 = -b/(m + 1/m)$$

$$X_0 = -8.0999/\left[0.24436 + (1/0.24436)\right]$$

$$X_0 = -1.86776$$

Substitute the value of X0 in the regression equation to solve for Y0:

$$Y_0 = 0.24436X_0 + 8.0999$$

$$Y_0 = (0.24436)(-1.86776) + 8.0999 = 7.64349$$

7. Determine ρ_0, the distance at closest approach.

To do this, use the distance formula:

$$d = \sqrt{(X_0 - x_2)^2 + (Y_0 - y_2)^2}$$

Since (x_2, y_2) is at the origin (0,0), the formula simplifies to

$$\rho_0 = \sqrt{X_0^2 + Y_0^2}$$

$$\rho_0 = \sqrt{(-1.86776)^2 + (8.0999)^2}$$

$$\rho_0 = 7.87 \text{ arcsec}$$

8. Calculate θ_0, the angle (PA) at closest approach.

This calculation requires some attention to the quadrant. An initial angle can be easily calculated by taking the tangent of a right triangle with opposite and adjacent sides corresponding to X_0 and Y_0:

$$\tan(\theta) = 1.86776/7.64349 = 0.23967$$
$$\theta = 13.7º$$

The result assumes that the angle is in the first quadrant (+,+) measured from zero . However, (X_0, Y_0) is in the third quadrant. If we plot (X_0, Y_0) we see that it is about 14º west of south, so this angle is added to 180º.

$$\theta_0 = 13.7º + 180º = 193.7º$$

9. Calculate T_0, the epoch of closest approach.

This calculation takes advantage of the formulas used in the CRE to calculate the ephemeris:

$$x = X_A(t - T_0) + X_0$$
$$y = Y_A(t - T_0) + Y_0$$

Calculation requires slopes X_A and Y_A, which are the slopes (m) of x versus Epoch and y versus Epoch respectively. For STF648AC values of $X_A = 0.107959$ and $Y_A = 0.026608$ were obtained. It is then a simple matter to substitute numbers for terms: pick an Epoch with (x,y) coordinates and solve the equation. The results are a function of the residuals, so I picked a recent measure with a small residual: 2009.152, with $(x,y) = (44.230, 18.903)$:

$$y = Y_A(t - T_0) + Y_0$$

Rearranged:

$$T_0 = (y - Y_A t - Y_0)/Y_A$$

$$T_0 = \left[18.903 - (0.026608 \bullet 2009.152) - 7.64349\right]/0.026608$$

$$T_0 = 1585.99$$

The other calculation, with X_A and X_0, resulted in $T_0 = 1582.16$; the difference of which I take as a measure of the range of the estimate and the average as the estimate of T_0. An alternative method, perhaps superior, would be to calculate a T_0-value for each observation and then take the average of all estimates.

10. Errors

Errors are a complex matter because the parameters estimated may fall outside the data. For example, calculating an error for time of closest approach is not intuitively obvious given that we are asked to calculate an error for the Epoch 1586 when our oldest measure is from Epoch 1907. The only errors easily derived from the analysis are the errors of X_A and Y_A, which are simply the errors of their respective slopes. Estimates of errors of X_0 and Y_0 are not intuitive. I have

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Table 4: A comparison of rectilinear elements for three optical pairs using methods in the paper (EOW) and values from the Catalog of Rectilinear Elements (CRE). "D" is the absolute difference between the measures; other abbreviations as in the text. Numbers are rounded, R^2 varies between 97.5-99.9%. Note: the 1910.95 measure of 06553+1235 measure not included in EOW calculations due to high residuals.

	06553+1235 J273AC			01190-0856 ENG5			08453+4140 STF1263AB		
Element	EOW	CRE	D	EOW	CRE	D	EOW	CRE	D
X0	1.793	1.832	0.039	77.67	77.15	0.52	−1.993	−1.983	0.01
XA	0.0505	0.0515	0.001	0.229	0.232	0.003	0.2695	0.2751	0.006
Y0	2.14	2.26	0.12	39.393	39.447	0.051	−0.8556	−0.842	0.014
YA	−0.0422	−0.0417	0.001	−0.452	−0.453	0.001	−0.6272	−0.644	0.017
T0	2082.5	2088.7	6.2	1636.6	1637.3	0.7	1823.8	1822	1.8
Rho0	2.792	2.909	0.12	87.089	86.654	0.435	2.168	2.156	0.012
Theta0	140.1	140	0.1	117	117	0	293.7	293	0.7

chosen one approach, but I do not claim that the results are more than an informal estimate.

In Minitab one can calculate a predicted value of the dependent variable given the independent variable. The only true independent variable in the analysis is Epoch, so I performed OLS on X|Epoch and Y|Epoch and asked the program to predict X and Y respectively at time T_0. I took the difference between the predicted value derived from the respective analysis and the calculated value as a measure of error. It is then an easy matter to add (or subtract) the error to the calculated X_0 and Y_0-values, recompute ρ_0 and θ_0, and subtract the results from the original measures to obtain errors.

So, for example, the upper bound of ρ_0 was recalculated by adding the errors to the original X_0 and Y_0 and then using the new values to calculate a perturbated ρ_0. The difference between the original ρ_0 and the new value was taken as an error term. The error of T_0 is simply half of the difference between the estimates given above. The alternative would be to take the standard deviation of all estimates if T_0 is averaged over all observations.

Comparing the Protocol with Known Optical Pairs

I calculated elements for three pairs that appear in the CRE in order to access the validity of the protocol presented here. While a tiny sample, the results shown in Table 4 indicate that this method finds similar results using techniques available to amateurs as does the professional algorithm used by Dr. Hartkopf and colleagues at the USNO. They are not likely to be identical because elements in the CRE are based on a weighted analysis that takes into account the many factors considered in weighting orbital elements.

05110+3203STF 648AB, a Physical Pair

Intuitively, the relative motions of CPMs and binaries should yield different results from optical pairs since their motions are not rectilinear. Regression analysis of Cartesian coordinates (OLS $y|x$) of the 62 measures of STF 648AB resulted in a significant regression model (p <.05) but with little explanatory power (R^2 = 20.1%). The positions of the secondary relative to the primary are tightly clustered but show some indication of motion (Figure 3). The combination of tight clustering and low explanatory power suggests that this pair has a common origin and is either a CPM pair or perhaps a binary with a very long period. For this pair, the result is not surprising given the other physical data mentioned above.

Residuals derived from OLS $y|x$ of a CPM pair should show random residuals; changes of theta and rho should vary over time only as functions of measurement error. Residuals derived from OLS $y|x$ of a bi-
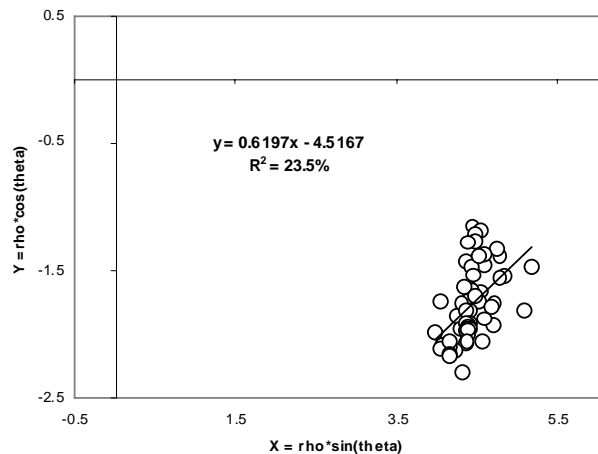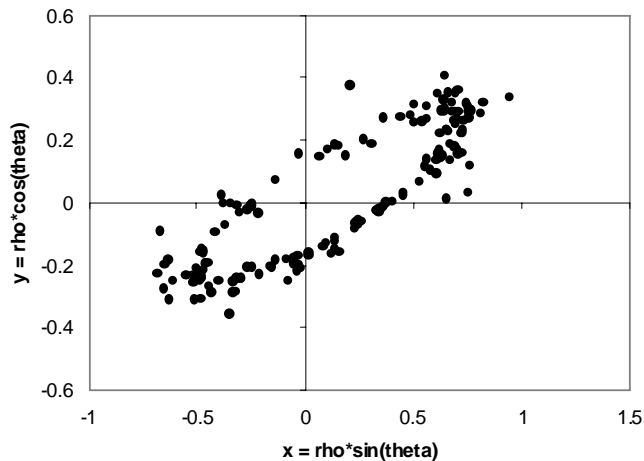

Figure 3. Relative motion of 05110+3203STF 648AB.

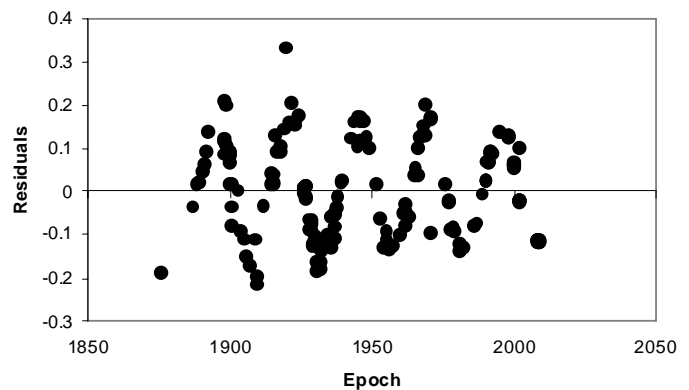**Relative Motion of the WDS 05110+3203 STF 648 System, ...**

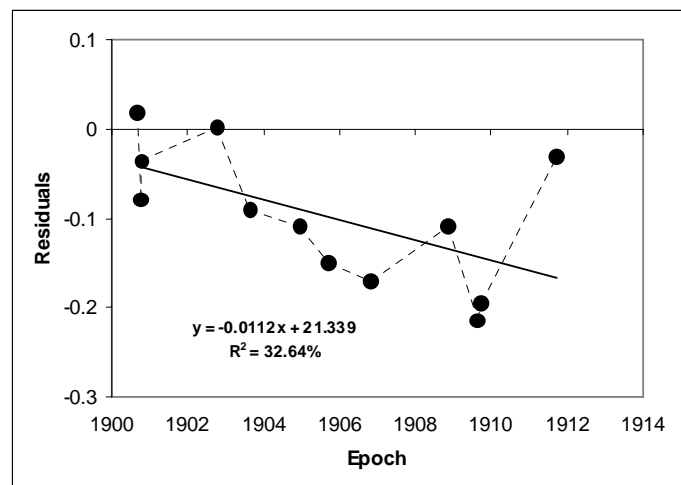Figure 4. Scatter plot of relative motion of the binary 00373 –2446BU 395.

Figure 5: Regression of residuals versus Epoch of 00373–2446BU 395 shows systematic behavior and suggests orbital period of ca. 25 years over the history of observations.
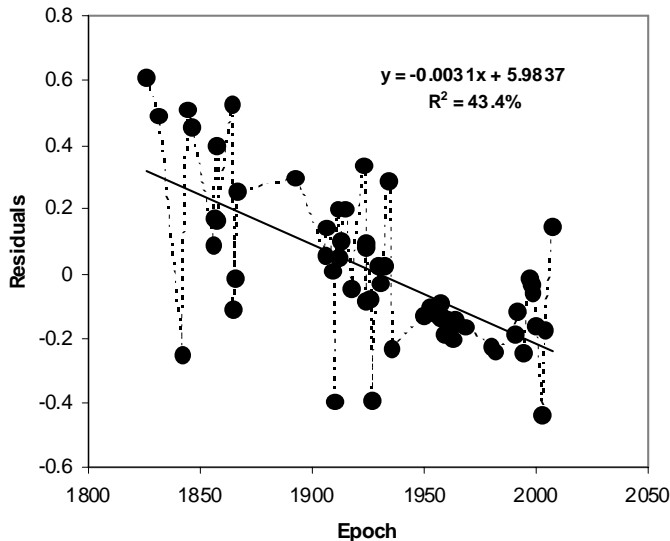

Figure 6: Residuals versus Epoch for the dates 1900.69 to 1911.73 for 00373–2446BU 395. This represents observations taken from approximately one–half of an orbit. Note the similarity in residual variation to 05110+3203STF 648AB in the next figure albeit over a much longer period of the orbit.

nary pair should show systematic, rather than random; changes of theta and rho over time would be functions of orbital motion (Aitken, 1935). To see if the techniques used here could uncover such behavior, I analyzed data for the known binary 00373-2446BU 395. The scatter plot is a nice picture of the orbital motion (Figure 4). Further, a plot of the residuals versus Epoch of observation shows the systematic variation of the residuals (Figure 5). One can even roughly estimate the orbital period of about 25 years. I then analyzed the residuals from only part of one orbit. This shows that the residuals display a non-random pattern of residual variation (Figure 6) with later measures following earlier measures in a coherent time series in the *y* versus *x* scatter plot. STF 648AB shows a similar non-random residual pattern (Figure 7). This suggests that STF 648AB may be binary albeit with a very long period. However, unlike BU 395, the order of measures in the *y* versus *x* scatter plot do not form a consistent time series (later dates do not always follow earlier dates along the apparent path), so the resulting non-random behavior of the residuals may simply be tracking noise. How to parse this out without using weighting functions will require study of additional pairs.

Discussion

In cases where the proper motions of a pair are known and different, classifying a pair as optical is trivial. However, there are many pairs for which the proper motion is known only for only one of the components. These techniques can be used to investigate such pairs. However, there are known caveats.

First, a true binary may display rectilinear motion if the companion is on one leg of a highly eccentric orbit. If the orbit is highly eccentric the residuals may not show systematic variation. This has long been recognized (e.g., Burnham, 1906; Hartkopf et al. 2008, CRE). Thus if other factors such as proper motion and parallax are similar, these data may trump the conclusion that the pair is optical based on relative motion studies. The second caveat was suggested by Dr. Hartkopf and concerns the geometry of approach. A direct approach along either of *x*-axis or *y*-axis will result in

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$$y = -0.0031x + 5.9837$$
$$R^2 = 43.4\%$$

Figure 7: Regression of residuals versus Epoch for 05110+3203STF 648AB suggests systematic variation of residuals over time.

either a failure to model (direct approach along the *y*-axis) or a model of $y = 0$ (direct approach along the *x*-axis). I have not seen either geometries but they are sure to exist. θ_0 could be calculated based on a simple distance measure (how long to the origin given the relative motion), but ρ_0 and θ_0 would simply be "zero."

I am currently investigating the characteristics of common proper motion pairs and binaries using these techniques. What I am predicting is that (1) CPM pairs can be distinguished by $y|x$ scatter plots that are tightly clustered, regression models with low R^2, and with $y|x$ regression residuals that are random relative to epoch of observation and that (2) binaries with observations covering only part of their periods (that is, those not yet characterized as binaries) will have $y|x$ scatter plots correlated with the inclination of their apparent orbits, R^2-values related to the eccentricity of their orbits (e.g. a circular orbit would have a relatively low R^2 value), and OLS $y|x$ regression residuals that vary systematically.

Acknowledgements

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