Determination of the Nature of Visual Double Stars Using Probability Theory

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Abstract: Nowadays the nature of many visual double and multiple stellar systems remain unknown and probably several tens of thousands of stellar systems could be optical. This situation prevents a better understanding of the formation of stars and stellar systems. So, it is very important to determine the true nature of the pairs we study. There are many tests used by professionals during the last decades. These tests can be used by amateurs because no important physical or mathematical knowledge is needed. In this work I comment in detail on tests based on probability theory that allow us determine the nature of visual pairs.

Introduction

Nowadays the nature of many double and multiple stellar systems remain unknown and probably several tens of thousands of stellar systems could be optical. This situation prevents a better understanding of the formation of stars and stellar systems.

Professional astronomers study relatively close visual double stars because they are physical in nature with orbital parameters or with important Keplerian motions. Astronomers do not have enough time to investigate the nature of wide double stars (mainly with rho > 10") and the human and technical resources would be poorly used because of those wide double stars only a few percent (about 10%) would be physical pairs. Amateurs have an important role, because we can spend time in determination of the nature of wide visual double stars.

Of the different criteria that help to determine the nature of visual double stars, there is a type of criterion that is based on probability theory analyzing the stellar distribution. This type of criterion is classified into two subtypes: (1) criteria that use the area distribution of stars with a determined bright (mainly brighter than secondary) and (2) criteria that uses the area density of stars with a determined proper motion (mainly with values between primary and secondary).

Historical background

After Bernadetto Castelli discovered Mizar to be a double star in 1617, astronomers thought that such pairs of stars were chance coincidence. During that time, astronomers thought that all stars have the same brightness, so dimmer stars were located at larger distances than brighter ones.

A work of John Mitchell (1768) disagreed rather strongly with Herschel over the question of how apparent magnitude is related to distance, but even more importantly, it contained a very simple statistical calculation on the question of the probability of finding two stars very close together on the sky, presuming them to be randomly distributed and given the total numbers of stars of any particular apparent magnitude. Mitchell's calculation showed that this probability is exceedingly low, so the fact that many such pairs are observed must imply that they are not chance coincidences in direction, but real physical pairs. It was not a difficult calculation and Herschel could easily verify it for himself, yet he seems to have purposely chosen to ignore it and proceeded with his observations to determine the first stellar parallaxes.

John Mitchell argued that the probability of observing two stars by chance closer than a certain apparent separation, was related to the arc of a disk with radius equal to that limit. The probability $p(\rho_{max})$

that two given stars are closer than the separation $\rho_{\max}$ is given by

$$P(\rho_{\max}) = \pi(\rho_{\max})^2 / (4\pi)$$

where $\rho_{\max}$ is assumed to be small and expressed in radians. The probability of these stars having a separation wider than $\rho_{\max}$ is then $1 - P(\rho_{\max})$. If we now consider N stars randomly distributed in the sky, the probability that no pair is closer than $\rho_{\max}$ is

$$[(1 - P(\rho_{\max}))^N]^N$$

Mitchell applied this relation in order to calculate the probability that no pair like β Capricorni could exist in the sky. He estimated that 230 stars were at least as bright as these stars. He calculated that the probability of finding no star with a companion at least as close and at least as bright as the secondary component of β Capricorni was 1-1/80. Then, the probability that a system like β Capricorni could appear by chance was 1/80, and Mitchell concluded that β Capricorni was a system of stars bound by gravitation, a hypothesis that has yet to be refuted.

Struve (1827 - 1852) calculated $n(\rho_{\max})$, the number of optical pairs with an apparent separation less than a certain limit $\rho_{\max}$ that should occur among N stars counted in a given area, A:

$$n(\rho_{\max}) = \frac{N(N-1)\pi{\rho_{\max}}^2}{2A}$$

Kubikowski, et al. (1959) considered the probability of finding a field star with magnitude m in the neighborhood of a given star given by

$$P(\rho_{\max}) = \frac{N(m)\pi{\rho_{\max}}^2}{A}$$

where N(m) is the number of stars with magnitude m. He assigned a given level (1%) to P, and he derived $\rho_{\max}$ as a function of the magnitude of secondary components. For example, he found that 1% of stars with magnitude 10 should have an optical bright companion. This 1% level is certainly not the proportion of optical systems among double stars, but only a means of deriving the contribution of optical pairs in the total number of binaries.

Van Albada(1968) and Bahcall & Soniera (1981) used another method, one based upon the distribution of the nearest neighbors. The probability that the nearest optical companion of a star is at a separation s is stated by a Poisson's law:

$$P(\rho \pm d\rho / 2) = \frac{2\pi\rho N}{A^{-\pi\rho^{2N/A}}} d\rho$$

They compared the histogram of the separation of the observed stars to this theoretical distribution and found that both were similar with regard to the larger separation in their samples. For close separations, they obtained an excess of systems due to physical binaries.

Bahcall et al. (1986) preferred the "near neighbors" method. They considered the separations between stars and all their neighbors closer than a given limit. The number of optical systems with separations $\rho \pm d\rho/2$ is then a linear function of ρ. Another formulation of this method is the so-called "two point correlation function" (Bahcall and Soniera, 1981). It consists of calculating $w(\rho)$, which is the relation between physical and optical pairs with the separation ρ.

Modern references in literature.

I carried out a search in astronomical professional literature for modern papers that have used these criteria. In papers where new wide companions or new wide stellar systems are discovered, it is the usual practice that authors use probability theory to obtain the probability that two stars were not chance coincidence.

The Mexican astronomers Poveda, Allen & Parrao (1982) in their work titled "*Statistical studies of visual double and multiple stars. I. Incompleteness of the IDS, intrinsic fraction of visual doubles and multiples, and number of optical systems*" filtered the Index Catalogue of Visual Double Stars (IDS) to eliminate optical systems in a statistical study.

To reduce the number of optical companions they applied a "1% filter" to each of the systems; the 1% filter consisted in testing if the faint companions (or secondaries) satisfy the following equation:

$$\pi s^2 N(m_2)_{l,b} < 0.01$$

Here, s is the angular separation of a secondary of magnitude m_2 from the primary, and $N(m_2)_{l,b}$ is the expected number of field stars per unit area brighter than apparent magnitude m_2 in the direction of the primary, which has galactic coordinates *l,b*.

Ciardullo et al. (1999) published a paper titled "*A Hubble Space Telescope Survey for Resolved Companions of Planetary-Nebula Nuclei*". In this work they used the following equation:

$$P = 1 - \left(1 - \frac{\pi\rho^2}{A}\right)^N$$

where ρ is the angular separation for both components in arcseconds, A is the area of the sky where we have searched for stars brighter that secondary (expressed in the same unit as ρ), and N is the number of stars brighter that secondary found in A. The expression $\pi\rho^2$ gives the circular area of the sky with radius equal to ρ. This professional team considered as physical pairs those that have a probability to be optical less than 5%.

G. Duchêne et al. (2001) published a paper titled "Visual binaries among high-mass stars". In this work they discovered new, very close visual double stars. They used probability theory and took into account the resolution power of their instruments depending on the magnitude of the star. They rejected in the calculation the area of the sky not resolved for their instruments and calculated the probability that a random star brighter than the secondary and weaker than the primary ($mag_{primary} \leq mag \leq mag_{secondary}$) is detected within separation ρ from the primary:

$$P = \sum_{K-K_p}^{K_S} n_k W_k$$

where n_k is the surface density in the considered field and W_k is the detectability area of stars at a given magnitude K (we must take into account that they used K band photometry). The value of W_k has the value $\pi(\rho^2 - \rho_{lim}(K))$ instead of the value $\pi\rho^2$. The term $\rho_{lim}(K)$ is the resolution power as a function of the K magnitude. So we can express

$$P = \sum_{K-K_\rho}^{K_S} \frac{N_k}{A} \pi(\rho^2 - \rho_{\text{lim}}(K))$$

where A is the area where we have counted the stars with $mag_{primary} \leq mag \leq mag_{secondary}$. G. Duchêne et al. summed the probability for each magnitude because the resolving power changes with the secondary magnitude.

Using the Proper Motions of the Components

Grocheva & Kiselev (1998) proposed using the real distribution of proper motions for estimation of P instead of using the distribution of stars brighter than a given magnitude. The probability that two stars with proper motions between $\mu(A)$ and $\mu(B)$ with angular distance of ρ is:

$$P = \frac{\pi\rho^2 S}{A}$$

where $\pi\rho^2$ is the area of the sky with radius ρ, A is the studied area of the sky, and S is the number of stars with proper motions between $\mu(A)$ and $\mu(B)$ found in an area A.

Grocheva & Kiselev chose assumed binaries from the same catalog. The sample must be restricted to pairs whose ρ are limited by same quantity. For example it is possible use Aitken's criterion (Rica 2006). They studied a sample with few known optical and physical pairs to obtain a definite criterion for the identification. The distribution of proper motions was derived from the PPM catalog for stars North-polar area. Analyzing the resulting probabilities they conclude that only the probability of random proximity of proper motion $P_\mu = S/N$ (where N is the total stars in the area A) can be used to identify true physical binaries. Grocheva & Kiselev observed that for physical pairs $P < 0.01$ whilst for optical ones are larger.

Results of the Method Based on Probability Theory

I initially calculated the probability P for two or three clear optical visual double stars studied by LIADA. The result made me suspect that the limit of probability P used by same professionals was not the best. The value of P that I obtained for clear optical pairs is significantly smaller than what some astronomers think. Poveda, Allen & Parrao (1982) used a limit of P = 1%; Ciardullo et al. (1999) used a limit of P = 5%.

So a further study was needed to determine the distribution of P values for optical and physical pairs. I selected a sample of 48 optical pairs and 25 physical pairs studied by LIADA Double Star Section.

For each visual pair, I counted stars as bright or brighter than secondary component in an area of 5 x 5 degrees around primary star. I used the VizieR tool from the website of Centre de Données Astronomiques de Strasbourg. We must take special care in the selection of the catalogue. In my sample some secondary components were dimmer than 13th magnitude. So we must select a catalogue that lists dimmer stars. Some catalogs don't cover the whole sky (for example UCAC-2). We must too take into account the sensitivity of the photometric data listed in the catalogs. UCAC-2 magnitudes are sensitive to red band. GSC-I, in the southern hemisphere magnitudes are sensitive to blue band, etc.

Finally I used the followed catalogues :

For sky region in $\delta < 50$ degrees I used the UCAC-2 (Zacharias et al. 2004) catalog. This catalog covers the sky with $\delta < 50$ degrees. UCAC-2 is compiled by astrometric and photometric reductions of CCD images. The photometric response is between V and R bands. I added 0.3 magnitude to the photometry of the catalogue to obtain a value closer to that of V band. The 0.3 value was chosen without any criterion. We have taken into account that if we use CDS web page, this catalog has a supplement for bright stars.

For region with $\delta > 40$ degrees, I was used the GSC-I catalogue. GSC used several combinations of photometric filters and photographic emulsions. The photometry for the north hemisphere is slightly brighter than the V band. So, to compensate for the red response of GSC-I for northern hemisphere, the photometric limit in the count of stars was the secondary component plus 0.3 magnitudes.

Taking into account the dependence of stellar density with respect the galactic latitude, in this study we listed the galactic latitudes for the stars analyzed.

Results

The values of P for the 48 optical pairs range from 0.09 % to 9.07 %. Figure 1 shows the distribution of P values for the optical pairs.

A sample of 25 physical pairs was used to determine the distribution of P values (Figure 2). Although there was an overlapping range in distribution of P for optical and physical pairs, the values of P were much smaller for physical pairs. The values of P for the physical sample range from 0.003 % to 1.54 %. Of the physical pairs only one binary had orbital parameters calculated. A sample of orbital pairs must show very much smaller values of P (very near of zero) than wide physical pairs. The overlapping region ranges from 0.09 % to 1.54 % and so, initially, the determination of the nature of visual pairs will not be very clear in many cases.

Filtering Double Star Catalogs

Maybe the best way to use this method is filtering a double star catalogue, such WDS (Mason et al. 2003) to obtain a sample with high probability of being

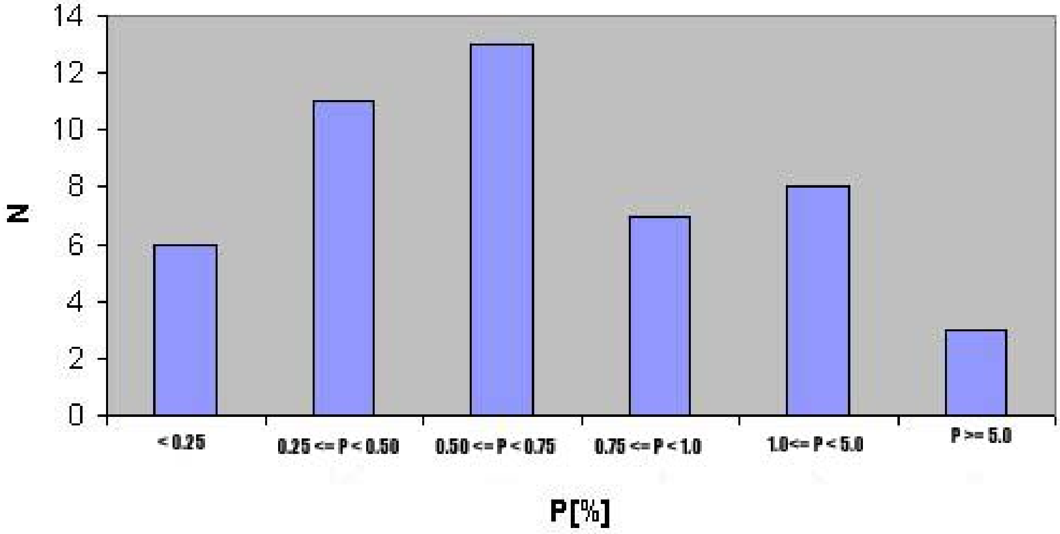

Figure 1: Distribution of P values for 48 optical pairs studied by LIADA Double Star Section.

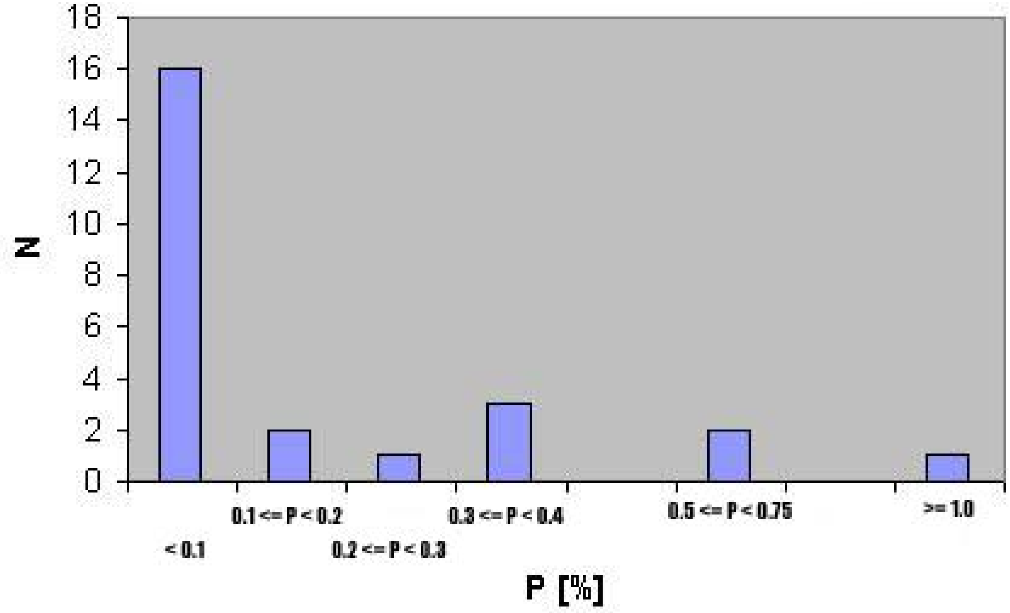

Figure 2: Distribution of P values for 25 physical pairs studied by LIADA Double Star Section.

physical or at least of common origin.

Ciardullo et al. (1999) used a value of $P = 5$ % to separate physical and optical candidates. They defined an optical pair to have $P > 5$ %. I used this 5 % filter to obtain candidates to be physical in my two samples of optical and physical pairs. The 5 % filter selected all physical pairs in the physical sample, but only rejected the 6 % of optical pairs in the optical sample. So this filter considered about 94 % of the optical pairs as candidate physical pairs.

Poveda, Allen & Parrao (1982) used a 1% filter to obtain an IDS-filtered with physical pairs from IDS catalog. They calculated that in this filtered sample must exist about 1% of optical pairs. The 1% filter considered as physical pairs about 98% of the systems in the LIADA physical sample and reject about 27% of optical pairs in the LIADA optical sample. For example, using the 1% filter, an artificial catalog with the same number of optical and physical double stars would have about 43 % of optical pairs after the filter process. Abt studied the list of 285 trapezium stellar systems published by Poveda in 2000. Abt, who knew the 1% filter of Poveda, was surprised, because he only found 14 physical trapezium stellar systems confirming our suspicion about the incorrect filters used in the literature.

As we can see, the best value for *P* is not so easy to choose and the value chosen depends on if we want to obtain a sample with only physical pairs or if we want not to reject many physical pairs. Table I shows the percent of physical and optical pairs in an initial artificial catalog. The first column gives the value of the filter *P* to consider a pair as physical. The second column lists the composition of the artificial catalog. The physical pairs (percent) considered to be physical by the filter are given in the next column. Column 4 gives the percent of optical pairs rejected by the filter. And finally the last column gives the composition of the filtered catalog

Example Calculation of the Probability of a Physical Pair

The visual double star FMR 5 was discovered by the author (Rica 2005) of this article. It is composed of 12.0 and 13.5 magnitude stars with an angular separation of 4.62 arcsec. There is very little astrophysical information in literature for this pair and I could not determine its nature using astrophysical tests. So in these cases we use probability theory to calculate the probability of chance projection of a star within 4.62 arcsec for the primary.

The value of the probability of a chance projection resulted be of 0.08 %. We must not interpret this value alone and must taken account the typical P val-

Filter	Initial artificial catalogue	Physical pairs	Rejected opticals	Filtered catalogue
P ≤ 5%	50,000 optical (50%) 50,000 physical (50%)	100%	6%	48,000 optical (49%) 50,000 physical (51%)
P ≤ 1%	50,000 optical (50%) 50,000 physical (50%)	98%	27%	36,500 optical (43%) 49,000 physical (57%)
P ≤ 0.5%	50,000 optical (50%) 50,000 physical (50%)	94%	65%	17,500 optical (27%) 47,000 physical (73%)
P ≤ 0.35%	50,000 optical (50%) 50,000 physical (50%)	92%	79%	10,500 optical (19%) 46,000 physical (81%)

Table 1: Results for different values of *P*.

ues for optical and physical pairs. About 80 percent of physical sample have $P \leq 0.08$ %. There was no optical pair in the optical sample with P <= 0.08 %. So in LIADA optical and physical samples all of pairs with $P \leq 0.08$ % were physical one and we can conclude that FMR 5 is surely a physical double star.

Conclusions

In this work I have described and analyzed the tests based in probability theory to obtain the probability of chance projection of a pair of stars. The analysis showed that the distribution range of values for *P* in optical and physical samples have an overlapped region that makes difficult the clear classification of visual double stars in optical or physical pairs. The values of *P* used in professional literature to discriminate optical and physical pairs were far to be correct. This method could be useful to filter visual double star catalogs obtaining a filtered-catalog with a high percent of physical pairs. Other use of this test is to obtain the probability that a pair of stars be a chance projection.

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