

Astrometric Measurements and Analysis of Four Star Systems

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Abstract

This paper presents astrometric measurements of double star systems 09483+2410 DAM 1758 AC, 07222-2412 HJ 9002 AB, 05067+5121 SMA 47, and 06440+7218 HJ 2323 AB. New measurements of the systems are presented and are shown to be similar to recent measurements and to listings in Gaia Data Release 3. In addition, each system's historical data are plotted and relative motions are calculated. We find that DAM 1758 AC, HJ 9002 AB, and SMA 47 are physically related, although unlikely to be bound, and that HJ 2323 AB is very likely to be bound.

1. Introduction

Four target double star systems investigated in this study are shown in Figure 1. Gaia Data Release 3 (DR3) magnitude and color data for the component stars are given in Table 1, with Absolute Gaia G-filter magnitudes calculated using Equation 1 (Babusiaux et al., 2023; Prusti et al., 2018; Vallenari et al., 2022).

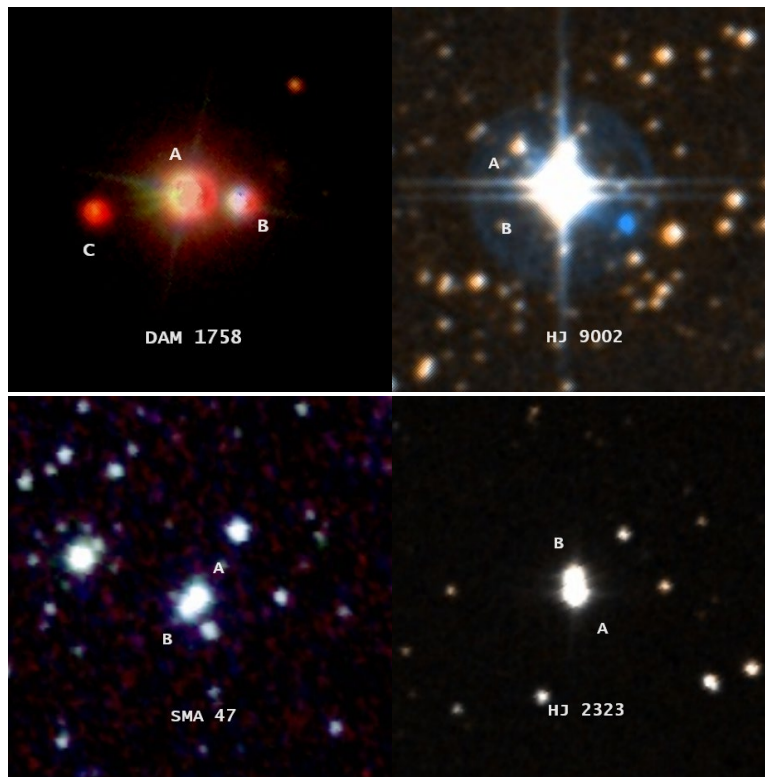


Figure 1: DAM 1758 AC, HJ 9002, SMA 47, and HJ 2323, from the Aladin Lite Sky Atlas.

Table 1. Gaia DR3 Magnitudes and Parallaxes

System	Primary Gmag	Secondary Gmag	Primary Parallax (mas)	Secondary Parallax (mas)	Absolute Primary Gmag	Absolute Secondary Gmag	Primary Color	Secondary Color
DAM 1758 AC	11.3000	16.3889	5.2619	5.2716	4.9	10.0	0.913	2.630
HJ 9002 AB	9.3346	9.8578	2.8449	2.8195	1.6	2.1	0.270	0.178
HJ 9002 AC	*	14.3086	*	0.3228	*	1.8	*	0.961
HJ 9002 AD	*	14.5544	*	0.7958	*	4.0	*	0.790
SMA 47	11.3525	11.8999	0.9090	0.9560	1.1	1.8	0.696	0.548
HJ 2323 AB	10.8307	11.5697	4.1150	4.1054	3.9	4.6	0.712	0.793

*HJ 9002 AC and AD share a primary star with AB.

Absolute G magnitude in Table 1 above is computed from Equation 1 where p is parallax in milliarcseconds and m is non-absolute G magnitude.

$$M_g = m + 5 \cdot \left(\log\left(\frac{p}{1000}\right) + 1 \right)$$

Equation 1: Absolute G Magnitude formula

Each of the four systems has noteworthy characteristics. DAM 1758 AC shares a primary star with POU 3058 AB, an optical double. However, as seen in Table 1 above, the primary A star has Gaia G-filter apparent magnitude of 11.3, while C has a G-filter magnitude of 16.38. (Babusiaux et al., 2023; Prusti et al., 2018; Vallenari et al., 2022). This makes DAM 1758 AC a high-delta mag star, which requires use of an infrared filter for the red C star to be visible in the same image as the blue A star.

Like DAM 1758, HJ 9002 is a physical double with additional optical doubles in its field. The system was first observed in 1835 by John Herschel (Herschel, 1847). Using luminosity and temperature estimates from two studies, the primary star is likely spectral type A1 or A2 and the secondary is spectral type A4 or A5 (McDonald et al., 2012; Zari et al., 2018). The stars have very similar parallaxes and are certainly physically related. However, the tertiary and quaternary stars, SHT 8AC and SHT 8AD, are thousands of parsecs away, as is evident from their parallax values in Table 1. Therefore, they are not physically related to the primary. These two stars were previously thought to be spectral class A0/1 (Shatsky et al., 1999).

Our third system is SMA 47, a pair of stars similar in temperature, magnitude, and color to each other. Combined with the fact that these stars are co-located and co-moving, these similarities may suggest a common origin. The stars' absolute magnitudes in Table 1 above are consistent with main sequence stars of types F3 and F6. From Earth, these stars are seen to be close but not touching and have remained this way for the century that they have been observed (Williams, 2010).

Much like SMA 47, HJ 2323 is a pair of extremely similar stars of near-solar mass and temperature. The pair was identified as a candidate comoving pair by an *Astronomical Journal* paper in 2017, in their catalog

of candidate comoving pairs. (Oh, 2017) Later the same year, in a Monthly Notices of the Royal Academic Society paper, the star was catalogued as a wide binary (Andrews, 2017).

All four pairs are placed on the Gaia HR-diagram in Figure 2 (Babusiaux et al., 2023; Prusti et al., 2018; Vallenari et al., 2022). This placement enables estimation of the stars' spectral types for stars where these are previously unknown. Once spectral types are determined, masses of each system's components are estimated using Dr. Siobahn Morgan's "Spectral Type Characteristics" (Morgan 2023). These estimated masses are tabulated in Table 2.

→ GAIA'S HERTZSPRUNG-RUSSELL DIAGRAM

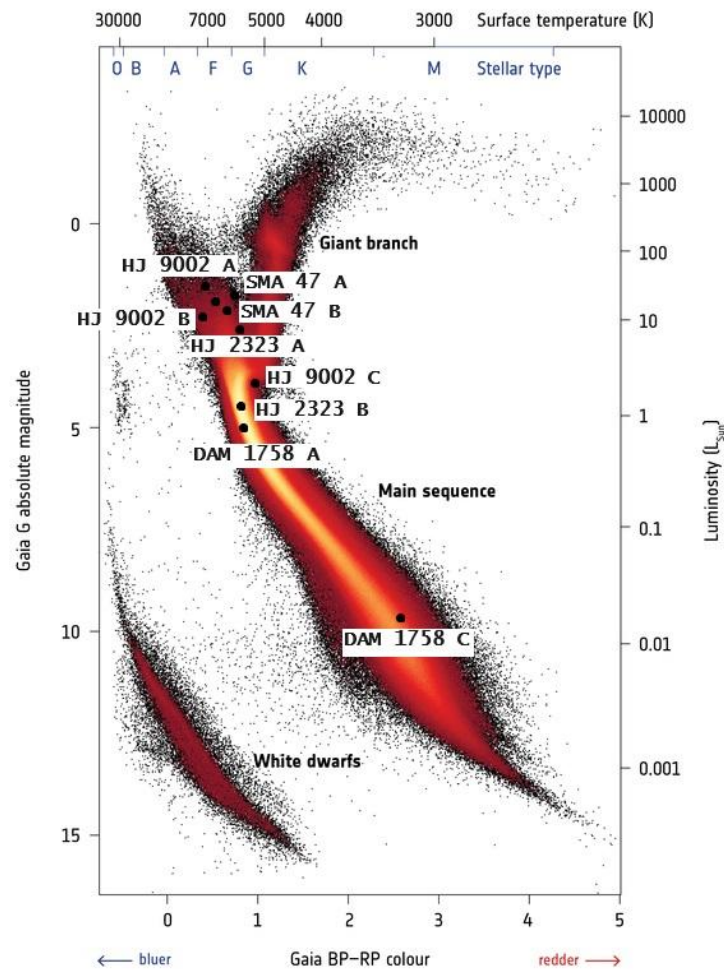


Figure 2: Gaia's HR diagram, used to estimate spectral types

Table 2. Estimated stellar types and masses.

System	Primary Stellar Type	Primary Mass (M_{\odot})	Secondary Stellar Type	Secondary Mass (M_{\odot})
DAM 1758 AC	G2	1.00	M5	0.40
HJ 9002 AB	A1	2.06	A5	1.73
HJ 9002 AC	*	*	A3	1.86
HJ 9002 AD	*	*	F6	1.20
SMA 47	F6	1.20	F3	1.29
HJ 2323 AB	G1	1.04	G2	1.00

2. Target Selection

There were several criteria that were used to select this system for study. First, to be visible in the late winter when the images were requested, the system needed an RA between 5 and 13 hours. The astronomical magnitude of the secondary star was intended to be less than 13, to be visible to the 0.35m Delta Rho telescopes at Las Cumbres Observatory Global Telescope (LCOGT) network, and the difference in magnitude was intended to be less than 3 so that the component stars could be exposed together in the same image. Although DAM 1758's secondary star had a V-filter magnitude of 17.4, which did not satisfy the brightness and delta mag criteria, this system was able to be imaged because of the color difference between its red secondary and blue primary. The final criterion was for the stars to have a separation of between 5 and 15 arcseconds, as a small separation also makes it difficult to resolve the individual stars, and a large one would decrease the likelihood of a strong physical relationship.

3. Instruments Used

The images taken for this paper were taken with a 0.35-meter Delta Rho telescope at the Las Cumbres Observatory Global Telescope (LCOGT) site at Siding Spring, New South Wales, Australia, and Teide Observatory at Tenerife, Canary Islands, Spain. This telescope uses a QHY600 CMOS camera. QHY600 CMOS cameras in central mode have a field of view (FOV) of $0.5^{\circ} \times 0.5^{\circ}$ and a pixel size of $0.73''$.

4. Measurements

Example measurements for each system are in Figure 3. Dam 1758 AC was imaged using a Sloan IR filter (Fukugita, M. et al., 1996). HJ 9002 and SMA 47 were imaged in a Bessel-V filter (Bessell, M. S., 1990). HJ 2323 was imaged in a PanSTAARS-w filter (Tonry, J.L. et al., 2012). Each image had its position angle and separation measured, using the software AstroImageJ. Exposure times are summarized in Table 3, and sample measurements are shown in Figure 3.

Table 3. Image Exposure Times, Filters, and Number of Images.

System	Exposure time	Filter	Number of images
DAM 1758	42	Sloan IR	10
HJ 9002 AB	3.5	Bessel-V	10
SMA 47	8.28	Bessel-V	10
HJ 2323	7	PanSTAARS-w	11

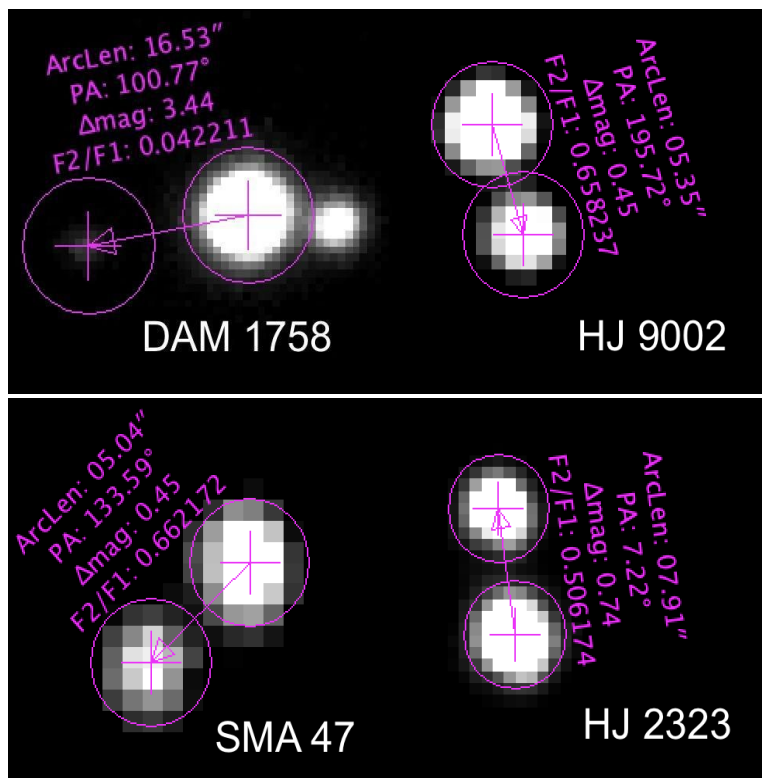


Figure 3: Astrometric measurements of DAM 1758, HJ 9002, SMA 47, and HJ 2323, made in AstroImageJ

5. Results

Table 4 shows the average position angle and separation measurements for each star system, along with the standard errors.

Table 4. Summary of Position Angle (PA) and Separation (Sep) Measurements.

System	Date	Number of Images	PA (°)	Standard Error PA	Sep (")	Standard Error Sep
DAM 1758 AC	2024.052	10	101.032	0.073	16.779	0.024
HJ 9002 AB	2024.0327	10	195.107	0.208	5.354	0.022
SMA 47	2023.0414	10	134.758	0.380	5.061	0.081
HJ 2323 AB	2024.0246	11	7.337	0.114	7.917	0.019

6. Relative Velocity and Escape Velocity Calculations

Astronomical objects are gravitationally bound if their relative velocity is less than terminal velocity, given by Equation 2, as derived in the appendix of Bonifacio, et. al., 2020.

$$v = \sqrt{\frac{2GM}{r}}$$

Equation 2. Escape Velocity Formula

Thus, to determine if a given system is bound, the relative velocity, distance between the stars, and mass must be known.

Relative velocity will be calculated first. To achieve this, both transverse and radial velocity must be known so that total relative velocity can be derived.

Transverse velocity is the vector sum of two component vectors: the relative proper motion in Right Ascension (RA) and relative proper motion declination (Dec). The total relative Proper Motion (PM) vector is calculated as the vector sum of these two perpendicular components, as shown in Equation 3. The pmRA and pmDEC variables stand for the proper motion in RA and Dec respectively, and the subscripts denote the primary or secondary stars. Their vector sum is termed relative proper motion. Proper motion components from Gaia DR3 are tabulated together with the relative proper motion vector magnitudes for each system in Table 5.

$$pm_{relative} = \sqrt{(pmRA_p - pmRA_s)^2 + (pmDEC_p - pmDEC_s)^2}$$

Equation 3: Relative PM Formula

Table 5. Relative Proper Motion Data.

System	Proper Motion RA Primary (mas/yr)	Proper Motion RA Secondary (mas/yr)	Proper Motion Dec Primary (mas/yr)	Proper Motion Dec Secondary (mas/yr)	Magnitude of Relative PM Vector (mas / yr)
DAM 1758 AC	-11.40646 ±0.04463	-10.90118 ±0.07278	-27.33307 ±0.03464	-28.65729 ±0.05729	1.42
HJ 9002 AB	-2.42788 ±0.00983	-1.90801 ±0.01223	-4.03981 ±0.01682	-2.95131 ±0.02310	1.21
HJ 9002 AC	-2.42788 ±0.00983	-3.57321 ±0.01077	-4.03981 ±0.01682	3.41594 ±0.01922	7.54
HJ 9002 AD	-2.42788 ±0.00983	3.78226 ±0.01288	-4.03981 ±0.01682	1.21226 ±0.02128	8.13
SMA 47	1.62144 ±0.03244	1.37719 ±0.02265	-5.6275 ±0.02723	-5.63733 ±0.01938	0.24
HJ 2323 AB	4.05227 ±0.008365	4.06422 ±0.009548	-47.27112 ±0.012406	-46.99337 ±0.013053	0.28

Now, to calculate the relative transverse velocity in physical units rather than angular units, the relative PM vector and parallax will be used. Table 6 gives parallaxes from Gaia (Babusiaux et al., 2023; Prusti et al., 2018; Vallenari et al., 2022), which can be used to find the distances using Equation 4:

$$d = \frac{1}{p}$$

Equation 4: Distance-Parallax formula

With the distance from earth, the PM vector can be converted from an angular quantity to a physical one, yielding the relative transverse motion through space. Equation 5 gives the details of the calculation, where rPM_v is the magnitude of the relative proper motion vector and plx is the parallax of the primary. Table 6 displays the numerical calculations, and the parallax of the secondary is also present for later reference.

$$\frac{rPM_v \cdot mas}{yr} \cdot \frac{1''}{1000 mas} \cdot \frac{1^\circ}{3600''} \cdot \frac{2\pi rad}{360^\circ} \cdot \frac{1}{plx} pc \cdot \frac{3.086 \cdot 10^{13} km}{pc} \cdot \frac{1 yr}{3.1636 \cdot 10^7 s}$$

Equation 5: Relative Transverse Motion formula

Table 6. Relative Transverse Motions of the Component Stars in Physical Units..

System	Parallax of Primary (mas)	Parallax of Secondary (mas)	Relative Transverse Motion Through Space (m/s)
DAM 1758 AC	5.2619 ± 0.04048	5.2716 ± 0.06553	1277
HJ 9002 AB	2.8449 ± 0.01637	2.8195 ± 0.01905	2010
HJ 9002 AC	*	0.3228	12568
HJ 9002 AD	*	0.79588	13551
SMA 47	0.9091 ± 0.02670	0.9561 ± 0.01881	1275
HJ 2323 AB	5.4391 ± 0.04401	7.1139 ± 0.19454	320

*Primary star values are the same.

The relative transverse motion is the stars' relative motion in the plane perpendicular to the earth, so to calculate the total relative 3D velocity the relative radial velocity (RV) must also be considered. Gaia supplies the radial velocity of all stars in the systems from which the relative velocity can be calculated as simply the absolute value of the primary's RV minus the secondary's. Then, since the transverse and radial velocity vectors are perpendicular, the total relative velocity can be calculated according to Equation 6, where with the results shown in Table 7 below.

$$\sqrt{(v_{pt} - v_{st})^2 + (v_{pr} - v_{sr})^2}$$

Equation 6: Total relative velocity formula, where *p* stands for primary, *s* stands for secondary, *t* stands for transverse, and *r* stands for radial.

Table 7. Relative Transverse Motion data.

System	Radial Velocity of Primary (km/s)	Radial Velocity of Secondary (km/s)	Relative Radial Velocity (m/s)	Relative 3D Space Velocity (m/s)
DAM 1758 AC	11.8 ± 0.22	-	-	1277*
HJ 9002 AB	-	3.0**	-	2010*
HJ 9002 AC	-	-	-	12568*
HJ 9002 AD	-	-	-	13551*
SMA 47	-12.0 ± 3.47	-9.0 ± 5.78	3070	3328
HJ 2323 AB	28.5 ± 0.33	28.8 ± 0.58	-0.271	320

*Radial velocity is assumed to be zero for the sake of calculation because it is unknown.

**Uncertainty not given.

Lastly, to find the escape velocity, the distance between the two stars must be calculated. The separation and distance (as an inverse of parallax) are known and the transverse separation can be calculated by solving for x in Equation 7. This follows because of the trigonometry depicted in Fig. 4, where Sep is the angular separation of the stars, and Transverse Sep is their transverse physical separation in space.

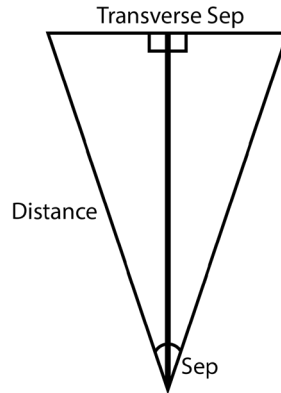


Figure 4: Trigonometric Intuition behind Transverse Sep Calculation

$$\sin\left(\frac{sep}{2}\right) = \frac{x}{D}$$

Equation 7: Transverse Sep formula

Using the small angle approximation listed in Equation 8, this can be rewritten as Equation 9.

$$\sin\left(\frac{sep}{2}\right) \approx \frac{sep}{2}$$

Equation 8: Small angle Approximation

$$D \cdot sep = x$$

Equation 9: Simplified transverse separation formula

Radial separation is simply calculated as the difference of distances of the two stars, which, substituting for parallax, is formulated in Equation 10. The radial and transverse distances together combine in a vector sum, yielding the total distance between the two stars. The calculations are summarized in Table 8 and parallax measurements are given from Table 6.

$$R_d = \left| \frac{1}{p_p} - \frac{1}{p_s} \right|$$

Equation 10: Radial Distance Formula

Table 8. Star Physical Separations

System	Angular Separation (arcsec)	Transverse Separation (pc)	Radial Separation (pc)	Total Separation (pc)
DAM 1758 AC	16.9	0.016	0.35	0.35
HJ 9002 AB	5.5	0.009	3.17	3.17
HJ 9002 AC	5.5	0.009	2746.39	2746.39
HJ 9002 AD	11.6	0.020	904.97	904.97
SMA 47	5.3	0.028	54.04	54.04
HJ 2323 AB	7.9	0.009	0.57	0.57

Finally, the escape velocity of each of the systems can be compared to the relative velocities and the boundedness of the systems can be determined. Escape velocity is determined via the calculation in Equation 2. If a system is bound, then its relative velocity will be less than the escape velocity. Note the masses and separations are in solar masses and pc, so they must be converted to kg and m respectively.

Table 8. Boundedness Calculation Results

System	Primary Star Mass (solar masses)	Secondary Star Mass (solar masses)	Separation (pc)	Relative Velocity (m/s)	Escape Velocity (m/s)	Bound?
DAM 1758 AC	1.00	0.402	0.35004	1277	840	N
HJ 9002 AB (se.)	1.73	1.97	3.17033	2010	1897	N
HJ 9002 AB (te.)	**	2.06	2746.39093	12568	1923	N
HJ 9002 AB (qu.)	**	1.25	904.96832	13551	1191	N
SMA 47	1.20	1.29	54.04281	3328	19.75	N
HJ 2323 AB	1.04	1.00	0.56832	420	1373*	Y

*Using 2D separation, as parallax uncertainties overlap

**Same values as above

Although all these quantities have some uncertainty, the data suggests that three out of the four stars are not gravitationally bound. The HJ 2323 AB system is potentially bound, although within parallax uncertainty, there is also a possibility that the star is not gravitationally bound after all. The other three systems may not

be gravitationally bound, although DAM 1758 AC and HJ 9002 AB likely have strong gravitational influence, due to their high escape velocity. Even SMA 47 could be bound, given that the sum of the parallax uncertainties, $0.0267+0.01881$ is almost equal to the difference in parallaxes, $0.95605-0.90908$. Thus, if the stars' parallaxes are assumed to be closest within the margin of error, the stars may be closer and more bound than initially suggested by the data.

7. Plots

Figure 10 shows the historical data plots for DAM1758, HJ9002, SMA47, and HJ2323. Given the escape velocities calculated above, the only plot we expect to show evidence of curvature is HJ 2323 AB. However, all four plots have few data points of which many of the earliest measurements are outliers, and are therefore inconclusive, though HJ 2323 AB and SMA 47 do show trends that appear somewhat linear.

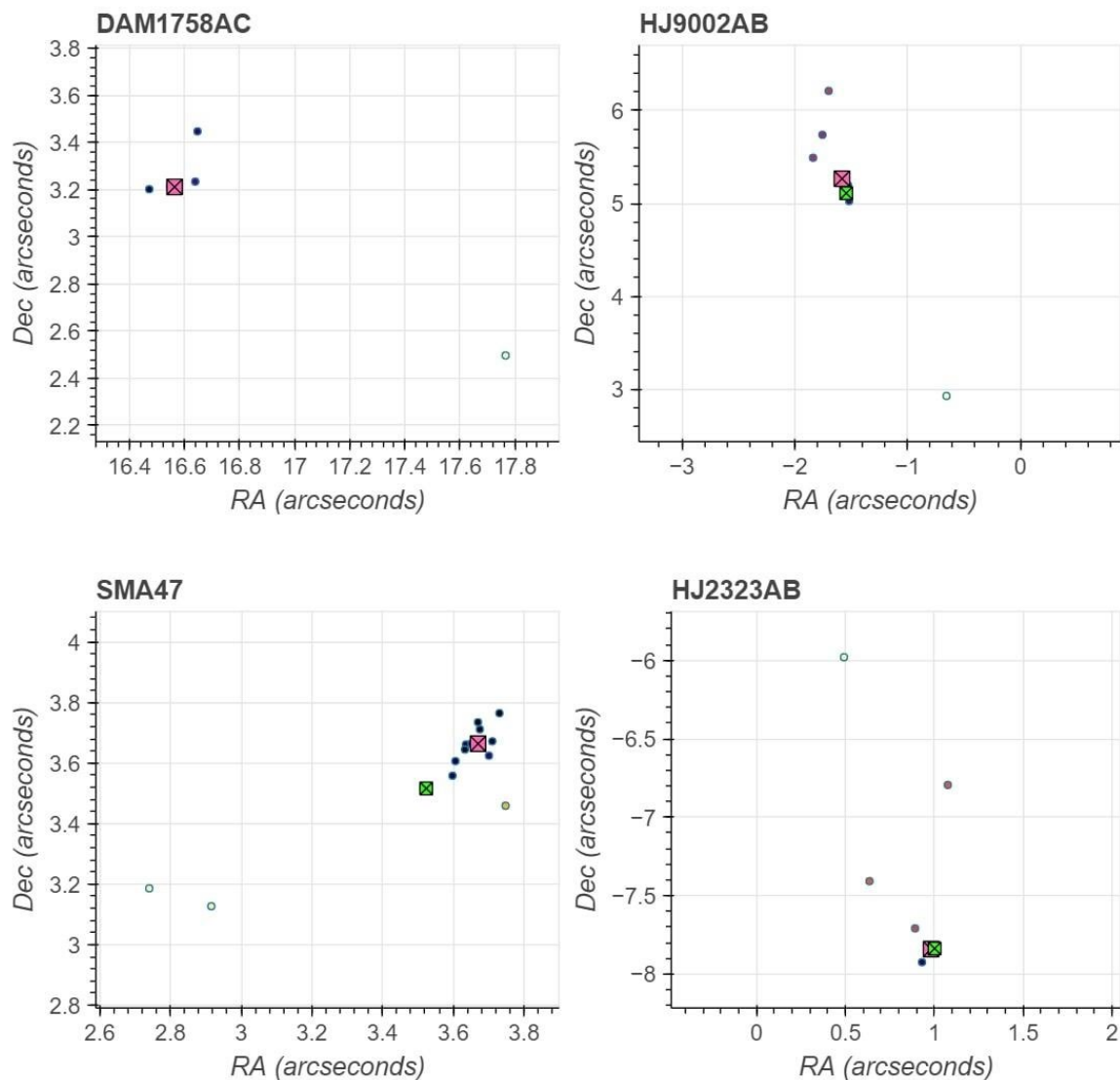


Figure 5: Historical data plots for DAM 1758, HJ 9002, SMA47, and HJ 2323. Gaia measurements are represented by green squares, and our measurements are represented by red squares. More recent measurements are darker.

8. Discussion

DAM1758 AC is a comoving physical double, with three-dimensional separation less than half a parsec in Table 8 and a relative PM vector almost 30 times smaller than its component PM vectors. We only have access to 4 previous observations of the system, so we do not have enough data to determine whether the relative motion is curving over time. However, our estimated relative velocity is around 35% greater than our estimated escape velocity for the system, suggesting that the stars, although physically related, are not gravitationally bound.

HJ9002 AB is also a physical double, while parallax shows that the C and D stars are optical. Historical data shows a trend, but we lack enough observations to confidently say whether the trend is linear. Our estimated relative velocity is greater than our estimated escape velocity, but only by around 5%. We can conclude that HJ9002 AB is physically related, but unlikely to be bound.

SMA47 AB is also a physical double, as indicated by the matching parallax and by the similarity of the PM vectors. However, the stars are almost certainly not bound, as the trend in the historical data is linear, and the estimated relative velocity is over 160 times the estimated escape velocity.

HJ2323 AB is very likely bound, as shown by matching parallax and by its almost-identical PM vectors. There is a clear trend in historical data, although curvature is not clearly visible. The stars have a relative 3d velocity of only 320 m/s, only a quarter of our estimated escape velocity. Therefore, the data we have suggests that HJ2323 AB is bound, and we can expect a curving trend to emerge over time.

9. Conclusion

This paper has examined four wide physical double star systems, and analyzed our own observations as well as available data to assess the strength of these systems' physical relationships.

We confirm DAM 1758 AB to be an optical double, and determine that DAM 1758 AC is physically related, but unlikely to be bound. The C and D stars in HJ 9002 are also optical doubles, and unrelated to HJ 9002 AB. We determine that the A and B stars of HJ 9002 are physically related but unlikely to be bound, given the combination of the historical data plot and escape velocity calculations. We find that SMA 47 AB is also physically related, but almost certainly not bound. Finally, we find that HJ 2323 AB is almost certainly a binary system. To better understand these stars, and our cosmos in general, more time and observations are needed.

Acknowledgments

This research was made possible by the Washington Double Star catalog maintained by the U.S. Naval Observatory, the Stelledoppie catalog maintained by Gianluca Sordiglioni, Astrometry.net, and AstroImageJ software which was written by Karen Collins and John Kielkopf.

This work has also made use of data from the European Space Agency (ESA) mission Gaia (<https://www.cosmos.esa.int/gaia>), processed by the Gaia Data Processing and Analysis Consortium (DPAC, <https://www.cosmos.esa.int/web/gaia/dpac/consortium>). Funding for the DPAC has been provided by national institutions, in particular the institutions participating in the Gaia Multilateral Agreement.

This work makes use of observations taken by the Planewave Delta Rho 350 + QHY600 CMOS camera systems of Las Cumbres Observatory Global Telescope Network located in Siding Spring, New South Wales, Australia.

The team would like to thank the JDSO reviewer for reading our study carefully and giving us helpful feedback.

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