

R.G. Aitken's Criterion to Detect Physical Pairs

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Abstract: One of the main goals in double star research is the calculation of stellar masses. So a double star observer must study the nature of double stars rejecting optical pairs due to their lack of astrophysical interest. LIADA's Double Star Section used several professional methods to classify visual double stars in optical or physical pairs. In this work the Aitken criterion is explained in detail and its effectiveness studied. The Aitken criterion is possibly the easiest of several possible criteria to apply, but may be one of the less efficient and rigorous. This criterion is very useful when an observational program is being designed because it allows us to increase the astrophysical interest of stars to be included in the program.

1. Introduction

One of the main goals of double star research is the calculation of stellar masses in order to obtain more data for the luminosity-mass relation. Until several years ago, binaries were the only way to determine stellar masses (now it is possible determine stellar masses using gravitational lenses).

But, before we can calculate stellar masses, we must know the orbital parameters for the binaries. We can determine orbital parameters only for physical double stars, so we must study the nature of double stars, rejecting optical pairs due to their lack of astrophysical interest.

But mass-determination should not be considered the only interest of double star astronomy. Today, research on the origin and the evolution of wide systems are wanted, not only for the understanding of the evolution of the stellar medium, but also for a better knowledge of galactic dynamics. So it is important to obtain a census of wide systems and therefore it is necessary to differentiate optical pairs and physical pairs.

Until the middle of the eighteenth century astronomers thought double stars were just two stars close by chance. In 1767 John Mitchell used formulae based in probability theory and argued that the probability of observing two stars by chance closer than a certain apparent separation, was related to the area of a disk with a radius equal to that separation. The

probability, p , that two given stars are closer than the separation ρ_{\max} is given by

$$p(\rho_{\max}) = \pi (\rho_{\max})^2 / (4\pi)$$

(ρ_{\max} is assumed to be small and expressed in radians). The probability of these stars having a separation wider than ρ_{\max} is then $1 - p(\rho_{\max})$. If we now consider N stars randomly distributed in the sky, the probability that no pair is closer than ρ_{\max} is $((1 - p(\rho_{\max}))^N)^N$. Michell applied this relation in order to calculate the probability that no pair like β Capricorni could exist in the sky. The separation between the components of beta Capricorni was 3.33" and he estimated that 230 stars were at least as bright as these stars. He calculated that the probability of finding no star with a companion at least as close and at least as bright as the secondary component of β Capricorni was 1-1/80. Then, the probability that a system like β Capricorni could appear by chance was 1/80, and Michell concluded that β Capricorni was a system of stars bound by gravitation, a hypothesis that has yet to be refuted.

William Herschel confirmed the orbital motion in double stars observing the curvature in the motion of the secondary with respect to the primary star. After John Mitchell, other astronomers tried to obtain a criterion to select physical pairs. Struve (1827, 1852) calculated $n(\rho_{\max})$, the number of optical pairs with an apparent separation less than a certain limit ρ_{\max} that

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should occur among N stars counted in a given area, A :

$$n(\rho_{\max}) = N(N - 1)\pi(\rho_{\max})^2 / (2A) \quad [2]$$

Camille Flammarion (1887) considered those visual double stars wider than $25''$ as optical doubles. Since then, there are several professionals that have designed their own methods to separate optical pairs from physical pairs.

In this article we are going to explain R.G. Aitken's method, which is defined empirically. The Aitken criterion may be one of the easiest to apply, but may be one of the less efficient.

2. The criterion

R.G. Aitken made a statistical study to obtain his empirical criterion. In this criterion, the combined magnitudes of the components with the maximum angular separation of a physical pair are related. Aitken then used his criterion to list double stars in his "New General Catalogue of Double Stars", which he published in the German journal *Astron. Nachr.*, 188, 281 in 1911 and again in 1932. The mathematical expression is:

$$\log(\rho_{\max}) = 2.8 - 0.2 * \text{combined_magnitude}$$

2.1. Combined magnitude

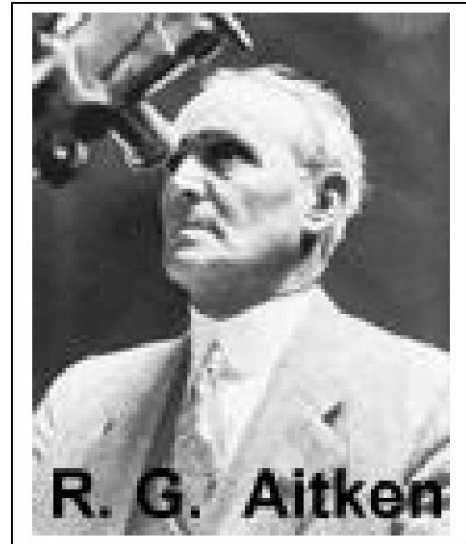
To use the Aitken criterion we must sum the fluxes of the components. We can transform the magnitudes to fluxes by this formula:

$$\text{Stellar_Flux} = 10^{-(\text{magnitude}/2.5)}$$

The combined magnitude is calculated using this formula:

$$\text{Combined_magnitude} = -2.5 * \log(A_flux + B_flux)$$

The Aitken Double Star Catalogue (ADS) is widely thought to contain mostly physical pairs. Not all ADS entries satisfy Aitken's criterion. In fact, out of 22,000 entries, 4,500 fail Aitken's criterion, while 1054 have no magnitude listed for either primary or secondary. According to the astronomer D. Sinachopoulos, for magnitudes greater than about 9 - 10 magnitudes, only a small fraction of the entries in ADS can be considered as surely physical. On the other hand, for $m \leq 7$ this percentage is very high.



2.2. An example

Rafael Benavides, an active member of LIADA's (Ligo Iberoamericana de Astronomia) Double Star Section, found an uncatalogued pair composed by GSC 3089-1053 and GSC 3089-2355 with visual magnitudes of 10.78 and 12.5. Rafael measured an angular separation of $16.87''$. Using expressions (2) and (3) a combined magnitude of 10.58 is calculated. The Aitken criterion (expression (1)) gave a maximum separation of $4.8''$. The angular separation of this pair is more than three times greater than the Aitken's limit, so according to this criterion stars do not constitute a physical pair.

3. Study of the Effectiveness

3.1. Introduction

One of the main goals of this article is study the effectiveness of the Aitken criterion to separate optical and physical pairs. Two samples were analyzed: an orbital binaries sample and a common proper motion binaries sample.

3.2. Effectiveness in the orbital binaries sample

The 1996 version of the Washington Double Star Catalog was consulted using Vizier from the CDS website. The selected double stars were orbital binaries with magnitudes for both components listed. Theta and rho for selected double stars had to be listed. Out of 70,000 double stars in WDS_1996, 492 orbital binaries were selected. The Aitken criterion was calculated automatically for the binaries in the sample using Microsoft Excel 97 and Microsoft Access 97.

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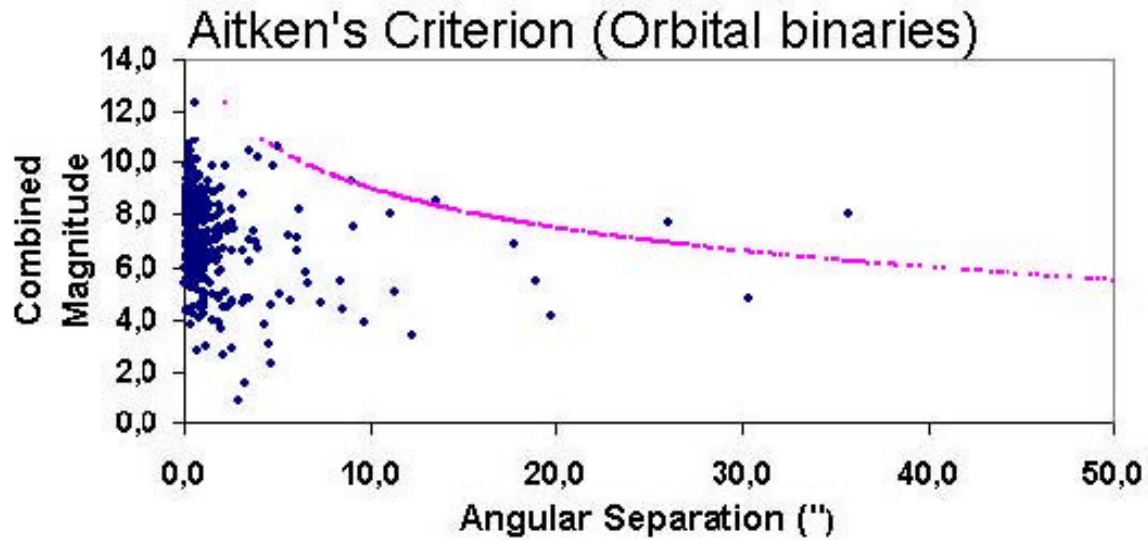


Figure 1: Aitken's Criterion and Orbital Binaries. The pink curve is the RHO limit for Aitken's criterion. The blue data points below the pink curve are physical pairs. Only systems with $\rho < 50''$ are plotted.

Figure 1 shows the curve of Aitken's criterion that relates the combined magnitude (vertical axis) with angular separation, ρ (horizontal axis). The points for orbital binaries are filled circles. Those points below the Aitken curve correspond to physical pairs according to Aitken's criterion. Most of the systems are considered to be physical pairs.

Of the 492 orbital binaries, 487 (99%) were considered to be physical pairs according to Aitken's criterion. So the effectiveness of Aitken's criterion for orbital binaries are excellent and only 1% of orbital pairs are considered optical.

Table 1 shows the 5 orbital binaries considered optical pairs. As we can see, members of these systems are intrinsically very weak stars composed by cool red dwarfs and white dwarfs. The angular separa-

tion for these orbital pairs are large compared with the typical angular separation in orbital pairs, but the orbital periods are typical for many orbital pairs. In my opinion, it seems that in the sample that Aitken used there were few very red and white dwarfs and statistically they are considered far away stars.

Except for GRB 34 AB, the orbital pairs are very near the limit in ρ of the Aitken criterion. So if members of these systems were a little hotter than they are (one or two subclasses), they would be considered physical pairs.

3.3. Common proper motions pairs

In addition to the intrinsic weakness of the stars, we suspect that Aitken's criterion could depend on the angular and real separation (semi-major axis).

To study the dependence of angular and true separa-

Double	Mg.B	Mg.A	Rho	Rho (max.)	Period	Spectral
GRB 34 AB	8.07	11.04	35.7''	15.8''	2600	M2V - M6V
STF 518 BC	9.52	11.17	8.9''	8.7''	252	DA - M4V
STF2398 AB	8.94	9.69	13.6''	12.6''	408	M4V - M5V
STF2434 BC	8.38	8.60	26.0''	23.5''	403	
WIR 1	10.7	13.2	5.0''	4.8''	359	M4V

Table 1: Orbital Pairs Considered Optical Using Aitken Criterion

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ration, we used a catalog of 122 common proper motion stars ("Wide binaries among high-velocity and metal-poor stars") compiled by C.Allen, A. Poveda and M.A.Herrera (National University of Mexico) and published in *Astronomy & Astrophysics* in 2000. The list was compiled by searching for common proper-motion companions to the more than 1200 high-velocity and metal-poor stars of Schuster and collaborators.

One of the main goals of the investigation of C. Allen *et al.* was to study the true separation distribution for these wide binaries. So they considered these pairs as physical ones. But Allen, *et al.* did not confirm their physical nature and they only commented: "We emphasize, however, that all of our listed wide companions are, in fact, common-proper-motion companions, and hence very likely to be physically associated.". Our experience tells us that some of these pairs could be only common origin pairs which have never been bounded or have been disrupted by galactic clouds.

Only 49 systems were considered as physical, about 41 % of the sample (see Figure 2). But, at what point is Aitken's criterion no longer reliable?

3.4. Dependence of Aitken's criterion on angular distance.

To analyze the correlation of Aitken's criterion with angular distance (ρ), the ratio between the angular separation of the system and the maximum ρ according to Aitken's criterion was plotted against ρ

(Figure 3). Systems with $\rho(\text{aitken})/\rho(\text{double}) > 1.0$ are physical pairs according to Aitken's criterion. A dependence is clearly visible and an important difference can be seen for $\rho > 10''$ and $\rho < 10''$. For pairs with $\rho > 10''$ only 13% (10 of 79 systems) are considered physical pairs. For systems with $\rho < 10''$, 38 of 41 doubles are considered as physical pairs and so the effectiveness of the Aitken criterion is about 93% for $\rho < 10''$.

Against the orbital binaries used in this last analysis, 7 of 10 binaries with $\rho > 10''$ are considered as physical pairs.

A more clear dependence of Aitken's criterion is observed with respect to the expected semimajor axis, $E(a)$, in A.U. To calculate the expected semimajor axis Paul Couteau's (1960) formula was used. This formula statistically relates angular separation with semimajor axis. The Paul Couteau results were confirmed by more recent investigations.

Figure 4 shows the dependence of the Aitken criterion on semimajor axes. The y axis is the ratio between the angular separation of the system and the maximum, according to Aitken's criterion. Again, systems with $\rho(\text{aitken}) / \rho(\text{double}) > 1.0$ are physical pairs according to Aitken's criterion. A dependence is clearly shown. Against the binaries with $E(a) > 1,000$ -1,500 A.U. only 3 of 68 systems were considered as physical. But for binaries with $E(a) < 1,000$ A.U. 92 % were considered as physical pairs.

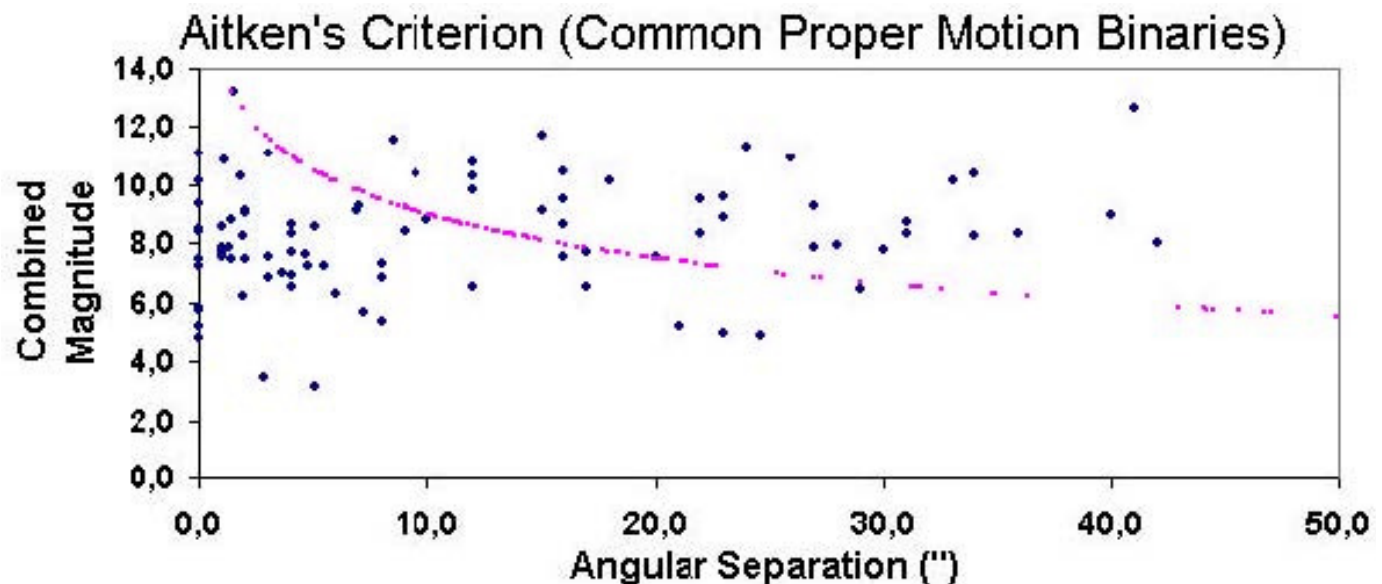


Figure 2: This figure shows the common proper motion pairs and the curve of Aitken's criterion. The blue data points below the curve of Aitken are physical pairs according to this criterion. Only systems with $\rho < 50''$ are included.

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Aitken's Criterion and RHO

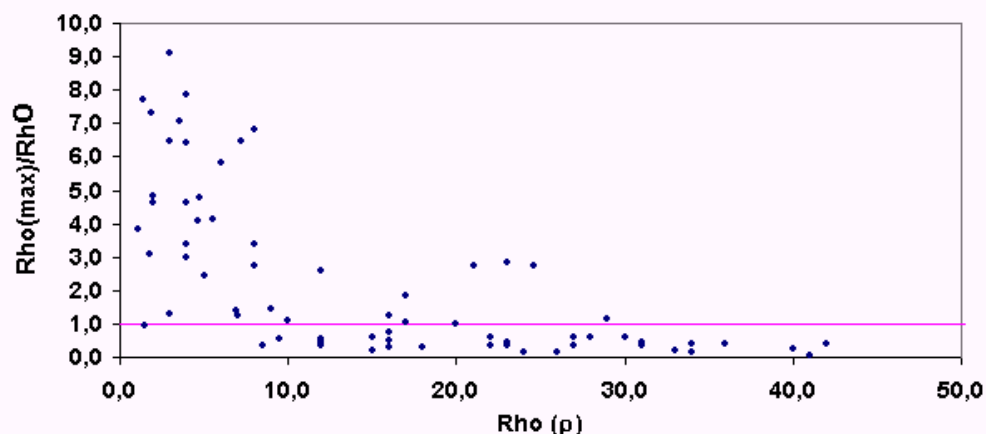


Figure 3: This graphic shows the dependence of Aitken's criterion with angular distance (rho). The y axis is the ratio between the angular separation of the system and the maximum rho according to Aitken's criterion. Systems with $\rho(\text{Aitken}) / \rho(\text{double}) > 1.0$ are physical pairs according to Aitken's criterion. A dependence is clearly showed and an important difference can be seen for $> 10''$ and $r < 10''$. Only those systems with $\rho < 50''$ were plotted.

4. Aitken's Criterion for the 100,000 Visual Double Stars in the WDS Catalog

Nowadays many double star observers know that in the Washington Double Star Catalogue (WDS) there are many thousands of optical double stars which have no astrophysical interest. For the majority of double stars in WDS their nature is unknown. How can we separate optical from physical pairs?

An important task in this study was to obtain Aitken's criterion for the 100,000 double stars catalogued in WDS. Today this task is not difficult due to the internet and data processing tools. Accessing the USNO web page for WDS catalog, I obtained the WDS in text format updated for 2003. The WDS catalog was imported to Microsoft Access 7.0 allowing me to study and consult any double star in the catalog. Designing several macros in Visual Basic, we could calculate the Aitken criterion for 100,000 double stars in a few minutes.

To calculate the Aitken criterion, the magnitudes and the angular separation must be known. If the secondary magnitude is unknown then $mgB = mgA + 0.5$. If the primary magnitude is unknown then $mgA = mgB - 0.5$. The 0.5 value was chosen randomly. The most recent value for the angular separation was used (which corresponds to the second epoch). The Aitken criterion was not calculated for about 1-2% of double stars in the WDS due to the lack of both magnitudes or angular separation.

Table 2 shows the results for different double star populations. Column 1 lists the name of the double star population; column 2 contains the total number for the double star population; column 3 lists the number of double stars for which the Aitken criterion was applied; column 4 has physical pairs according to Aitken's criterion; column 5 has the percent of physical pairs for the population.

The first three double star populations make reference to all double stars in the WDS, the double stars closer than $10''$ and wider than $10''$. About 61% of double stars in WDS are physical and so about 40,000 double stars are optical.

About 90% of the double stars included in the observational program of LIADA's Double Star Section are those unconfirmed and discovered by John Herschel between 1820-1830. So I studied the Aitken criterion for about 6,000 double stars discovered by John Herschel. The results indicate that about 25% of them are physical. This value is about two or three times greater than the percent of physical for unconfirmed John Herschel's double stars.

The double star population composed by Hipparcos (HDS pairs) and Tycho-2 (TDS pairs) were studied. About 91% and 98% of Hipparcos and Tycho-2 pairs are considered as physical pairs according to Aitken's criterion confirming the good criterion of these missions to detect new true pairs. For pairs with $\rho > 10''$, only 9% were considered physical.

The pairs discovered recently were studied. For double stars discovered since 1995, about 68% are considered physical pairs (a similar percent obtained for the whole WDS). For double stars discovered in 2001 and 2002, only 11% and 7% respectively, are considered physical. The low percentage of physical pairs is due to not using astrophysical or statistical criteria to detect binaries in the new pairs. This is a worrisome situation for those who want to study binary systems.

In my opinion, Aitken's criterion is useful for designing observational programs listing double stars with higher astrophysical interest, but it is a weak criterion to determine the nature of individual pairs.

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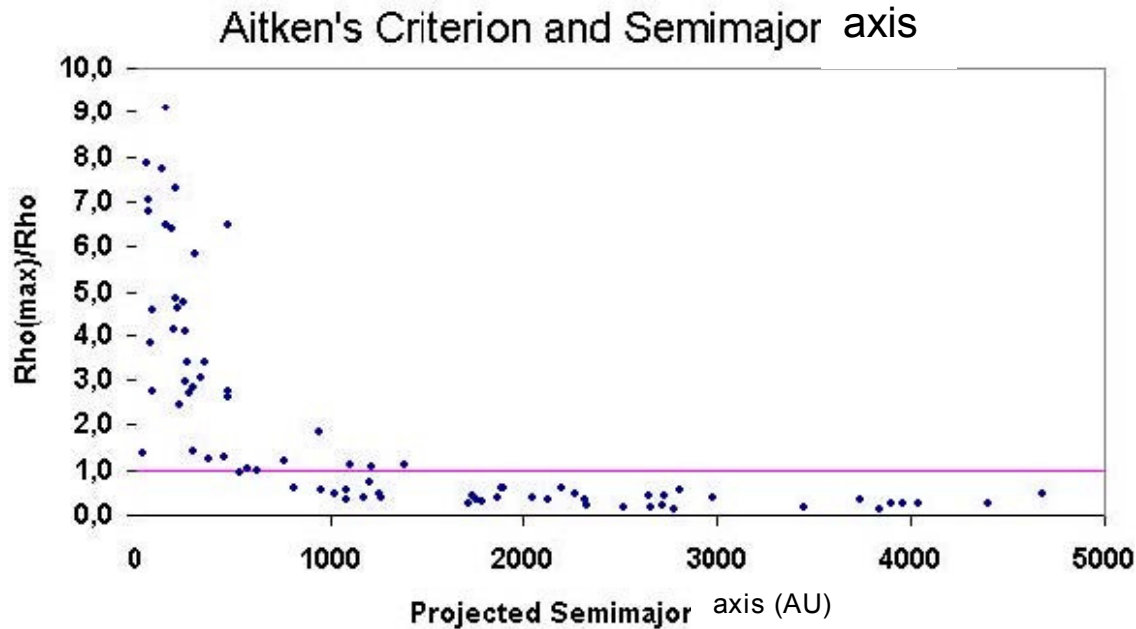


Figure 4: This graphic shows the dependence of the Aitken's criterion with semimajor axis. The y axis is the ratio between the angular separation of the system and the maximum ρ according to Aitken's criterion. Systems with $\rho(\text{aitken})/\rho(\text{double}) > 1.0$ are physical pairs according to Aitken's criterion. A dependence is clearly showed and an important difference can be seen for $a < 1000$ to 1500 AU.

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(1) Double Star Population	(2) Double stars	(3) Doubles with Aitken's criterion calculated	(4) Physical Pairs	(5) % of Physical pairs
ALL double stars in WDS	99,394	97,682 (98 %)	59,191	61 %
ALL pairs with $\rho < 10''$	67,778	67,225 (99 %)	56,071	83 %
ALL pairs with $\rho \geq 10''$	30,743	30,427 (99 %)	3129	10 %
John Herschel's (HJ) pairs	5,874	5858 (100 %)	1485	25 %
Hipparcos' (HDS) pairs	3395	3395 (100 %)	3101	91 %
HDS pairs with $\rho \geq 10''$	268	268 (100 %)	24	9 %
Tycho-2's (TDS) pairs	4312	4312 (100%)	4242	98 %
Pairs discovered since 1994	709	613 (86 %)	417	68 %
Pairs discovered in 2001	61	57 (93 %)	6	11 %
Pairs discovered in 2002	57	55 (97 %)	4	7 %

Table 2: Aitken's Criterion applied to double stars in the WDS.