# Journal of <br> Double Star Observations 

## Inside this issue:

| Catalog Access and New Lists of Neglected Doubles <br> Brian D. Mason | 3 |
| :--- | :---: |
| CCD Astrometric Measurements and Historical Data Summary of Double Stars <br> WDS 05548-2527 and WDS 00177+2630 <br> Ana Parra, Alexander Beltzer-Sweeney, Irena Stojimirovic, Pat Boyce, and Grady Boyce | 5 |
| Double Star System WDS 02229+5835 BLL 7 (S Per) <br> Olivia Ho, Kieran Saucedo, Alani Bayha, Eliana Meza-Ehlert, Shakara Tilghman, Brian Delgado, <br> Pat Boyce, and Grady Boyce |  <br> Double Star Photometry - March 2019 <br> Wilfried R.A. Knapp <br> Double Star Photometry - April 2019 <br> Wilfried R.A. Knapp <br> Orbit Determination of Close Binary Systems <br> F. M. Rica and H. Zirm <br> The Number of Binaries in the Sky Compared to a Random Distribution of Similar Stars <br> T. V. Bryant, III <br> Counter-Check of Reported Common Origin Pairs <br> Wilfried R.A. Knapp <br> TYC 2036-1173-1: An Optical Triple Star System in Corona Borealis? <br> Trygve Prestgard <br> Measurement of Rasalgethi with a DSLR Camera <br> Blake Nancarrow <br> CCD and Gaia Measurements Indicate that WSD 12095 + 3356 is a Physical System <br> Alexa Brammer, Jessica Padron-Loredo, Charmain Brammer, and Cameron Pace <br> Astrometric Measurement of WDS 12459-7511 HJ 4545 <br> Isabel Zheng, Yael Brynjegard-Bialik, Jackie Roche, Pat Boyce, and Grady Boyce <br> Astrometric Measurements of OSO 51 AB <br> Shreya Goel, Shabdika Gubba, Pat Boyce, and Grady Boyce <br> Discovery of a Wide Binary in the Solar Neighborhood <br> Wilfried R.A. Knapp <br> 48 |
| 68 |  |

## Inside this issue:

| UCAC4 337-189531, Discovery of Stellar Duplicity During Asteroidal Occultation <br> by (3130) Hillary <br> Carles Perello, Eric Frappa, Tomas Janik, Bjoern Kattentidt, Jiri Polak, Michal Rottenborn, and Antoni Selva | 87 |
| :--- | :---: |
| Astronomical Association of Queensland 2017 Program: <br> Blue Star Observatory Measurement of Six Neglected Southern Multiple Stars <br> Peter N. Culshaw, Diane Hughes, John Hughes, Des Janke, and Graeme Jenkinson | 93 |
| Astrometric Measurement and Analysis of Celestial Motion for <br> Double Star WDS 02176+5920 <br> Marielle Cooper, Theophilus Human, Grady Boyce, Pat Boyce, and Jae Calanog | 98 |

# Catalog Access and New Lists of Neglected Doubles 

Brian D. Mason<br>U.S. Naval Observatory<br>3450 Massachusetts Avenue, NW, Washington, DC, 20392-5420<br>brian.d.mason@navy.mil

## 1. Catalog Access

The US Naval Observatory Websites are undergoing modernization and will be offline starting Thursday, 24 October 2019. The expected completion of work and return of service is estimated as 30 April 2020. Until that time, the only access to double star catalogs will be via our website mirrors:

- The Washington Double Star Catalog:
http://www.astro.gsu.edu/wds/
- Sixth Catalog of Orbits of Visual Binary Stars: http://www.astro.gsu.edu/wds/orb6.html
- Second Catalog of Rectilinear Elements: http://www.astro.gsu.edu/wds/lin2.html
- Fourth Catalog of Interferometric Measurements of Binary Stars:
http://www.astro.gsu.edu/wds/int4.html
- The Third Photometric Magnitude Difference Catalog:
http://www.astro.gsu.edu/wds/dm3.html
- IAU Commission G1 (Binary and Multiple Stars) webpage:
http://www.astro.gsu.edu/wds/bsl/
- Double Star Astronomy at the U.S. Naval Observatory:
http://www.astro.gsu.edu/wds/ds_history.html


## 2. Growth of the WDS and Data Mining

The availability of large astrometric catalogs and the admirable acumen of users has led to the republishing of the same measures and identification of the same "new" systems by multiple data-miners. This has significantly increased the amount of work needed to properly incorporate these data into the USNO double star catalogs. Therefore, in the future, data mining results will be added to the Washington Double Star (WDS) and Washington Double Star Supplement
(WDSS) Catalogs at the discretion of the catalogers.
Furthermore, preference will be given to data prepared by those specifically associated with the original catalog project.

As can be seen in Figure 1, the WDS and the other catalogs we maintain are being added to at a prodigious rate. A great deal of this work is coming from data mining, most recently from Gaia (DR2). While this can be useful, it is always there to be mined and based on some private discussions it is possible that the best and final Gaia astrometric solution will not be producted until DR4 or later, so to avoid current data mining efforts being eventually superseded and replaced, data mining of Gaia results is not recommended at this time.

## 3. What Needs to be Done?

Observe. Actual observations cannot be replicated. The observations you make tonight cannot be made tomorrow night or next week. Due to the slow motion of many of the pairs in the WDS and WDSS, to first order, the claim is absurd: the motion of most known visual pairs are insignificant and well below the measurement error on consecutive nights.

However, it does get to the crux of the issue: your observations are a unique dataset which cannot be replicated.

As a result, lists have been generated of pairs which need to be observed. These lists include pairs which either are unconfirmed or pairs which have not been measured in many years ("many" set arbitrarily at 20 years). In the initial formulation two lists have been generated:

- https://ad.usno.navy.mil/wds/Webtextfiles/ neglected list1.txt: List 1: Unconfirmed or (date last) $>20$ yrs., $V_{a}<12$, No X or K code systems.
- https://ad.usno.navy.mil/wds/Webtextfiles/ neglected list2.txt: List 2: as above, but no magnitude restrictions.


## Catalog Access and New Lists of Neglected Doubles



Figure 1. Growth of the WDS. The solid blue line and dots indicates the number of mean positions in the WDS, indicated on the left margin, at certain key dates. Indicated are publication of the IDS (1961), the major WDS data releases (1984, 1996, 2001, 2006.5), more recent dates corresponding to IAU General Assemblies (2009.5, 2012.5, 2015.5, 2018.5) and now (2019.75). The dashed blue line and open circles indicates the number of systems on those same dates and is indicated on the right margin. The solid/dashed red lines and filled/ open red dotes indicates growth of the new WDS Supplement at inception and later (2017.5, 2018.0, 2019.41, 2019.75), on the same scales as the WDS plots.

The above lists are in WDS summary line format and are also available at the WDS mirror website at the weblinks below. These files will be automatically updated from the WDS as new observations and systems are added. The update of the lists will occur at least monthly, but may occur more often.

- http://www.astro.gsu.edu/wds/Webtextfiles/ neglected_list1.txt
- http://www.astro.gsu.edu/wds/Webtextfiles/ neglected_list2.txt

For these neglected pairs, even a non-detection can be useful if your observing capability is much greater than the parameters of the pair in question. For the neglected pairs where (date - last) is a very large number, the pair may be lost or miscataloged, and it may involve detective work or the perusal of old articles. This type of investigative work may be found especially appealing.

Good observing!

# CCD Astrometric Measurements and Historical Data Summary of Double Stars WDS 05548-2527 and WDS 00177+2630 

Ana Parra ${ }^{1}$, Alexander Beltzer-Sweeney ${ }^{1}$, Irena Stojimirovic ${ }^{1}$, Pat Boyce ${ }^{2}$, and Grady Boyce ${ }^{2}$<br>1. San Diego Mesa Community College<br>2. Boyce Research Initiatives and Education Foundation (BRIEF)


#### Abstract

We report CCD astrometric measurements of the double star system WDS 005482527 (B 92AB) and WDS 00177+2630 (BUP 5AB) using the iTelescope network. A position angle of $309.7^{\circ} \pm 0.03^{\circ}$ and an angular separation of $7.33^{\prime \prime} \pm 0.004 \prime$ " was determined for B 92 AB . A position angle of $200.7^{\circ} \pm 0.007^{\circ}$ and angular separation of $77.53^{\prime \prime} \pm 0.02^{\prime \prime}$ was determined for BUP 5 AB . Based on our new measurements and historic data on the systems we see no clear evidence that either system is binary.


## Introduction

Astrometry is a branch of astronomy that measures a celestial body's position in the sky and its movement. Through astrometry, we can track the movement of the secondary star relative to the primary star of a double star system by recording its position angle (theta), measured in degrees from the celestial north, and angular separation (rho) between the two stars in arcseconds. By fixing the position of the primary star, the secondary star's movement can be tracked based on the pair's relative position angle and angular separation over time. The linear separation between binary pairs determines the time needed to complete one orbit by the secondary star (third Kepler's law). Binary components in a pair with a large separation may exhibit a linear motion with respect to each other over a long period of time. In this case even if the stars are true binaries, we may not see the signature of elliptical orbits in a few hundred years of observing. Small separation between binary components may quickly yield curvature in the motion of the secondary component confirming their binary nature. Optical pairs can display a linear/flat trend or random motion over extended time. Additionally, a double star's properties, such as common proper motion, parallax, and spectral type, can be inspected to further investigate its binary or optical identity.

We selected two double star systems with unknown status: WDS 00548-2527 (B 92AB) and WDS $00177+2630$ (BUP 5AB). The pairs meet the following


Figure 1. SIMBAD's optical image of B 92AB.
criteria per the Washington Double Star (WDS) Catalog, accessible through Stelle Doppie: (a) observable during the fall in the northern and southern hemisphere by having a right ascension between 00 and 06 hours, (b) a magnitude difference ( $\Delta \mathrm{m}$ ) of 6 or less, (c) and an angular separation of $6^{\prime \prime}$ or more. We then (1) measured position angle and angular separation for each selected pair by the analysis of CCD images provided by the iTelescope network, and (2) investigated the double star's binary or optical properties by examining its historical data provided by The United States Naval Observatory (USNO), the WDS catalog, and SIMBAD, an astronomical database.

CCD Astrometric Measurements and Historical Data Summary of Double Stars WDS 05548-2527 ...

|  |  | Historical Measurements of B 92AB |  |  |
| :---: | :---: | :---: | :---: | :--- |
| Epoch | Position <br> Angle (deg) | Angular <br> Separation <br> (arcseconds) | Source | Measurement Type |
| 1911.01 | $314.1^{\circ}$ | $7.88^{\prime \prime}$ | WFC1998 | Pa - Photographic with an astrograph |
| 1926.00 | $310.2^{\circ}$ | $7.30^{\prime \prime}$ | B_1928b | Ma - Micrometer with refractor |
| 1965.44 | $310.0^{\circ}$ | $7.10 "$ | Knp1996a | Ma - Micrometer with refractor |
| 1965.67 | $309.8^{\circ}$ | $7.28^{\prime \prime}$ | B_1968 | Ma - Micrometer with refractor |
| 1999.02 | $309.2^{\circ}$ | $7.38^{\prime \prime}$ | TMA2003 | E2 - 2MASS Survey |
| 1999.14 | $309.2^{\circ}$ | $7.32^{\prime \prime}$ | UC_2013b | EU - UCAC Catalog |

Table 1. Historic measurements and data available on B 92AB; measurements are courtesy of the WDS catalog.

## B 92AB Historical Background

B 92AB, Figure 1, found in Lepus, was discovered by Willem Hendrik van den Bos, a Dutch-South African astronomer, using a micrometer and a 0.7 -meter refracting telescope in 1926. However, there was an earlier observation in 1911 not noted until 1998 by the Washington Fundamental Catalog (Wycoff, Mason, Urban 2006). Van den Bos recorded a position angle of $309.8^{\circ}$ and angular separation of $7.288^{\prime \prime}$. The most recent observation was by USNO CCD Astrograph Catalog (UCAC) in 1999 and recorded a position angle of $309.2^{\circ}$ and an angular separation of $7.322^{\prime \prime}$. There are currently seven observations of B 92AB in the WDS and it has been 18 years since its last observation. The historic observations for B 92 AB are summarized in Table 1 (Mason, Hartkopf 2015).

B 92AB's A component is a star similar to our Sun, with a spectral class $\mathrm{G} 1 / 2 \mathrm{~V}$ and a magnitude of 8.67 , and its B component has magnitude of 11.20 (Mason, Hartkopf 2015). Gaia DR2 reported the A component to have a proper motion of [-98.428-33.771] and a parallax 15.1665 ( $\pm 0.0353$ ) milli-arcseconds (Gaia 2018b). For the B component Gaia DR2 reported a proper motion of $[-96.864-23.442]$ and a parallax 16.0971 ( $\pm 0.1873$ ) milli-arcseconds (Gaia 2018b). The small angular separation of $7.3^{\prime \prime}$ may appear challenging to image, Figure 1, however its 2.53 difference in magnitude grants leverage to the resolution capabilities of the iTelescope network.

## BUP 5AB Historical Background

BUP 5AB, Figure 2, located in Andromeda, was discovered by Sherburne Wesley Burnham, an American astronomer, using a micrometer and a 1 -meter refractor telescope of the Yerkes observatory in 1910. There was an earlier observation in 1895; however, it was not noted until 1998 by the Washington Funda-
mental Catalogue (Wycoff, Mason, Urban 2006). Burnham recorded a position angle of $199.7^{\circ}$ and angular separation of $80.2^{\prime \prime}$ for BUP 5 AB . The most recent observation was by USNO CCD Astrograph Catalog (UCAC) in 1999 and recorded a position angle of $200.5^{\circ}$ and an angular separation of $77.90^{\prime \prime}$. There are currently seven observations of BUP 5AB in the WDS and it has been 16 years since its last observation. We summarize the historic observations for BUP 5AB in Table 2 (Mason, Hartkopf 2015).

BUP 5AB's A component has magnitude of 10.52 and was reported from Gaia DR2 to have a proper motion of $[+40.024-10.851]$ and a parallax 3.2695 ( $\pm 0.0570$ ) milli-arcseconds (Gaia 2018b). The B component has a 12.27 magnitude and was reported from Gaia DR2 to have a proper motion of [+33.598 $+16.084]$ and a parallax 4.5776 ( $\pm 0.0557$ ) milliarcseconds (Gaia 2018b). Having an $77.9^{\prime \prime}$ angular separation and 1.75 delta magnitude makes it a good candidate to observe without challenge to the iTelescope network. Different proper motions and parallaxes suggest an optical system.


Figure 2. SIMBAD's optical image of BUP $5 A B$.

CCD Astrometric Measurements and Historical Data Summary of Double Stars WDS 05548-2527 ...

| Historical Measurements of BUP 5AB |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Epoch | Position <br> Angle (deg) | Angular <br> Separation <br> (arcseconds) | Source | Measurement Type |
| 1895.76 | $199.2^{\circ}$ | $80.49^{\prime \prime}$ | WFC1998 | Pa - Photographic with an astrograph |
| 1897.77 | $199.6^{\circ}$ | $79.8^{\prime \prime}$ | WFC1998 | Pa - Photographic with an astrograph |
| 1907.78 | $199.6^{\circ}$ | $80.41^{\prime \prime}$ | WFC1998 | Pa - Photographic with an astrograph |
| 1910.56 | $199.7^{\circ}$ | $80.02^{\prime \prime}$ | Bu_1913 | Ma - Micrometer with refractor |
| 1991.47 | $200.5^{\circ}$ | $78.3^{\prime \prime}$ | TYC2002 | Ht - Tycho |
| 1997.80 | $200.5^{\circ}$ | $78.08^{\prime \prime}$ | TYC2003 | E2 - 2MASS Survey |
| 2001.58 | $200.5^{\circ}$ | $77.90^{\prime \prime}$ | UC_2013b | EU - UCAC Catalog |

Table 2. Historic measurements and data available on BUP 5AB; measurements are courtesy of the WDS catalog.

## Equipment

B 92 AB images were acquired by Telescope T32, Figure 3, located in Siding Springs, Australia at an elevation of 1,122 meters. The CCD camera for T32 is a FLI Proline 16803 with a resolution of 0.63 " per pixel housing an array 4096 by 4096 with a FOV of 43.2 by 43.2 arcminutes. The CCD camera is mounted on a Planewave 17" Corrected Dall-Kirkham (CDK) with a focal length of $2,912 \mathrm{~mm}$, an aperture of 431 mm , and a focal ratio of $\mathrm{f} / 6.8$.

BUP 5AB images were acquired by Telescope T11, Figure 4, located in Mayhill, New Mexico at an elevation of 2,225 meters. The CCD camera for T11 is a FLI Proline PL1102M with a resolution of 0.81 " per pixel housing an array 4008 by 2672 with a FOV of 36.2 by 54.3 arcminutes. The CCD camera is mounted on a Planewave 20" CDK with a focal length of $2,280 \mathrm{~mm}$, an aperture of 510 mm , and a focal ratio of $\mathrm{f} / 4.5$. All images were saved as FITS files.

## Methods and Procedures

Through the iTelescope network, we requested images of B 92 AB and BUP 5 AB at various exposure times and filters. The images B 92 AB were observed at an exposure times of 60 seconds and 90 seconds with the Blue, OIII, and Hydrogen-alpha filters, and 90 seconds for the Ionized Sulfur filters. The images of BUP 5 AB were captured at an exposure length of 45 seconds and 90 seconds with two filters: Luminance and Red.

The FITS files were individually uploaded to Astrometry.net for an astrometric calibration. The Right Ascension and Declination coordinates were calculated for the stars in the FITS images by comparing them to catalog images. Right Ascension and Declination grid of the image creates World Coordinate System (WCS) and was saved in the FITS file header. The down-


Figure 3. T32 17" PlaneWave f/6.8 CDK Astrograph with FLI Proline 16803 CCD in Siding Springs, Australia.


Figure 4. T11 20" PlaneWave f/4.58 CDK Astrograph with FLI Proline PL1102M CCD Mayhill, New Mexico.

CCD Astrometric Measurements and Historical Data Summary of Double Stars WDS 05548-2527 ...


Figure 5. B 92AB under Hydrogen Alpha Filter-60 seconds. Image was processed in Mira Pro x64.


Figure 6. BUP 5AB under Luminance Filter-45 seconds. Image was processed in Mira Pro x64
loadable calibrated images were transferred to Mira Pro x64, an image processing software developed by Mirametrics, Inc.

Each image was processed in Mira Pro x64 (Mira) to calculate position angle and angular separation. The WDS catalog and SIMBAD were referenced to verify the A and B component's location on the calibrated images. Mira's Vertical Transfer Function and Vertical Palette granted better star visibility. With Mira's Distance and Angle tool, a line was drawn to connect the primary and secondary star's centroid and astrometric measurements were generated.

## Results

Nine images of B 92 AB were acquired on November 22, 2017, Table 3, and a sample image (Hydrogen Alpha filter with 60 second exposure) is provided in Figure 5. Four images of BUP 5AB were acquired on November 11, 2017 by T11, Table 4. A sample image (Luminance filter with 45 second exposure) of BUP 5 AB is provided in Figure 6. Mira measurement of position angle and angular separation for each exposure and filter were exported to Microsoft Excel. Excel was used to calculate the mean, standard deviation, and

| WDS 00548-2527 |  | (B 92AB) Astrometry |
| :---: | :---: | :---: |
| Telescope T32 | Epochs 2017.89 |  |
| Filter Type-Exposure <br> Time (seconds) | Position <br> Angle <br> (degrees) | Angular <br> Separation <br> (arcseconds) |
| Red-60s | $309.58^{\circ}$ | $7.3^{\prime \prime}$ |
| Red-90s | $309.65^{\circ}$ | $7.33^{\prime \prime}$ |
| OIII-60s | $309.91^{\circ}$ | $7.31^{\prime \prime}$ |
| OIII-90s | $310.11^{\circ}$ | $7.4^{\prime \prime}$ |
| Hydrogen Alpha-60s | $309.08^{\circ}$ | $7.32^{\prime \prime}$ |
| Hydrogen Alpha-90s | $310.00^{\circ}$ | $7.33^{\prime \prime}$ |
| Blue-60s | $309.87^{\circ}$ | $7.32^{\prime \prime}$ |
| Blue-90s | $309.76^{\circ}$ | $7.35^{\prime \prime}$ |
| SII-90s | $309.66^{\circ}$ | $7.37^{\prime \prime}$ |
| Mean | $309.7^{\circ}$ | $7.33^{\prime \prime}$ |
| Std. Deviation | 0.3 | $0.04^{\prime \prime}$ |
| SEM | 0.03 | $0.004^{\prime \prime}$ |

Table 3. Position angle, angular separation and uncertainties for B $92 A B$.

| WDS 05548-2527 (BUP 5AB) Astrometry |  |  |
| :---: | :---: | :---: |
| Telescope T11 | Epochs 2017.87 |  |
| Filter Type-Exposure Time (seconds) | Position Angle (deg) | Angular Separation (arcsec) |
| Luminance-45s | $200.69^{\circ}$ | 77.47" |
| Luminance-90s | $200.68^{\circ}$ | 77.52" |
| Red-45s | $200.75^{\circ}$ | 77.53" |
| Red-90s | $200.72^{\circ}$ | 77.62" |
| Mean | $200.7^{\circ}$ | $77.53 "$ |
| Std. Deviation | 0.03 | 0.06" |
| SEM | $0.007^{\circ}$ | 0.02" |

Table 4. Position angle, angular separation, and uncertainties for BUP $5 A B$.
standard error of mean for position angle and angular separation. The results are summarized for B 92 AB in Table 3, and BUP 5AB in Table 4.

To determine if the pair's components had similar proper motion, we generated a Richard Harshaw (Harshaw 2014) rating, a classification system for common proper motion pairs (CPMs), by dividing the difference in the vectors by the sum of the vectors. The

# CCD Astrometric Measurements and Historical Data Summary of Double Stars WDS 05548-2527 ... 

| Harshaw Rating for B 92AB |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| WDS | COMP | PM Ra (mas) | PM Dec (mas) | Rating \% |  |  |
| $05548-2527$ | A | $-98.428 \pm 0.038$ | $-33.771 \pm .051$ |  |  |  |
|  | B | $-96.864 \pm 0.198$ | $-23.442 \pm .272$ | 0.05128 |  |  |

Table 5. Proper motion of B 92AB Components; measurements are courtesy of Gaia DR2.

| Harshaw Rating for BUP 5AB |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| WDS | COMP | PM Ra (mas) | PM Dec (mas) | Rating \% |  |  |
| $00177+2630$ | A | $+40.024 \pm 0.061$ | $-10.851 \quad \pm 0.062$ | 0.351773 |  |  |
|  | B | $+33.598 \pm 0.062$ | $+16.084 \pm .061$ |  |  |  |

Table 6. Proper motion of BUP 5AB Components; measurements are courtesy of Gaia DR2.
rating value range from 0 for CPMs and 1 for optical pairs. In the interest of obtaining accurate data with cited sources, the proper motion values for B 92 AB , Table 5, and BUP 5AB, Table 6, were obtained from Gaia DR2 rather than the WDS catalog.

## Discussion

## Is B 92AB a physical pair?

Historic trends for the angular separation, Figure 7, range from $7.1^{\prime \prime}$ to $8^{\prime \prime}$. In 1999, the 2MASS Survey in 1999 reported a $7.38^{\prime \prime}$ separation and UCAC reported a 7.32" separation (Mason, Hartkopf 2015) and our measurement is at $7.33^{\prime \prime}$ of separation. Within the limits of our camera resolution, these data agree indicating constant separation between components and not much motion, not just the last twenty years but possibly from the very first measurement in 1911.

Position angle vs. time, Figure 7 bottom panel, shows somewhat random behavior. The largest position angle is reported in the first measurement (1911) to be around 314 degrees and in the following measurement (1926) measured 310 degrees consistent with our measurements. Most historical observations of this pair were performed using a micrometer, which has lower precision than our current measurement or the 1999 measurements ( 2 MASS Survey and UCAC Catalog). Both 2MASS Survey and UCAC report the same measurements for the position angle of $309.2^{\circ}$, very close to our measurement is $309.7^{\circ}$. With the first historic observational point excluded, we would get flat line fit, indicating no change in position angle in the last hundred years.

The orbital plot for the B 92 AB system is shown in


Figure 7. Angular separation versus time (top) and position angle versus time of $B 92 A B$. The yellow diamond symbol represents our measurement.

Figure 8. In the top panel B 92 A is fixed in the origin $(0,0)$ position, and motion of the B component is shown by blue dots. As expected, based on position and angular separation vs time plots, not much motion is apparent. Most points cluster in the same spot, with first his-

## CCD Astrometric Measurements and Historical Data Summary of Double Stars WDS 05548-2527 ...

toric point slightly offset. In the bottom panel of Figure 8 , we zoom into the motion of the secondary component and each historic point is labeled. The trend displays somewhat random movement over time.

By analyzing B 92A's A parallax ( 15.1665 milliarcseconds) and its error ( $\pm 0.0353$ ), we calculated its minimum distance ( 214.45 light years), mid-point distance (214.95 light years), and maximum distance ( 215.45 light years) from our Sun. We analyzed component B's parallax ( 16.0971 milli-arcseconds) and its error ( $\pm 0.1873$ ) and calculated a minimum distance (200.19 light years), mid-point distance ( 202.52 light years), and maximum distance ( 204.91 light years) from our Sun. Even at three standard deviations from the mean measurements, the two stars would be almost four light years apart. A 0.051 Harshaw rating per Table 5 , a value closer to 0 than 1 is suggestive of a common proper motion pair. A large separation suggests that they may be a physical pair traveling in the same direction in space, Table 5.

## Is BUP 5AB a physical pair?

Using historical data and our new measurements, position angle and separation of the A and B components are plotted as a function of time, Figure 9. The data indicate clear trend in the decrease in the separation, Figure 9 top, and increase in position angle, Figure 9 top, since 1907.

In Figure 10 we show the orbital plot for the BUP 5 AB system. In the top panel, A is fixed in the origin $(0,0)$ position, and motion of the B component is shown by blue dots. This image doesn't reveal a long stretch of the secondary star's path relative to the primary star. Most data points are clustered close together. In the bottom panel of Figure 10, we zoom more into the motion of the secondary component, where we label each historic data point. The secondary star appears to be moving from southeast to northwest direction. Data points fluctuate around the line that connects first and last position and this may be attributed to the measurement errors. Overall the secondary star appears to be moving along the straight line with respect to the primary which is consistent with both long term binary and/or physically unrelated systems.

By analyzing BUP 5A's parallax ( 3.2695 milliarcseconds) and its error ( $\pm 0.057$ ), we calculated its minimum distance ( 980.01 light years), mid-point distance ( 997.09 light years), and maximum distance (1014.79 light years) from our Sun. We analyzed component B's parallax ( 4.5776 milli-arcseconds) and its error ( $\pm 0.0557$ ) and calculated a minimum distance (703.60 light years), mid-point distance ( 712.16 light years), and maximum distance (720.94 light years) from our Sun.


Figure 8. XY plot of the historical and new astrometric data of $B 92 A B$ with B 92A at $(0,0)$ (top). The difference between these two panels is the level of zoom applied. In the bottom panel B 92A is not present, but details of $B 92 B$ motion are more obvious.

A Harshaw rating of 0.352, a value settled between 0 and 1 , is suggestive of not being a common proper motion pair, Table 6. Our astrometric data in combination with historical data, its Harshaw rating and very wide distance between components essentially eliminates the possibility that these are a physical pair.

## Conclusion

B 92AB's separation and position angle did not change much in the last hundred years. In our astrometric data we see no indication of the gravitationally bound orbit of the secondary star B around the primary star A. However, with similar parallaxes and proper motion, they may be physical with orbital motion not observable in a few hundred years but possibly tens of

CCD Astrometric Measurements and Historical Data Summary of Double Stars WDS 05548-2527 ...


Figure 9. Angular separation vs time (top) and position angle versus time of BUP 5AB. The diamond symbol represents our measurement (bottom).
thousands of years from present day. Therefore, additional observational methods are encouraged to reveal the target's true nature.

BUP 5AB's 2017 measurements are close to its most recent WDS values from 2001 and combined with the historic data we see some consistent trends in change of position angle and separation. Common proper motion values are not suggestive of traveling in the same direction. This and parallax data from Gaia suggest these have no physical relationship.

## Acknowledgements

We thank Brian Mason from USNO for helping the Fall 2017 semester research teams and past teams obtain the historical data of our elected double star systems. Additionally, we thank Pat and Grady Boyce of the Boyce Research Initiative and Education Foundation (B.R.I.E.F) for their instructional support and financial donation that allowed us access to the iTelescope network and the software tools on the BARC server. This research has made use of the Washington Double Star catalog maintained by the USNO, Aladin Sky Atlas software developed at CDS, Strasbourg Observatory, France, and the SIMBAD database operated by CDS, Strasbourg Observatory, France. This work has made use of data from the European Space Agency (ESA) mission Gaia (https:www.cosmos.esa.int/


Figure 10. XY plot of the historical and new astrometric data of BUP $5 A B$ with BUP $5 A$ at $(0,0)$ (top). The difference between the two panels is the level of zoom applied. In the bottom panel BUP 5A is not present, but details of BUP 5B motion is more obvious.
gaia), processed by the Gaia Data Processing and Analysis Consortium (DPAC, https://www.cosmos.esa.int/ web/gaia/dpac/consortium). Funding for the DPAC has been provided by national institutions, in particular the institutions participating in the Gaia Multilateral Agreement. This research made use of data provided by Stelle Doppie and Astrometry.net.

## References

Gaia Collaboration, 2016, "The Gaia misson", Astronomy \& Astrophysics, 595, A1.

Gaia Collaboration, 2018b, "Gaia Data Release 2: Summary of the contents and survey properties", $A \& A$, 616, A1.

Genet, R., Johnson, J., Buchheim, R., and Harshaw, R, 2016, Small Telescope Astronomical Research Handbook. Ed. Collins, D.
Harshaw, R., 2014, "Another Statistical Tool for Evaluating Binary Stars", Journal for Double Star Observations, 10 (1), 32-51.
Mason, B. and Hartkopf, W., 2015, The Washington Double Star Catalog, Astrometry Department, U.S. Naval Observatory.
Skrutskie, M.F., et al., 2006, "The Two Micron All Sky Survey (2MASS)", The Astronomical Journal, 131, 1163-1183. http://adsabs.harvard.edu/ abs/2006AJ....131.1163S

Wycoff, G.L., Mason, B.D., and Urban S. E., 2006, "Data Mining for Double Stars in Astrometric Catalogs", The Astronomical Journal, 132, 50-60. http://adsabs.harvard.edu/abs/2006AJ....132...50W

# Double Star System WDS 02229+5835 BLL 7 (S Per) 

Olivia $\mathrm{Ho}^{1}$, Kieran Saucedo ${ }^{1}$, Alani Bayha ${ }^{1}$, Eliana Meza-Ehlert ${ }^{1}$, Shakara Tilghman ${ }^{1}$, Brian Delgado ${ }^{1}$, Pat Boyce ${ }^{2}$, and Grady Boyce ${ }^{2}$

1. High Tech High School
2. Boyce Research Initiatives and Education Foundation (BRIEF)


#### Abstract

Research regarding the Double Star System WDS 02229+5835 BLL 7 (S Per)] was conducted to contribute to the previous observations of the system, determine the nature of this system, and to further the science and data regarding double stars. Data was collected through careful observations of BLL 7, using Charge-Coupled Device cameras from the Las Cumbres Observatory. Researchers were able to find the current theta $20.34^{\circ}$ and rho $69.10^{\prime \prime}$, on epoch 2018.832. The collected measurements and data show that this system is most likely an optical double.


## Introduction

Referencing the Washington Double Star Catalog (WDS), students searched through lists of Double Star systems that met the following criteria: the right ascension (RA) of the systems were between 00 and 08 hours, the stars had a separation of at least $5^{\prime \prime}$ arcseconds, and magnitudes between 7 and 12. Data was requested for star systems that met these requirements, and the system WDS $02229+5835$ BLL 7 was picked from those candidates.

This star was chosen due to the uncertainty regarding the gravitational nature of BLL 7 based on research using GAIA data. Thus, this unknown, as well as the extensive historical value of the system, made it an intriguing system to study. It's stellar location with an RA of 02 h , and a declination (Dec) of about $+57^{\circ}$ made it visible during the time of year this study was conducted. The delta magnitude is 0.63 , with S Persei as the primary star having a magnitude of 10.76 and the secondary star having a magnitude of 11.39 . The separation angle (theta) from the last measurement taken in 2010 is about $69.3^{\circ}$, changing by $1.1^{\circ}$ from the initial measurement in 1880 of, $68.2^{\circ}$.

## Methods and Materials

The images were taken using a 0.4 -meter telescope with an SBIG CCD requested through Las Cumbres Observatory (LCO) Observing Portal (LCO Web).


Figure 1. SDSS r' Filtered Image of BLL 7 processed through the OSS Pipeline.

LCO is a conglomeration of twenty-one telescopes across eight global locations, tasked with keeping a constant eye on the night sky. An SBIG CCD camera was used to take the images of BLL 7, Figure 1, in order to enable precise measurements.

Thirty images in total were requested from LCO, 15 using the SDSS r' filter, isolating red wavelengths, with an exposure time of eight seconds, and 15 using

Double Star System WDS 02229+5835 BLL 7 (S Per)


Figure 2. Image of AIJ Measurement.
the SDSS g' filter, isolating green wavelengths, with an exposure time of 12 seconds. The images were all taken on BJD 2458422.7025 using the Teide 1 telescope in Tenerife.

The images were taken and initially processed through LCO, then through the OSS Pipeline, (Fitzgerald, 2018 accepted). After getting the files from the OSS Pipeline, AstroImageJ (AIJ) was used as astrometric software that provided further information on all 30 images. Starting from the center of the primary star, a line was drawn to the rough center of the secondary star. The position angle, separation in arcseconds, RA and Dec, were then calculated by AIJ, Figure 2. This process was repeated, with each image being measured with the same method.

## Data and Results

The data is exhibited in Tables $1-5$. Table 1 shows the average position angle (theta) of the 15 SDSS r' filtered images, the 15 SDSS g' filtered images, and the compiled data from all 30 of the images together. Table 2 shows the average length of separation in arcseconds (rho). Table 3 displays the historical data on this double star with this paper's 2018 measurement included for reference. Table 4 provides Effective Temperature and Luminosity Values reported by GAIA. Figure 3 provides Proper Motion vectors provided by ALADIN10 using GAIA data.

## Discussion

After an in-depth analysis, using new data brought forth by this study compared with previous measurements of this system, it was concluded that this system is most likely an optical double star system with no gravitational bond. Gravitationally bound double stars display similar parallax and proper motions, whereas these values for BLL 7 are significantly different, Table 4 , when comparing the minimum, middle, and maximum possible values of both stars parallaxes. This il-

| Filter | SDSS r' | SDSS g' | Across All <br> Filters / <br> Images |
| :---: | :---: | :---: | :---: |
| Mean | $20.32^{\circ}$ | $20.36^{\circ}$ | $20.34^{\circ}$ |
| Standard Deviation | 0.02 | 0.03 | 0.05 |
| Standard Deviation <br> of the Mean | 0.013 | 0.003 | 0.009 |

Table 1. Theta measurements

| Filter | SDSS r' | SDSS g' | Across All <br> Filters/ <br> Images |
| :---: | :---: | :---: | :---: |
| Mean | $69.08^{\prime \prime}$ | $69.12^{\prime \prime}$ | $69.10^{\prime \prime}$ |
| Standard Deviation | $0.06^{\prime \prime}$ | $0.06^{\prime \prime}$ | $0.06^{\prime \prime}$ |
| Standard Deviation <br> of the Mean | $0.015^{\prime \prime}$ | $0.015^{\prime \prime}$ | $0.011^{\prime \prime}$ |

Table 2. Rho Measurements

| Epoch | Position Angle | Separation |
| :---: | :---: | :---: |
| 1880 | $20^{\circ}$ | $68.2^{\prime \prime}$ |
| 1908.05 | $19.6^{\circ}$ | $69.193^{\prime \prime}$ |
| 1908.87 | $19.8^{\circ}$ | $69.04^{\prime \prime}$ |
| 1919.95 | $19.5^{\circ}$ | $69.008^{\prime \prime}$ |
| 1929.89 | $19.4^{\circ}$ | $69.583^{\prime \prime}$ |
| 1956.77 | $20.2^{\circ}$ | $67.206^{\prime \prime}$ |
| 1989 | $20.2^{\circ}$ | $68.751^{\prime \prime}$ |
| 1991.64 | $20.1^{\circ}$ | $69.08^{\prime \prime}$ |
| 1999.71 | $20.1^{\circ}$ | $69.16^{\prime \prime}$ |
| 2000.87 | $20.5^{\circ}$ | $69.42^{\prime \prime}$ |
| 2003.652 | $20.5^{\circ}$ | $69.3^{\prime \prime}$ |
| 2018.832 | $20.34 \pm$ | $.009^{\circ}$ |
| $69.10^{\prime \prime} \pm 0.011^{\prime \prime}$ |  |  |

Table 3. Historical Data.

| Star | Temperature <br> Effective Value | Luminosity Value |
| :---: | :---: | :---: |
| Primary | 3293 Kelvin | Not Reported |
| Secondary | 5899.50 Kelvin | 25.366 |

Table 4. Luminosity / Temperature

Double Star System WDS 02229+5835 BLL 7 (S Per)


Figure 3. Image of Aladin 10 Proper Motion Vectors.
lustrates that the smallest possible distance between these two stars is 1818.83 parsecs (calculated by taking the minimum parallax of the primary star, maximum parallax of the secondary star, and finding the difference between the two). Two stars with such a vast amount of (minimum) distance between them are most


Figure 4. Historical data star positions.

| Star | Right Ascension <br> (Proper Motion) | Declination <br> (Proper Motion) |
| :---: | :---: | :---: |
| Primary | $-0.010 \pm 0.295$ | $-2.570 \pm 0.307$ |
| Secondary | $-0.177 \pm .081$ | $-1.617 \pm 0.083$ |

Table 5. Proper Motion
likely not gravitationally bound-especially when other data regarding this system is taken into account.

Considering the proper motion measurements from GAIA under 3 mas/year, Table 5, the analysis indicates the probability that these stars have different proper motions as well. While there is a possibility that the RA motion of both stars could be equal, the GAIA data, in its current state, points to the possibility that there may not be an overlap between the Dec motion of both stars.

Viewing the historical measurements, Table 3, shows that from the first recorded measurement in 1880 to the most recent measurements in 2018, there is only a minor difference in the position angle ( $20^{\circ}$ to $20.34^{\circ}$ $\pm 0.009^{\circ}$ ) with common variations through measurement history (e.g. from 1908 to 1929 the position angle drops from $19.8^{\circ}$ to $19.4^{\circ}$, before increasing to $20.5^{\circ}$ in 2000). These variations can be attributed to variations in astronomers and the methods they employed. The separation shows a change in $1^{\prime \prime}$ between the first recorded measurement in 1880 to the most recent measurements in 2018 ( $68.2^{\prime \prime}$ to $69.10^{\prime \prime} \pm 0.011$ "), providing too little data to discern a distinguishable pattern. The totality of data points is plotted, Figure 4, showing no evi-


Figure 5. Historical data plotted with conducted observation marked in red.

## Double Star System WDS 02229+5835 BLL 7 (S Per)

dence of a trend line. When the 2018 data was plotted along with the historical data, shown in Figure 5, it appears evident that there has been little change over time.

The data garnered from the historical measurements as well as the recent measurement are confirmed with data from GAIA - showing that the parallax of the primary star and the parallax of the secondary star are different (putting the stars over 1818.83 parsecs apart), the proper motion of the primary star and the proper motion of the secondary star are different (the RA have a small overlap but the Dec are definitely different), the systems position angle is constantly changing (decreasing and increasing seemingly randomly), and the system's separation shows no discernable pattern (decreasing and increasing seemingly randomly)-this data helps to provide us with the conclusion that this double star system is not gravitationally bound or a common proper motion pair, but optical.

## Conclusion:

WDS $02229+5835$ BLL 7 is a double star system that was first measured in 1880 and most recently measured in 2018. Using CCD camera technology and the assistance of several heavily experienced mentors, this double star system was measured and researched. Considering this new data as well as historical data, it is unlikely these two stars are gravitationally bound given both stars' RA, Dec and proper motions.

## Acknowledgments

The authors of this paper would like to thank Dr. Michael Fitzgerald, Richard Harshaw, and Bob Buchheim for their contributions to the seminar classes provided by BRIEF. They would also like to thank the L.C.O. network for the time and resources they spent helping make the writing of this paper a possibility. Lastly, the authors of this paper would like to thank Brian Delgado, a physics teacher from High Tech High San Diego for introducing this scientific opportunity.

## References

Chapman, Allan, "Sir Robert Stawell Ball (1840-1913): Royal Astronomer in Ireland and Astronomy's Public Voice", Journal of Astronomical History and Heritage, 10, 198-210, 2007.
"About LCO." Parallax and Distance Measurement | Las Cumbres Observatory, http://www.lco.global/ about/.
"02229+5835 BLL 7 (S Per)." Stelle Doppie - Double Star Database, http://www.stelledoppie.it/ index2.php?iddoppia=8633.
Mason, B., Wycoff, G., and Hartkopf, W. The Washington Double Star Catalog. Astrometry Department, U.S. Naval Observatory.

# Double Star Photometry - March 2019 

Wilfried R.A. Knapp<br>Vienna, Austria<br>wilfried.knapp@gmail.com


#### Abstract

The WDS catalog contains per June 2019 about 148,500 objects. About 50,000 of these come with a magnitude for the primary with single digit precision indicating rather an estimation than a precise measurement and over 16,000 objects are listed with magnitudes in the blue or red band (WDS note codes $\mathrm{B} / \mathrm{K} / \mathrm{R} / \mathrm{I}$ ) thus in need of a measurement in the V band. After eliminating all objects not suited for resolution with the tools currently available to me (too small separation, too faint, too bright) about 26,000 objects remained as targets of interest for this project. The selection criterion for the objects for a specific report is then at a given point of time simply the currently highest given altitude to eliminate atmospheric effects as far as possible - so this is then a more or less random selection out of the mentioned 26,000 objects. This report covers the first 37 such objects from images taken end of March 2019 with V-filter to allow for visual magnitude measurement by differential photometry. All objects were additionally checked for potential gravitational relationship using GAIA DR2 parallaxes.


## Introduction

With few exceptions one single image was taken for all selected WDS objects with iTelescope iT24 with V-filter and 20 seconds exposure time and the imaging conditions were despite several due to bad weather cancelled sessions overall quite favourable. The number of objects in this report is somewhat smaller than planned as in several cases the secondary was too faint to be resolved with 20 seconds exposure time - useful lesson for the next imaging sessions to use for objects expected to be fainter than 15 Vmag longer exposure times.

The images were plate solved with Astrometrica using the URAT1 catalog with reference stars in the Vmag range of 8.5 to 16.5 giving not only RA/Dec coordinates but also photometry results for all reference stars used including an average Vmag error. The objects were then located in the center of the image and astrometry/photometry was then done by the rather comfortable Astrometrica procedure with point and click at the components delivering RA/Dec coordinates and Vmag measurements based on all reference stars used for plate solving.

## Results of Photometry and Catalog Checking

The measurement results are given in table 1 below with the following structure:

| WDS | $=$ WDS ID |
| ---: | :--- |
| Disc | $=$ Discoverer code |
| C | $=$ Components (AB if blank) |
| RA/Dec | $=$ Positions for primary and secondary in |
|  | HH:MM:SS.sss/DD.MM.SS.ss format |
| dRA/dDec | $=$ Plate solving errors for RA and Dec |
|  | in arcseconds |
| Sep | $=$ Calculated separation in arcseconds |
| eSep | $=$ Separation error |
| PA | $=$ Calculated position angle in degrees |
| e_PA | $=$ Position angle error |
| Mag | $=$ Vmags for both components measured |
|  | by differential photometry |
| e_Mag | $=$ Magnitude errors |
| SNR | $=$ Signal to noise ratio for both |
|  | components |
| dVmag | $=$ Plate solving error in Vmag |
| Date | $=$ Julian observation epoch |
| Notes | $=$ Additional comments listed below |
|  | Table 1 |

WDS = WDS ID
Disc = Discoverer code
$\mathrm{C} \quad=$ Components (AB if blank)
RA/Dec $=$ Positions for primary and secondary in HH:MM:SS.sss/DD.MM.SS.ss format

Sep $=\begin{gathered}\text { in arcseconds } \\ \text { Calculated separation in arcseconds }\end{gathered}$
e_Sep = Separation error
PA $=$ Calculated position angle in degrees
e_PA = Position angle error
Mag $=$ Vmags for both components measured by differential photometry
e_Mag = Magnitude errors
SNR = Signal to noise ratio for both components
dVmag $=$ Plate solving error in Vmag
Date $=$ Julian observation epoch
Notes $=$ Additional comments listed below Table 1

## Double Star Photometry - March 2019

| WDS | Disc | c | RA |  | Dec |  | dRA | dDec | Sep | e_Sep | PA | e_PA | Mag | e_Mag | SNR | dVmag | Date | Notes |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $11401+3758$ | CBL 349 |  | 114007.409 |  | 5743 | 43.20 | 0.08 | 0.08 | 18.92309 | 0.11314 | 197.269 | 0.343 | 12.240 | 0.061 | 93.34 | 0.06 | 2019.24395 | 1)3) |
|  |  |  | 114006.934 | 37 | 5725 | 25.13 |  |  |  |  |  |  | 16.086 | 0.112 | 10.97 |  |  |  |
| $11283+3144$ | CRB 89 |  | 112815.738 | 31 | 4346 | 46.94 | 0.05 | 0.05 | 25.30060 | 0.07071 | 12.405 | 0.160 | 11.356 | 0.050 | 186.80 | 0.05 | 2019.24104 | 1)3) |
|  |  |  | 112816.164 | 31 | 4411 | 11.65 |  |  |  |  |  |  | 16.243 | 0.098 | 12.41 |  |  |  |
| $11472+3812$ | CRB 90 |  | 114712.717 | 38 | 122 | 24.78 | 0.09 | 0.07 | 36.55991 | 0.11402 | 192.304 | 0.179 | 15.974 | 0.115 | 10.51 | 0.06 | 2019.24394 | 1) 3) |
|  |  |  | 114712.056 | 38 | 114 | 49.06 |  |  |  |  |  |  | 16.586 | 0.180 | 5.92 |  |  |  |
| $11289+3206$ | CVR 614 |  | 112853.718 | 32 | 062 | 21.27 | 0.04 | 0.04 | 28.12077 | 0.05657 | 130.734 | 0.115 | 15.850 | 0.086 | 13.87 | 0.04 | 2019.24110 | 1) 3) |
|  |  |  | 112855.395 | 32 | 060 | 02.92 |  |  |  |  |  |  | 16.614 | 0.297 | 3.21 |  |  |  |
| $11346+4052$ | ES 1401 |  | 113433.529 | 40 | 512 | 23.13 | 0.05 | 0.04 | 6.39762 | 0.06403 | 327.739 | 0.573 | 11.955 | 0.051 | 141.88 | 0.05 | 2019.24382 | 1) |
|  |  |  | 113433.228 | 40 | 512 | 28.54 |  |  |  |  |  |  | 12.538 | 0.051 | 105.44 |  |  |  |
| $11318+3513$ | FMR 101 |  | 113149.191 | 35 | 130 | 07.30 | 0.08 | 0.09 | 7.28388 | 0.12042 | 145.461 | 0.947 | 16.313 | 0.115 | 10.03 | 0.05 | 2019.24392 | 1)3) |
|  |  |  | 113149.528 | 35 | 130 | 01.30 |  |  |  |  |  |  | 16.342 | 0.220 | 4.58 |  |  |  |
| $11460+3149$ | GIC 102 |  | 114556.434 | 31 | 492 | 25.70 | 0.03 | 0.03 | 9.10242 | 0.04243 | 271.322 | 0.267 | 13.756 | 0.044 | 59.09 | 0.04 | 2019.24106 | 1) |
|  |  |  | 114555.720 | 31 | 492 | 25.91 |  |  |  |  |  |  | 14.092 | 0.046 | 45.65 |  |  |  |
| $11187+3759$ | GRV 833 |  | 111844.768 | 37 | 592 | 28.05 | 0.10 | 0.11 | 33.45107 | 0.14866 | 343.599 | 0.255 | 11.324 | 0.051 | 107.95 | 0.05 | 2019.24377 | 1) |
|  |  |  | 111843.969 | 38 | 000 | 00.14 |  |  |  |  |  |  | 11.785 | 0.051 | 91.84 |  |  |  |
| $11266+3946$ | GRV 835 |  | 112639.192 | 39 | 463 | 31.01 | 0.11 | 0.09 | 24.52560 | 0.14213 | 292.815 | 0.332 | 11.687 | 0.081 | 96.05 | 0.08 | 2019.24384 | 1) |
|  |  |  | 112637.231 | 39 | 464 | 40.52 |  |  |  |  |  |  | 11.836 | 0.081 | 81.94 |  |  |  |
| $11500+3612$ | GRV 842 |  | 115001.071 | 36 | 121 | 16.83 | 0.08 | 0.08 | 31.55400 | 0.11314 | 357.340 | 0.205 | 10.623 | 0.031 | 183.84 | 0.03 | 2019.24392 | 1) |
|  |  |  | 115000.950 | 36 | 124 | 48.35 |  |  |  |  |  |  | 10.775 | 0.031 | 166.02 |  |  |  |
| $11280+3403$ | HJ 498 |  | 112758.447 | 34 | 034 | 40.04 | 0.05 | 0.07 | 21.56549 | 0.08602 | 88.725 | 0.229 | 11.566 | 0.050 | 153.91 | 0.05 | 2019.24382 | 1) |
|  |  |  | 112800.182 | 34 | 034 | 40.52 |  |  |  |  |  |  | 12.280 | 0.051 | 112.27 |  |  |  |
| 11248+4128 | HJ 2570 | A | 112447.235 | 41 | 302 | 26.40 | 0.03 | 0.04 | 17.50494 | 0.05000 | 279.038 | 0.164 | 10.677 | 0.050 | 182.96 | 0.05 | 2019.24113 | 2) |
|  |  | B | 112445.696 | 41 | 302 | 29.15 |  |  |  |  |  |  | 13.869 | 0.056 | 43.62 |  |  |  |
| $11248+4128$ | HJ 2570 | A | 112447.235 | 41 | 302 | 26.40 | 0.03 | 0.04 | 46.08922 | 0.05000 | 210.916 | 0.062 | 10.677 | 0.050 | 182.96 | 0.05 | 2019.24113 | 2) |
|  |  | c | 112445.127 |  | 294 | 46.86 |  |  |  |  |  |  | 12.800 | 0.052 | 71.16 |  |  |  |
| 12006+3954 | HJ 2593 |  | 120038.682 |  | 545 | 57.64 | 0.07 | 0.07 | 24.77786 | 0.09899 | 333.840 | 0.229 | 10.657 | 0.071 | 127.90 | 0.07 | 2019.24118 | 1) |
|  |  |  | 120037.746 | 38 | 551 | 19.88 |  |  |  |  |  |  | 14.123 | 0.085 | 21.64 |  |  |  |
| 11150+3501 | KZA 12 | A | 111502.819 | 35 | 010 | 03.52 | 0.04 | 0.04 | 34.95900 | 0.05657 | 20.896 | 0.093 | 12.141 | 0.051 | 95.82 | 0.05 | 2019.24110 | 2) |
|  |  | B | 111503.834 |  | 013 | 36.18 |  |  |  |  |  |  | 14.661 | 0.065 | 25.98 |  |  |  |

## Double Star Photometry - March 2019

| WDS | Disc | c | RA |  | Dec | dRA | dDec | Sep | e_Sep | PA | e_PA | Mag | e_Mag | SNR | dVmag | Date | Notes |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 11152+3628 | KZA 13 |  | 111510.809 | 36 | 2811.13 | 0.09 | 0.07 | 22.06597 | 0.11402 | 280.444 | 0.296 | 13.130 | 0.093 | 45.33 | 0.09 | 2019.24117 | 1)3) |
|  |  |  | 111509.010 | 36 | 2815.13 |  |  |  |  |  |  | 15.163 | 0.125 | 12.10 |  |  |  |
| $11152+3521$ | KZA 15 | A | 111526.180 | 35 | 2008.84 | 0.07 | 0.05 | 18.83778 | 0.08602 | 138.004 | 0.262 | 13.820 | 0.058 | 36.86 | 0.05 | 2019.24114 | 1) |
|  |  | B | 111527.210 | 35 | 1954.84 |  |  |  |  |  |  | 14.212 | 0.063 | 28.09 |  |  |  |
| 11152+3521 | KZA 15 | A | 111526.180 | 35 | 2008.84 | 0.07 | 0.05 | 18.91869 | 0.08602 | 352.119 | 0.261 | 13.820 | 0.058 | 36.86 | 0.05 | 2019.24114 | 1) 3) |
|  |  | c | 111525.968 | 35 | 2027.58 |  |  |  |  |  |  | 15.086 | 0.082 | 16.32 |  |  |  |
| 12001+4107 | LDS5212 |  | 120011.131 | 41 | 0527.05 | 0.05 | 0.05 | 7.22402 | 0.07071 | 172.989 | 0.561 | 14.428 | 0.066 | 38.96 | 0.06 | 2019.24105 | 1)3) |
|  |  |  | 120011.209 | 41 | 0519.88 |  |  |  |  |  |  | 15.775 | 0.083 | 18.27 |  |  |  |
| 11512+3708 | NSN 50 |  | 115111.989 | 37 | 0749.24 | 0.13 | 0.08 | 4.20700 | 0.15264 | 10.979 | 2.078 | 11.871 | 0.050 | 212.14 | 0.05 | 2019.24391 | 1) 3) 4) |
|  |  |  | 115112.056 | 37 | 0753.37 |  |  |  |  |  |  | 16.458 | 0.228 | 4.39 |  |  |  |
| 11507+3312 | SKF 8 |  | 115042.735 | 33 | 1218.58 | 0.09 | 0.15 | 7.09669 | 0.17493 | 108.570 | 1.412 | 12.331 | 0.093 | 45.17 | 0.09 | 2019.24122 | 1) 5) |
|  |  |  | 1115043.271 |  | 1216.32 |  |  |  |  |  |  | 12.208 | 0.093 | 47.46 |  |  |  |
| $11478+3648$ | SKF2577 |  | 114747.326 | 36 | 4738.02 | 0.10 | 0.09 | 45.85906 | 0.13454 | 283.904 | 0.168 | 11.813 | 0.061 | 103.31 | 0.06 | 2019.24394 | 2) |
|  |  |  | 114743.620 | 36 | 4749.04 |  |  |  |  |  |  | 14.979 | 0.078 | 21.41 |  |  |  |
| 11554+3919 | SKF2579 |  | 115526.264 |  | 1919.62 | 0.14 | 0.12 | 6.51689 | 0.18439 | 342.059 | 1.621 | 12.519 | 0.072 | 64.04 | 0.07 | 2019.24380 | 2) |
|  |  |  | 115526.091 |  | 1925.82 |  |  |  |  |  |  | 12.731 | 0.073 | 56.02 |  |  |  |
| $12010+4058$ | SKF2641 |  | 120058.284 | 40 | 5732.21 | 0.07 | 0.12 | 5.36533 | 0.13892 | 65.793 | 1.483 | 14.027 | 0.054 | 51.98 | 0.05 | 2019.24104 | 2) |
|  |  |  | 120058.716 | 40 | 5734.41 |  |  |  |  |  |  | 14.997 | 0.064 | 26.80 |  |  |  |
| $11498+4024$ | SKF2692 |  | 114950.775 |  | 2414.47 | 0.11 | 0.09 | 77.68701 | 0.14213 | 27.390 | 0.105 | 10.233 | 0.060 | 186.54 | 0.06 | 2019.24379 | 2) 6) |
|  |  |  | 114953.904 | 40 | 2523.45 |  |  |  |  |  |  | 14.301 | 0.065 | 45.08 |  |  |  |
| 11527+3937 | SKF2693 |  | 115244.348 | 39 | 3638.05 | 0.12 | 0.13 | 12.38037 | 0.17692 | 4.014 | 0.819 | 12.800 | 0.091 | 85.35 | 0.09 | 2019.24383 | 2) 7) |
|  |  |  | 115244.423 | 39 | 3650.40 |  |  |  |  |  |  | 14.714 | 0.096 | 33.39 |  |  |  |
| $11141+3926$ | SLW 580 |  | 111403.465 |  | 2534.09 | 0.06 | 0.06 | 32.19528 | 0.08485 | 50.012 | 0.151 | 16.386 | 0.141 | 8.35 | 0.07 | 2019.24108 | 1)3) |
|  |  |  | 111405.594 |  | 2554.78 |  |  |  |  |  |  | 16.514 | 0.144 | 8.14 |  |  |  |
| 12069+3921 | SLW 684 |  | 120652.633 | 39 | 2058.10 | 0.03 | 0.04 | 8.28579 | 0.05000 | 19.717 | 0.346 | 15.918 | 0.092 | 15.10 | 0.06 | 2019.24101 | 1)3) |
|  |  |  | 120652.874 |  | 2105.90 |  |  |  |  |  |  | 15.945 | 0.093 | 14.88 |  |  |  |
| $11246+3752$ | UC 163 |  | 112435.262 | 37 | 5224.92 | 0.06 | 0.08 | 16.22781 | 0.10000 | 29.457 | 0.353 | 10.778 | 0.041 | 159.17 | 0.04 | 2019.24389 | 2) |
|  |  |  | 112435.936 | 37 | 5239.05 |  |  |  |  |  |  | 10.810 | 0.041 | 157.35 |  |  |  |
| $11390+3253$ | UC 163 |  | 113857.810 | 32 | 5308.64 | 0.07 | 0.10 | 34.16630 | 0.12207 | 270.989 | 0.205 | 12.549 | 0.061 | 93.95 | 0.06 | 2019.24376 | 2) |
|  |  |  | 113855.098 | 32 | 5309.23 |  |  |  |  |  |  | 13.717 | 0.018 | 59.36 |  |  |  |

## Double Star Photometry - March 2019

| WDS | Disc | c | RA |  | Dec | dRA | dDec | Sep | e_Sep | PA | e_PA | Mag | e_Mag | SNR | dVmag | Date | Notes |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 11281+3940 | UC 2144 |  | 112803.377 | 39 | 4021.25 | 0.05 | 0.09 | 25.96301 | 0.10296 | 92.252 | 0.227 | 10.713 | 0.091 | 91.22 | 0.09 | 2019.24386 | 2) |
|  |  |  | 112805.624 | 39 | 4020.23 |  |  |  |  |  |  | 13.404 | 0.093 | 43.78 |  |  |  |
| 11414+3624 | UC 2190 |  | 114123.277 | 36 | 2331.45 | 0.15 | 0.11 | 11.02058 | 0.18601 | 169.329 | 0.967 | 11.761 | 0.061 | 84.87 | 0.06 | 2019.24396 | 1) 3) |
|  |  |  | 114123.446 | 36 | 2320.62 |  |  |  |  |  |  | 14.610 | 0.091 | 15.26 |  |  |  |
| 11431+4040 | UC 2196 |  | 114304.505 | 40 | 4014.94 | 0.08 | 0.16 | 20.25504 | 0.17889 | 113.634 | 0.506 | 14.393 | 0.027 | 40.09 | 0.05 | 2019.24384 | 1) 3) |
|  |  |  | 114306.136 | 40 | 4006.82 |  |  |  |  |  |  | 16.722 | 0.162 | 6.55 |  |  |  |
| 11454+3856 | UC 2205 |  | 114525.135 | 38 | 5612.46 | 0.06 | 0.06 | 33.13616 | 0.08485 | 12.506 | 0.147 | 13.997 | 0.046 | 46.10 | 0.04 | 2019.24393 | 1) 3) |
|  |  |  | 114525.750 | 38 | 5644.81 |  |  |  |  |  |  | 15.818 | 0.083 | 14.45 |  |  |  |
| 12019+4045 | UC 2253 |  | 120155.069 | 40 | 4528.89 | 0.03 | 0.03 | 31.20611 | 0.04243 | 276.477 | 0.078 | 11.029 | 0.040 | 227.30 | 0.04 | 2019.24103 | 2) |
|  |  |  | 120152.340 | 40 | 4532.41 |  |  |  |  |  |  | 15.714 | 0.063 | 21.72 |  |  |  |
| $11150+3501$ | wno 35 |  | 111502.821 | 35 | 0103.50 | 0.06 | 0.06 | 31.24440 | 0.08485 | 165.728 | 0.156 | 12.156 | 0.051 | 103.61 | 0.05 | 2019.24112 | 1) 3) |
|  |  |  | 111503.448 | 35 | 0033.22 |  |  |  |  |  |  | 15.551 | 0.083 | 15.97 |  |  |  |

[^0]
## Double Star Photometry - March 2019

| Object | Comp | PA | e_PA | Sep | e_Sep | Plx1 | e_Plx1 | Plx2 | e_Plx2 | Min_D_AU | Med_D_AU | Max_D_AU | LPGR |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| CBL 349 |  | 196.402 | 0.000 | 18.55914 | 0.00008 | 3.7158 | 0.0457 | 3.6504 | 0.0596 | 4921 | 1086968 | 5804054 | 9.58 |
| CRB 89 |  | 12.503 | 0.000 | 25.25153 | 0.00013 | 3.3730 | 0.0755 | 3.1610 | 0.1033 | 7500 | 4089259 | 15619502 | 1.66 |
| CRB 90 |  | 193.898 | 0.000 | 36.29887 | 0.00019 | 1.8739 | 0.1395 | 1.9167 | 0.0605 | 18056 | 6085613 | 58942228 | 1.79 |
| CVR 614 |  | 130.743 | 0.000 | 27.91903 | 0.00010 | 2.4359 | 0.0491 | 2.3833 | 0.0884 | 11126 | 2756750 | 20310406 | 4.03 |
| ES 1401 |  | 326.757 | 0.001 | 6.40594 | 0.00010 | 7.0785 | 0.0571 | 7.0926 | 0.0676 | 897 | 248338 | 1715373 | 41.19 |
| FMR 101 |  | 143.896 | 0.001 | 7.38382 | 0.00011 | 8.1703 | 0.0774 | 8.3056 | 0.0869 | 892 | 419687 | 2039336 | 23.36 |
| GIC 102 |  | 271.738 | 0.001 | 9.07326 | 0.00016 | 31.9093 | 0.0473 | 31.8181 | 0.1199 | 284 | 22454 | 157008 | 100.00 |
| GRV 833 |  | 343.783 | 0.000 | 33.57743 | 0.00014 | 4.4512 | 0.0822 | 4.5259 | 0.0714 | 7288 | 941913 | 5823795 | 11.31 |
| GRV 835 |  | 293.900 | 0.000 | 24.42271 | 0.00007 | 3.6815 | 0.0411 | 3.6886 | 0.0398 | 6539 | 590579 | 3871893 | 18.12 |
| GRV 842 |  | 356.931 | 0.006 | 31.76644 | 0.00311 | 1.8447 | 0.8246 |  |  |  |  |  |  |
| HJ 498 |  | 88.363 | 0.000 | 21.41405 | 0.00011 | 3.5047 | 0.0627 | 4.3205 | 0.1057 | 3863988 | 11107893 | 18294413 | 0.00 |
| HJ 2570 | AB | 279.188 | 0.000 | 17.61598 | 0.00006 | 3.2274 | 0.0509 | 0.2019 | 0.0249 | 588626193 | 957060342 | 1978988952 | 0.00 |
| HJ 2570 | AC | 211.069 | 0.000 | 45.88431 | 0.00007 | 3.2274 | 0.0509 | 2.6416 | 0.0335 | 8275657 | 14165474 | 19917679 | 0.00 |
| HJ 2593 |  | 332.966 | 0.000 | 25.03633 | 0.00006 | 4.2369 | 0.0439 | 0.9070 | 0.0277 | 151231586 | 178708930 | 216037356 | 0.00 |
| KZA 12 | AB | 20.630 | 0.000 | 34.91873 | 0.00007 | 2.6797 | 0.0504 | 1.0246 | 0.0427 | 91648787 | 124296459 | 169923241 | 0.00 |
| KZA 12 | AC | 215.953 | 0.000 | 43.76426 | 0.00007 | 2.6797 | 0.0504 | 1.3338 | 0.0416 | 57267221 | 77652259 | 106844634 | 0.00 |
| KZA 13 |  | 280.573 | 0.000 | 21.96325 | 0.00009 | 1.3184 | 0.0521 | 1.1586 | 0.0531 | 17035 | 21543491 | 71113371 | 0.15 |
| KZA 15 | AB | 138.168 | 0.000 | 18.84457 | 0.00006 | 1.9902 | 0.0415 | 2.2549 | 0.0443 | 385958 | 12181925 | 25175618 | 0.00 |
| KZA 15 | AC | 352.488 | 0.000 | 19.02812 | 0.00007 | 1.9902 | 0.0415 | 1.2637 | 0.0591 | 31623109 | 59550076 | 109554970 | 0.00 |
| LDS5212 |  | 171.832 | 0.000 | 7.22360 | 0.00006 | 7.7240 | 0.0319 | 7.6662 | 0.0494 | 932 | 211771 | 1049347 | 47.37 |
| NSN 50 |  | 10.142 | 0.001 | 4.77175 | 0.00009 | 7.2536 | 0.0666 | 7.1711 | 0.0935 | 653 | 394037 | 2384315 | 26.69 |
| SLW 580 |  | 50.394 | 0.000 | 31.99172 | 0.00011 | 3.5263 | 0.0749 | 3.4440 | 0.0806 | 8784 | 1638829 | 10305174 | 6.34 |
| SLW 684 |  | 19.450 | 0.000 | 8.34125 | 0.00007 | 4.6031 | 0.0380 | 4.6768 | 0.0422 | 1768 | 715578 | 3044818 | 12.66 |
| UC 163 |  | 29.473 | 0.000 | 16.27874 | 0.00008 | 4.0915 | 0.0682 | 3.2223 | 0.0712 | 5292104 | 13597107 | 20472816 | 0.00 |
| UC 164 |  | 270.989 | 0.000 | 34.22015 | 0.00006 | 5.6205 | 0.0372 | 5.6331 | 0.0238 | 6037 | 201796 | 1318002 | 49.61 |
| UC 2144 |  | 92.512 | 0.000 | 26.05041 | 0.00006 | 7.4464 | 0.0349 | 7.4163 | 0.0354 | 3477 | 148840 | 904244 | 63.66 |
| UC 2190 |  | 170.715 | 0.000 | 11.34537 | 0.00006 | 5.6596 | 0.0530 | 5.5420 | 0.0380 | 2009 | 773474 | 2693529 | 7.82 |
| UC 2196 |  | 112.689 | 0.000 | 20.47500 | 0.00011 | 2.2147 | 0.1038 | 1.8961 | 0.0502 | 10544 | 15642446 | 35926491 | 0.06 |
| UC 2205 |  | 12.741 | 0.000 | 33.09966 | 0.00008 | 25.5666 | 0.0579 | 25.4984 | 0.0502 | 1292 | 23432 | 122543 | 100.00 |
| UC 2253 |  | 276.525 | 0.000 | 31.19689 | 0.00006 | 11.2609 | 0.0385 | 11.0959 | 0.0534 | 2781 | 272109 | 741915 | 25.39 |
| WNO 35 |  | 165.930 | 0.000 | 31.36745 | 0.00008 | 2.6797 | 0.0504 | 0.6600 | 0.0510 | 154553143 | 235644851 | 382942367 | 0.00 |

Table 2: Results for cross-matched objects
(Continued from page 17)

## Cross-Match with GAIA DR2

All listed objects were additionally cross-matched with GAIA DR2 to check for potential gravitational relationship (PGR) - the results are given in Table 2 with the following structure:

| Object | Discoverer ID |
| :---: | :---: |
| Comp = | Components (AB if blank) |
| PA = | Position angle in degrees |
| e_PA | Error position angle |
| Sep | Separation in arcseconds |
| e_Sep | Error separation |
| Plx1 | Parallax 1 in mas |
| e Plx1 = | Error parallax1 |
| Plx 2 | Parallax 2 in mas |
| e_Plx2 | Error parallax 2 |
| Min_D_AU= | Minimum spatial distance in AU between components (see Appen dix) |
| Med_D_AU = | Median spatial distance in AU between components (see Ap pendix) |
| Max_D_AU = | Maximum spatial distance in AU between components (see Appendix) |
| LPGR = | Likelihood of potential gravitational relationship (see Appendix) |

To avoid redundant reporting some objects were deleted in Table 2 if already cross-matched with GAIA DR2 in other reports. For the objects UC 2205, UC 2144 and GIC 102 WDS code "T" is suggested for likely physical by common parallaxes. For the objects with LPGR <10 WDS code "U" for likely optical is suggested.

## Summary

A good part of the 37 measured objects shows the expected magnitude difference larger than 0.5 compared with the WDS catalog data especially for the secondary but for many objects the given WDS magnitudes were simply confirmed within the given error range. 3 objects have parallaxes and angular separations allowing for a higher than $50 \%$ likelihood for a spatial distance between the components of less than $200,000 \mathrm{AU}(\sim 1$ parsec) suggesting potential gravitational relationship.

## Acknowledgements

The following tools and resources have been used for this research:

- Washington Double Star Catalog
- GAIA DR2 catalog
- DSS2 images
- Aladin Sky Atlas v10.0
- iTelescope
- iT24: 610 mm CDK with 3962 mm focal length. Resolution 0.625 arcsec/pixel. Vfilter. No transformation coefficients available. Located in Auberry, California. Elevation 1405m
- AAVSO VPhot
- Astrometrica v4.10.0.427
- URAT1 catalog
- AstroPlanner v2.2
- MaxIm DL6 v6.08


## References

Knapp, Wilfried R. A., 2018, "A New Concept for Counter-Checking of Assumed Binaries", Journal of Double Star Observations, 14 (3), 487-491.

## Appendix

## Description of the PGR assessment procedure

GAIA DR2 data for RA/Dec and Plx are used for a Monte Carlo simulation assuming a normal distribution for these parameters with the given error range as standard deviation. The spatial distance between the components is (according to Knapp 2018) calculated from the inverted simulated parallax data and the simulated angular separation using the law of cosine

$$
\text { sep }=\sqrt{a^{2}-2 a b \cos \gamma+b^{2}}
$$

with $a$ and $b=$ distance vectors for the stars A and B in lightyears calculated as $(1000 / \mathrm{Plx}) * 3.261631$ and $\gamma=$ angular separation in degrees calculated as

$$
\gamma=\arccos [\sin (D E 1) \sin (D E 2)+\cos (D E 1) \cos (D E 2) \cos (|R A 1-R A 2|)]
$$

The likelihood for potential gravitational relationship (LPGR) is the percentage of simulation results $<200,000 \mathrm{AU}$ ( $\sim 1$ parsec) out of the simulation sample with a size of 120,000

The given smallest, median and largest spatial distance between the components is the smallest, median and largest result out of the simulation sample.

# Double Star Photometry - April 2019 

Wilfried R.A. Knapp<br>Vienna, Austria<br>wilfried.knapp@gmail.com


#### Abstract

The WDS catalog contains per June 2019 about 148,500 objects. About 50,000 of these come with a magnitude for the primary with single digit precision indicating rather an estimation than a precise measurement and over 16,000 objects are listed with magnitudes in the blue or red band (WDS note codes $\mathrm{B} / \mathrm{K} / \mathrm{R} / \mathrm{I}$ ) thus in need of a measurement in the V band. After eliminating all objects not suited for resolution with the tools currently available to me (too small angular separation, too faint, too bright) about 26,000 objects remained as targets of interest for this project. The selection criterion for the objects for a specific report is then at a given point of time simply the currently highest given altitude to eliminate atmospheric effects as far as possible - so this is then a more or less random selection out of the mentioned 26,000 objects. This report covers 46 such objects from images taken end of April 2019 with V-filter to allow for visual magnitude measurement by differential photometry. All objects were additionally checked for potential gravitational relationship using GAIA DR2 parallaxes.


## 1. Introduction

One single image was taken for all selected WDS objects with iTelescope iT24 with V-filter and 20 seconds exposure time and the imaging conditions were overall quite favourable.

The images were plate solved with Astrometrica using the URAT1 catalog with reference stars in the Vmag range of 8.5 to 16.5 giving not only RA/Dec coordinates but also photometry results for all reference stars used including an average Vmag error. The objects were then located in the center of the image and astrometry/photometry was then done by the rather comfortable Astrometrica procedure with point and click at the components delivering RA/Dec coordinates and Vmag measurements based on all reference stars used for plate solving.

## 2. Results of Photometry and Catalog Checking

The measurement results are given in Table 1 with the following structure:

- WDS = WDS ID
- $\quad$ Disc $=$ Discoverer code
- $\mathrm{C}=$ Components ( AB if blank)
- $\quad$ RA/Dec $=$ Positions for primary and secondary in HH:MM:SS.sss/DD.MM.SS.ss format
- $\quad \mathrm{dRA} / \mathrm{dDec}=$ Plate solving errors for RA and Dec in arcseconds
- $\quad$ Sep $=$ Calculated separation in arcseconds
- e_Sep = Separation error
- $\quad \mathrm{PA}=$ Calculated position angle in degrees
- e_PA = Position angle error
- $\quad$ Mag = Vmags for both components measured by differential photometry
- e_Mag = Magnitude errors
- $\quad$ SNR = Signal to noise ratio for both components
- $\quad \mathrm{dVmag}=$ Plate solving error in Vmag
- Date $=$ Julian observation epoch
- Notes = Additional comments below Table 1


## 3. Cross-Match with Gaia DR2

All listed objects were additionally cross-matched with Gaia DR2 to check for potential gravitational relationship (PGR) - the results are given in Table 2 with

## Double Star Photometry - April 2019

| WDS | Disc | c | RA | Dec | dRA | dDec | Sep | Err Sep | PA | Err PA | Mag | Err Mag | SNR | dVmag | Date | Notes |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $12596+4502$ | BVD 226 |  | 125936.300 | 450153.01 | 0.07 | 0.13 | 19.11155 | 0.14765 | 308.548 | 0.443 | 12.526 | 0.061 | 128.10 | 0.06 | 2019.31764 | 1) |
|  |  |  | 125934.890 | 450204.92 |  |  |  |  |  |  | 14.537 | 0.064 | 47.65 |  |  |  |
| $12359+3758$ | CBL 596 |  | 123556.596 | 375802.32 | 0.05 | 0.09 | 8.17342 | 0.10296 | 178.342 | 0.722 | 15.469 | 0.053 | 30.61 | 0.04 | 2019.31492 | 1) 2) |
|  |  |  | 123556.616 | 375754.15 |  |  |  |  |  |  | 16.296 | 0.074 | 17.03 |  |  |  |
| $13064+3042$ | CRB 93 |  | 130622.262 | 304142.65 | 0.04 | 0.03 | 25.97269 | 0.05000 | 107.061 | 0.110 | 13.808 | 0.043 | 73.11 | 0.04 | 2019.31770 | 1) |
|  |  |  | 130624.187 | 304135.03 |  |  |  |  |  |  | 14.476 | 0.045 | 51.05 |  |  |  |
| $13005+2900$ | CVR 131 |  | 130030.056 | 285939.00 | 0.08 | 0.09 | 35.28610 | 0.12042 | 310.208 | 0.196 | 15.444 | 0.073 | 25.53 | 0.06 | 2019.31765 | 1) 2) |
|  |  |  | 130028.002 | 290001.78 |  |  |  |  |  |  | 16.202 | 0.088 | 16.42 |  |  |  |
| $13038+3210$ | CVR 132 |  | 130349.959 | 320959.77 | 0.06 | 0.05 | 9.72754 | 0.07810 | 231.917 | 0.460 | 15.847 | 0.076 | 22.64 | 0.06 | 2019.31769 | 1) 2) |
|  |  |  | 130349.356 | 320953.77 |  |  |  |  |  |  | 16.209 | 0.085 | 17.64 |  |  |  |
| $12467+3327$ | DAM 709 |  | 124643.687 | 332649.29 | 0.03 | 0.03 | 5.11083 | 0.04243 | 65.985 | 0.476 | 15.943 | 0.060 | 20.62 | 0.03 | 2019.31500 | 1) 2) |
|  |  |  | 124644.060 | 332651.37 |  |  |  |  |  |  | 16.424 | 0.082 | 13.69 |  |  |  |
| $12353+3634$ | ES 2166 |  | 123515.841 | $3633 \quad 30.68$ | 0.09 | 0.03 | 4.69447 | 0.09487 | 357.499 | 1.158 | 12.433 | 0.041 | 106.71 | 0.04 | 2019.31491 | 1) |
|  |  |  | 123515.824 | 363335.37 |  |  |  |  |  |  | 12.691 | 0.041 | 103.47 |  |  |  |
| $12397+3714$ | ES 2167 |  | 123947.064 | 371119.54 | 0.05 | 0.05 | 6.19500 | 0.07071 | 272.313 | 0.654 | 11.555 | 0.060 | 191.44 | 0.06 | 2019.31495 | 1) |
|  |  |  | 123946.546 | 371119.79 |  |  |  |  |  |  | 12.485 | 0.061 | 132.57 |  |  |  |
| $13065+3240$ | ES 2471 |  | 130632.721 | 324000.16 | 0.05 | 0.08 | 8.34188 | 0.09434 | 39.140 | 0.648 | 10.104 | 0.060 | 308.25 | 0.06 | 2019.31771 | 1) |
|  |  |  | 130633.138 | 324006.63 |  |  |  |  |  |  | 13.862 | 0.063 | 61.51 |  |  |  |
| 13097+3355 | GRV 864 |  | 130944.701 | 335609.37 | 0.08 | 0.07 | 24.61192 | 0.10630 | 214.389 | 0.247 | 11.060 | 0.060 | 215.37 | 0.06 | 2019.31773 | 1) |
|  |  |  | 130943.584 | 335549.06 |  |  |  |  |  |  | 11.096 | 0.060 | 217.67 |  |  |  |
| $12525+3155$ | HJ 524 |  | 125230.300 | 315517.71 | 0.04 | 0.08 | 21.78332 | 0.08944 | 111.943 | 0.235 | 10.224 | 0.050 | 279.44 | 0.05 | 2019.31761 | 1) |
|  |  |  | 125231.887 | 315509.57 |  |  |  |  |  |  | 12.299 | 0.051 | 136.39 |  |  |  |
| $12319+4034$ | HJ 2614 |  | 123149.680 | $40 \quad 3501.78$ | 0.05 | 0.05 | 25.14680 | 0.07071 | 243.882 | 0.161 | 11.346 | 0.050 | 214.31 | 0.05 | 2019.31489 | 1) |
|  |  |  | 123147.698 | $40 \quad 3450.71$ |  |  |  |  |  |  | 12.632 | 0.051 | 126.51 |  |  |  |
| $12468+4125$ | HJ 2620 |  | 124648.115 | 412502.82 | 0.05 | 0.05 | 14.36222 | 0.07071 | 293.035 | 0.282 | 13.065 | 0.071 | 105.05 | 0.07 | 2019.31500 | 1) |
|  |  |  | 124646.940 | 412508.44 |  |  |  |  |  |  | 13.858 | 0.072 | 71.05 |  |  |  |
| $12308+3640$ | J 1023 |  | 123046.190 | 363935.07 | 0.06 | 0.05 | 5.16415 | 0.07810 | 172.502 | 0.866 | 11.373 | 0.050 | 183.29 | 0.05 | 2019.31489 | 1) |
|  |  |  | 123046.246 | 363929.95 |  |  |  |  |  |  | 11.860 | 0.051 | 124.31 |  |  |  |
| $18292+1742$ | J 2912 |  | 182917.382 | 174151.55 | 0.12 | 0.13 | 6.26732 | 0.17692 | 142.168 | 1.617 | 11.202 | 0.120 | 143.64 | 0.12 | 2019.31505 | 1) |
|  |  |  | 182917.651 | 174146.60 |  |  |  |  |  |  | 13.738 | 0.124 | 35.87 |  |  |  |
| Table 1. Results for measured WDS objects |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

## Double Star Photometry - April 2019

| wDS | Disc | c | RA | Dec | dRA | dDec | Sep | Err Sep | PA | Err PA | Mag | Err Mag | SNR | dVmag | Date | Notes |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 13024+4308 | KPP 856 |  | 130224.798 | 430757.42 | 0.08 | 0.09 | 7.12698 | 0.12042 | 194.499 | 0.968 | 11.871 | 0.090 | 152.07 | 0.09 | 2019.31766 | 1) |
|  |  |  | 130224.635 | 430750.52 |  |  |  |  |  |  | 12.339 | 0.090 | 115.37 |  |  |  |
| $12548+4105$ | KPP1796 |  | 125446.001 | 410502.15 | 0.06 | 0.17 | 13.93698 | 0.18028 | 237.881 | 0.741 | 10.963 | 0.060 | 238.49 | 0.06 | 2019.31762 | 1) |
|  |  |  | 125444.957 | 410454.74 |  |  |  |  |  |  | 13.424 | 0.061 | 84.04 |  |  |  |
| $12359+3600$ | KPP2139 |  | 123551.410 | 360019.58 | 0.06 | 0.06 | 17.44376 | 0.08485 | 2.991 | 0.279 | 11.457 | 0.070 | 198.19 | 0.07 | 2019.31491 | 1) |
|  |  |  | 123551.485 | 360037.00 |  |  |  |  |  |  | 14.014 | 0.072 | 64.89 |  |  |  |
| $12283+3710$ | KZA 34 |  | $12 \quad 2813.920$ | 371001.17 | 0.07 | 0.10 | 6.46396 | 0.12207 | 244.233 | 1.082 | 12.750 | 0.051 | 100.24 | 0.05 | 2019.31487 | 1) |
|  |  |  | 122813.433 | 370958.36 |  |  |  |  |  |  | 12.541 | 0.051 | 118.46 |  |  |  |
| $12285+3722$ | KZA 35 |  | 122827.166 | 372202.85 | 0.05 | 0.06 | 32.69580 | 0.07810 | 27.279 | 0.137 | 10.307 | 0.060 | 302.40 | 0.06 | 2019.31487 | 1) |
|  |  |  | 122828.423 | 372231.91 |  |  |  |  |  |  | 13.847 | 0.062 | 71.43 |  |  |  |
| $12473+2959$ | LDS4268 |  | 124714.513 | 300019.29 | 0.08 | 0.07 | 6.27032 | 0.10630 | 55.960 | 0.971 | 11.553 | 0.100 | 137.66 | 0.10 | 2019.31501 | 1) |
|  |  |  | 124714.913 | 300022.80 |  |  |  |  |  |  | 11.880 | 0.100 | 119.69 |  |  |  |
| $12576+3514$ | LDS5764 |  | 125739.821 | 351327.47 | 0.07 | 0.08 | 16.03069 | 0.10630 | 226.375 | 0.380 | 10.607 | 0.060 | 279.64 | 0.06 | 2019.31764 | 1) |
|  |  |  | 125738.874 | 351316.41 |  |  |  |  |  |  | 13.270 | 0.061 | 94.18 |  |  |  |
| $12523+3836$ | NI 30 |  | $12 \quad 5216.017$ | $38 \quad 3542.37$ | 0.07 | 0.07 | 10.70876 | 0.09899 | 158.889 | 0.530 | 14.644 | 0.055 | 44.58 | 0.05 | 2019.31761 | 1) |
|  |  |  | $12 \quad 5216.346$ | 383532.38 |  |  |  |  |  |  | 14.463 | 0.054 | 51.57 |  |  |  |
| $12556+2933$ | SKF 490 |  | $12 \quad 5534.367$ | 293232.70 | 0.02 | 0.02 | 4.61002 | 0.02828 | 179.838 | 0.352 | 14.689 | 0.039 | 43.57 | 0.03 | 2019.31762 | 1) |
|  |  |  | 125534.368 | 293228.09 |  |  |  |  |  |  | 14.948 | 0.041 | 37.85 |  |  |  |
| 13016+2924 | SKF 491 |  | 130134.415 | 292348.38 | 0.02 | 0.03 | 8.92912 | 0.03606 | 143.959 | 0.231 | 13.481 | 0.032 | 86.95 | 0.03 | 2019.31765 | 1) |
|  |  |  | 130134.817 | 292341.16 |  |  |  |  |  |  | 14.457 | 0.036 | 54.61 |  |  |  |
| $12424+3653$ | SKF2593 |  | 124225.055 | 365307.37 | 0.02 | 0.03 | 20.95542 | 0.03606 | 2.822 | 0.099 | 12.382 | 0.031 | 149.00 | 0.03 | 2019.31499 | 1) |
|  |  |  | 124225.141 | 365328.30 |  |  |  |  |  |  | 13.956 | 0.034 | 70.51 |  |  |  |
| 12463+3341 | SKF2594 | A | 124620.924 | 334122.04 | 0.02 | 0.04 | 6.83159 | 0.04472 | 156.187 | 0.375 | 10.400 | 0.040 | 268.53 | 0.04 | 2019.31499 | 1) 2) |
|  |  | c | 124621.145 | 334115.79 |  |  |  |  |  |  | 17.355 | 0.300 | 3.17 |  |  |  |
| $12473+3811$ | SKF2596 |  | 124715.102 | 381106.18 | 0.02 | 0.02 | 6.60055 | 0.02828 | 99.593 | 0.246 | 12.324 | 0.061 | 129.97 | 0.06 | 2019.31503 | 1) |
|  |  |  | 124715.654 | 381105.08 |  |  |  |  |  |  | 14.632 | 0.065 | 40.95 |  |  |  |
| $12487+3437$ | SKF2598 |  | 124843.645 | $34 \quad 3723.30$ | 0.02 | 0.02 | 20.54103 | 0.02828 | 299.805 | 0.079 | 11.990 | 0.031 | 146.83 | 0.03 | 2019.31503 | 1) |
|  |  |  | 124842.201 | 343733.51 |  |  |  |  |  |  | 15.033 | 0.045 | 31.32 |  |  |  |
| $12382+4212$ | SKF2652 |  | $12 \quad 3809.287$ | 421212.14 | 0.03 | 0.04 | 125.84146 | 0.05000 | 125.961 | 0.023 | 9.823 | 0.040 | 300.96 | 0.04 | 2019.31494 | 1) |
|  |  |  | 123818.453 | 421058.25 |  |  |  |  |  |  | 11.483 | 0.040 | 214.61 |  |  |  |

## Double Star Photometry - April 2019

| WDS | Disc | c | RA | Dec | dRA | dDec | Sep | Err Sep | PA | Err PA | Mag | Err Mag | SNR | dVmag | Date | Notes |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $12568+4029$ | SKF2656 |  | 125649.092 | 402901.18 | 0.03 | 0.03 | 16.65733 | 0.04243 | 113.492 | 0.146 | 11.328 | 0.030 | 222.81 | 0.03 | 2019.31763 | 1) |
|  |  |  | 125650.431 | 402854.54 |  |  |  |  |  |  | 15.633 | 0.041 | 25.99 |  |  |  |
| $12320+3406$ | SKF2708 |  | 123200.310 | 340530.74 | 0.05 | 0.08 | 24.69309 | 0.09434 | 114.067 | 0.219 | 11.543 | 0.050 | 184.14 | 0.05 | 2019.31490 | 1) |
|  |  |  | 123202.125 | 340520.67 |  |  |  |  |  |  | 12.907 | 0.051 | 108.06 |  |  |  |
| $12365+3135$ | SKF2709 |  | 123631.365 | 313522.22 | 0.04 | 0.03 | 6.12574 | 0.05000 | 340.378 | 0.468 | 9.767 | 0.040 | 341.16 | 0.04 | 2019.31493 | 1) |
|  |  |  | 123631.204 | 313527.99 |  |  |  |  |  |  | 13.437 | 0.045 | 54.32 |  |  |  |
| 13029+2935 | SLE 908 |  | 130256.274 | 293457.31 | 0.06 | 0.06 | 8.93717 | 0.08485 | 234.104 | 0.544 | 13.803 | 0.015 | 70.28 | 0.03 | 2019.31768 | 1) |
|  |  |  | 130255.719 | 293452.07 |  |  |  |  |  |  | 14.800 | 0.039 | 41.92 |  |  |  |
| 13065+2819 | SLE 911 |  | 130628.633 | 281923.87 | 0.08 | 0.04 | 6.84299 | 0.08944 | 151.787 | 0.749 | 12.786 | 0.051 | 116.49 | 0.05 | 2019.31770 | 1) |
|  |  |  | 130628.878 | 281917.84 |  |  |  |  |  |  | 14.099 | 0.053 | 58.94 |  |  |  |
| $13076+2822$ | SLE 914 |  | 130733.373 | 282209.95 | 0.03 | 0.03 | 8.56133 | 0.04243 | 32.132 | 0.284 | 14.539 | 0.047 | 44.75 | 0.04 | 2019.31772 | 1) |
|  |  |  | 130733.718 | $28 \quad 2217.20$ |  |  |  |  |  |  | 15.081 | 0.054 | 29.47 |  |  |  |
| $13086+2920$ | SLE 915 |  | 130835.570 | 291944.45 | 0.07 | 0.07 | 21.98829 | 0.09899 | 151.976 | 0.258 | 12.261 | 0.061 | 121.70 | 0.06 | 2019.31773 | 1) |
|  |  |  | 130836.360 | 291925.04 |  |  |  |  |  |  | 13.967 | 0.063 | 55.01 |  |  |  |
| $12400+4102$ | SMH 17 |  | 124000.643 | 410131.02 | 0.04 | 0.07 | 12.05396 | 0.08062 | 210.274 | 0.383 | 13.057 | 0.051 | 98.97 | 0.05 | 2019.31497 | 1) |
|  |  |  | 124000.106 | 410120.61 |  |  |  |  |  |  | 13.186 | 0.051 | 95.53 |  |  |  |
| $12525+2910$ | SMH 18 |  | 125227.217 | 290931.01 | 0.06 | 0.05 | 13.49652 | 0.07810 | 186.856 | 0.332 | 14.083 | 0.062 | 63.82 | 0.06 | 2019.31761 | 1) |
|  |  |  | 125227.094 | 290917.61 |  |  |  |  |  |  | 15.908 | 0.075 | 24.04 |  |  |  |
| $12406+3444$ | UC 2383 |  | 124035.811 | 344420.79 | 0.04 | 0.05 | 11.14705 | 0.06403 | 103.859 | 0.329 | 15.539 | 0.072 | 27.27 | 0.06 | 2019.31497 | 1) |
|  |  |  | 124036.689 | 344418.12 |  |  |  |  |  |  | 15.771 | 0.073 | 25.25 |  |  |  |
| $12494+4023$ | UC 2409 |  | 124921.736 | 402233.87 | 0.09 | 0.08 | 17.13774 | 0.12042 | 184.704 | 0.403 | 15.417 | 0.082 | 24.46 | 0.07 | 2019.31504 | 1) |
|  |  |  | 124921.613 | 402216.79 |  |  |  |  |  |  | 15.363 | 0.080 | 27.22 |  |  |  |
| $13026+3523$ | UC 2459 |  | 130237.424 | 352306.93 | 0.07 | 0.07 | 15.40424 | 0.09899 | 96.710 | 0.368 | 15.631 | 0.083 | 24.29 | 0.07 | 2019.31766 | 1) 2) |
|  |  |  | 130238.675 | 352305.13 |  |  |  |  |  |  | 16.154 | 0.094 | 16.86 |  |  |  |
| $13032+3944$ | UC 2461 |  | 130309.992 | 394427.66 | 0.08 | 0.09 | 23.74498 | 0.12042 | 29.189 | 0.291 | 14.796 | 0.065 | 44.29 | 0.06 | 2019.31768 | 1) 2) |
|  |  |  | 130310.996 | 394448.39 |  |  |  |  |  |  | 16.202 | 0.082 | 19.09 |  |  |  |
| $13048+3355$ | UC 2472 |  | 130449.713 | 335520.61 | 0.13 | 0.08 | 8.64258 | 0.15264 | 71.377 | 1.012 | 16.387 | 0.087 | 16.81 | 0.06 | 2019.31769 | 1) 3) |
|  |  |  | 130450.371 | 335523.37 |  |  |  |  |  |  | 16.740 | 0.104 | 12.35 |  |  |  |
| $12411+4316$ | UR 7 |  | 124105.007 | 431530.66 | 0.04 | 0.04 | 5.07058 | 0.05657 | 359.136 | 0.639 | 12.227 | 0.051 | 144.78 | 0.05 | 2019.31498 | 1) |
|  |  |  | 124105.000 | 431535.73 |  |  |  |  |  |  | 12.931 | 0.051 | 105.35 |  |  |  |
| $12576+3650$ | WRS 3 |  | 125736.820 | 364929.43 | 0.05 | 0.07 | 14.85286 | 0.08602 | 343.081 | 0.332 | 11.655 | 0.080 | 184.41 | 0.08 |  |  |
|  |  |  | 125736.460 | 364943.64 |  |  |  |  |  |  | 12.446 | 0.080 | 136.24 |  |  |  |

Table 1 Notes
$\begin{array}{ll}\text { 1) } & \text { 1) iT24 } 1 \times 20 \mathrm{~s} \\ \text { 2) } & \text { 2) } S N R B<20 \\ \text { 3) } & \text { 3) } S N R A \text { and }\end{array}$

| 会， | $\begin{aligned} & \circ \\ & \hline 0 \\ & 0 . \\ & 0 \\ & 0 . \\ & \hline . \end{aligned}$ | $\begin{aligned} & \text { N } \\ & \infty \\ & \vdots \\ & \vdots \\ & \vdots \\ & \vdots \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\infty$ $\stackrel{\infty}{\circ}$ O $\stackrel{3}{0}$ 0 | $\begin{aligned} & 0 \\ & 0 \\ & \infty \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \infty \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \text { N} \\ & \tilde{N} \\ & 0 \\ & \vdots \\ & i \end{aligned}$ |  | $\begin{aligned} & \stackrel{\rightharpoonup}{\infty} \\ & \infty \\ & \underset{~}{0} \\ & \vdots \\ & 0 \end{aligned}$ | $\left.\begin{aligned} & \underset{\sim}{N} \\ & 0 \\ & \infty \\ & \infty \\ & 0 \\ & 0 \end{aligned} \right\rvert\,$ |  | $\begin{gathered} \sim \\ \sim \\ \sim \\ 0 \\ 0 \\ 0 \\ 0 \end{gathered}$ | $\begin{aligned} & \tilde{y} \\ & \underset{\sim}{2} \\ & \tilde{0} \\ & \vdots \end{aligned}$ |  | $\begin{array}{\|c\|} \substack{1 \\ \hat{0} \\ 0 \\ \infty \\ 0 \\ 0 \\ \hline} \end{array}$ |  | $\begin{aligned} & \infty \\ & \infty \\ & 0 \\ & 0 \\ & \infty \\ & 0 \\ & 0 \end{aligned}$ |  |  |  | $\begin{aligned} & \text { N } \\ & \tilde{\sim} \\ & \tilde{\sim} \\ & \dot{0} \end{aligned}$ |  | $\begin{aligned} & \infty \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{array}{\|l\|l} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}$ | $\begin{aligned} & \text { Z } \\ & \stackrel{1}{A} \\ & \vdots \\ & \vdots \\ & \vdots \end{aligned}$ | $\begin{aligned} & \underset{\sim}{\infty} \\ & \infty \\ & \infty \\ & \stackrel{\infty}{0} \end{aligned}$ | $\begin{aligned} & \text { r} \\ & \stackrel{O}{0} \\ & \vdots \end{aligned}$ |  |  | $\left\lvert\, \begin{gathered} \stackrel{n}{0} \\ 0 \\ 0 \\ \vdots \\ 0 \end{gathered}\right.$ | $$ | 0 - - $\vdots$ $\vdots$ 0 | $\begin{aligned} & \underset{\sim}{\infty} \\ & \underset{\infty}{\infty} \\ & \stackrel{\alpha}{0} \end{aligned}$ | $\begin{aligned} & \stackrel{0}{\infty} \\ & \infty \\ & \\ & \\ & \dot{0} \end{aligned}$ | $\begin{array}{l\|l} 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ \dot{0} \\ \hline \end{array}$ | $\begin{aligned} & y_{1} \\ & \vdots \\ & \vdots \\ & \vdots \\ & \vdots \end{aligned}$ |  | $\begin{gathered} n \\ \infty \\ \infty \\ 0 \\ 0 \\ \vdots \end{gathered}$ | $\begin{gathered} 0 \\ \stackrel{n}{n} \\ \infty \\ \infty \\ 0 \\ 0 \end{gathered}$ | $\dot{\circ}$ | N O O． 0 0 | $\begin{gathered} \text { N} \\ \text { ু } \\ \vdots \\ \vdots \\ \vdots \end{gathered}$ | N <br> $\stackrel{8}{8}$ <br> O <br> － | $\begin{aligned} & \infty \\ & \infty \\ & \vdots \\ & \vdots \\ & \vdots \\ & 0 \end{aligned}$ | $\begin{gathered} \stackrel{\sim}{N} \\ \underset{\sim}{N} \\ \dot{0} \end{gathered}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\left\|\begin{array}{l} n_{1} \\ \Sigma_{1} \end{array}\right\|$ |  | $\begin{aligned} & \text { N } \\ & \infty \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \text { N } \\ & 7 \\ & \vdots \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \infty \\ & 0 \\ & 0 \\ & \vdots \\ & \\ & 0 \\ & 0 \end{aligned}$ |  | $\begin{gathered} \text { オ} \\ \text { ס } \\ \vdots \\ \vdots \\ 0 \end{gathered}$ |  | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \text { O} \\ & \text { O} \\ & 0 \\ & 0 \\ & \vdots \\ & \vdots \end{aligned}$ | $\begin{aligned} & \underset{\sim}{2} \\ & \stackrel{\rightharpoonup}{\lambda} \\ & \infty \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \underset{O}{O} \\ & 0 \\ & 0 \\ & \vdots \\ & - \end{aligned}$ |  | $\begin{gathered} 0 \\ \underset{\sim}{0} \\ \underset{\sim}{\infty} \\ 0 \\ 0 \end{gathered}$ | $\underset{ }{7}$ $\underset{\sim}{3}$ $\infty$ 0. 0 0 | $\begin{gathered} \underset{\sim}{n} \\ \underset{\sim}{n} \\ \underset{\sim}{i} \end{gathered}$ | $$ | $\left\lvert\, \begin{gathered} \infty \\ \infty \\ \infty \\ \infty \\ \vdots \\ \vdots \end{gathered}\right.$ | $\begin{gathered} 0 \\ \tilde{n} \\ \vdots \\ 0 \\ \vdots \end{gathered}$ | $\begin{gathered} \infty \\ \infty \\ 0 \\ 0 \\ 0 \\ \vdots \\ i \end{gathered}$ | $\begin{aligned} & 2 \\ & \tilde{n} \\ & \underset{\sim}{n} \\ & \vdots \end{aligned}$ | $\left\lvert\, \begin{aligned} & \stackrel{\rightharpoonup}{\circ} \\ & \stackrel{0}{0} \\ & \vdots \\ & \vdots \\ & 0 \end{aligned}\right.$ |  | $\begin{aligned} & \infty \\ & \infty \\ & \infty \\ & \infty \\ & \vdots \\ & \vdots \\ & 0 \end{aligned}$ | $\begin{aligned} & \underset{\sim}{n} \\ & \\ & \vdots \\ & \vdots \\ & i \\ & \hline \end{aligned}$ | $\begin{aligned} & 0 \\ & \stackrel{0}{0} \\ & \underset{\sim}{n} \\ & \underset{i}{-} \end{aligned}$ | $\begin{aligned} & N \\ & \underset{\sim}{2} \\ & \vdots \\ & \underset{\sim}{2} \end{aligned}$ | $\begin{aligned} & \stackrel{n}{0} \\ & 0 \\ & 0 \\ & \\ & \vdots \end{aligned}$ |  |  |  | $\begin{aligned} & \infty \\ & \infty \\ & \infty \\ & \infty \\ & \infty \\ & 0 \\ & 0 \end{aligned}$ | $\begin{gathered} 0 \\ \\ \\ 0 \\ \underset{\sim}{2} \end{gathered}$ |  | $\begin{aligned} & \underset{\sim}{n} \\ & \underset{\sim}{\Omega} \\ & \infty \\ & 0 \\ & 0 \end{aligned}$ | $\stackrel{\underset{\sim}{\lambda}}{\stackrel{\rightharpoonup}{\lambda}}$ | $\begin{aligned} & \underset{\sim}{\tilde{n}} \\ & \underset{\sim}{\tilde{n}} \\ & \stackrel{0}{0} \end{aligned}$ | $\begin{aligned} & n \\ & \\ & \vdots \\ & \vdots \\ & \vdots \\ & \vdots \end{aligned}$ | $\begin{aligned} & \infty \\ & \vec{~} \\ & \vec{n} \\ & \stackrel{r}{-} \\ & - \end{aligned}$ | $\begin{aligned} & \underset{2}{2} \\ & \underset{\sim}{o} \\ & \overparen{O} \\ & \vdots \end{aligned}$ | $\exists$ |  | $\begin{aligned} & 0 \\ & \stackrel{0}{\circ} \\ & \stackrel{0}{0} \\ & \dot{0} \\ & \hline \end{aligned}$ | $\begin{aligned} & 2 \\ & 0 \\ & \vdots \\ & \vdots \\ & \vdots \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & \vdots \\ & \vdots \\ & 0 \\ & 0 \end{aligned}$ | $\begin{gathered} \sim_{0}^{\infty} \\ \vdots \\ \vdots \\ \vdots \\ \vdots \end{gathered}$ | ¢ |
| $\left\lvert\, \begin{gathered} \underset{\mathscr{C}}{\substack{a}} \mid \\ \hline \end{gathered}\right.$ |  | $\begin{aligned} & \infty \\ & \stackrel{\infty}{\infty} \\ & \dot{\infty} \end{aligned}$ | $\begin{aligned} & \underset{\sim}{\mathrm{N}} \\ & \underset{\sim}{\dot{N}} \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & \dot{\sim} \\ & \stackrel{2}{2} \end{aligned}$ | $\begin{aligned} & \stackrel{\sim}{\mathrm{N}} \\ & \stackrel{\circ}{\sim} \end{aligned}$ | $\begin{aligned} & \dot{\infty} \\ & \infty \\ & \dot{m} \end{aligned}$ | $\left\|\begin{array}{c} \vec{~} \\ \dot{n} \\ i \end{array}\right\|$ | $\begin{aligned} & \underset{\sim}{\circ} \\ & \underset{\sim}{\infty} \end{aligned}$ | $\begin{aligned} & \infty \\ & \stackrel{\infty}{0} \\ & \stackrel{1}{2} \end{aligned}$ |  | $\begin{aligned} & \circ \\ & \vdots \\ & \dot{\circ} \end{aligned}$ | $\begin{aligned} & \circ \\ & \vdots \\ & \vdots \end{aligned}$ | $\stackrel{\circ}{\circ}$ |  | $\stackrel{\circ}{\circ}$ | $\left\lvert\, \begin{aligned} & 0 \\ & \dot{0} \\ & i \end{aligned}\right.$ | $\begin{aligned} & \circ \\ & \vdots \\ & 0 \end{aligned}$ | $\begin{aligned} & \circ \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ |  | $\begin{aligned} & \circ \\ & \vdots \\ & 0 \end{aligned}$ | $\begin{gathered} \hat{o} \\ \dot{\sim} \\ \underset{\sim}{2} \end{gathered}$ |  | $\begin{aligned} & \text { A} \\ & \dot{A} \\ & \dot{\sigma} \end{aligned}$ | $\begin{aligned} & \circ \\ & \stackrel{0}{0} \\ & \hline \end{aligned}$ | $\begin{aligned} & n \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{gathered} \tilde{N} \\ \underset{\sim}{n} \end{gathered}$ | ! | $\begin{gathered} \underset{\sim}{\grave{n}} \\ \dot{N} \end{gathered}$ | $\begin{aligned} & \circ \\ & \vdots \\ & 0 \end{aligned}$ | $\begin{aligned} & 0 \\ & \dot{m} \\ & \dot{m} \end{aligned}$ |  | $\begin{aligned} & \circ \\ & \AA \\ & \dot{0} \end{aligned}$ | $\circ$ | $\begin{aligned} & \circ \\ & \vdots \\ & \vdots \end{aligned}$ | $\begin{aligned} & \dot{0} \\ & \dot{0} \\ & \dot{-} \end{aligned}$ | $\begin{aligned} & \circ \\ & \vdots \\ & \vdots \end{aligned}$ | $\begin{aligned} & \circ \\ & \vdots \\ & 0 \end{aligned}$ | $\begin{aligned} & \circ \\ & \vdots \\ & \dot{0} \end{aligned}$ | $\stackrel{\circ}{\circ}$ | $\begin{aligned} & \stackrel{\circ}{2} \\ & \stackrel{i}{2} \end{aligned}$ | $\begin{aligned} & \circ \\ & \vdots \\ & \vdots \end{aligned}$ | $\begin{aligned} & \stackrel{n}{\sim} \\ & \stackrel{n}{\sim} \\ & \hline \end{aligned}$ |  | $\begin{aligned} & \overrightarrow{7} \\ & \dot{a} \end{aligned}$ | $\begin{aligned} & \circ \\ & \vdots \\ & \vdots \end{aligned}$ | $\stackrel{\square}{\circ}$ |
| $\left\|\begin{array}{l} p_{1} \\ a_{1} \\ x_{1} \\ \stackrel{x}{2} \end{array}\right\|$ | $\begin{aligned} & \infty \\ & \infty \\ & \infty \\ & 0 \\ & 0 \\ & 0 \\ & \hline \end{aligned}$ | $\begin{gathered} \underset{\infty}{\infty} \\ \underset{\sim}{N} \\ \underset{\sim}{\infty} \\ \sim \end{gathered}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & \infty \\ & \infty \\ & \infty \\ & \underset{\sim}{\infty} \end{aligned}$ | $\begin{aligned} & \infty \\ & \underset{\sim}{\infty} \\ & \underset{\sim}{\alpha} \\ & \infty \\ & \infty \\ & \cdots \end{aligned}$ |  | $\begin{aligned} & \text { N } \\ & \cline { 1 - 1 } \\ & 0 \\ & \underset{\sim}{2} \\ & \text { on } \end{aligned}$ | $\begin{aligned} & \text { n } \\ & \vdots \\ & N \\ & \\ & \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & \vdots \\ & \sim \end{aligned}$ | $\left\lvert\, \begin{aligned} & 0 \\ & 0 \\ & \underset{\sim}{N} \\ & \omega \\ & \sim \end{aligned}\right.$ | $\begin{gathered} \underset{\sim}{N} \\ \underset{y}{4} \\ \underset{\infty}{\infty} \end{gathered}$ |  |  | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & \end{aligned}$ | $\begin{aligned} & \stackrel{0}{n} \\ & \stackrel{n}{\sim} \\ & \underset{\sim}{n} \\ & \end{aligned}$ | $\begin{gathered} 0 \\ 0 \\ n \\ \underset{\sim}{1} \\ \infty \\ \underset{\sim}{m} \\ \underset{m}{2} \end{gathered}$ | $\infty$ <br> 0 <br> 0 <br> 0 <br> 0 <br> 0 <br> $\cdots$ <br> $\cdots$ | $\left\lvert\, \begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & \infty \\ & 0 \\ & - \\ & - \\ & \hline \end{aligned}\right.$ | $\begin{aligned} & N \\ & \\ & \underset{\sim}{2} \\ & \\ & \end{aligned}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{\hat{N}} \\ & \stackrel{\rightharpoonup}{\circ} \\ & 0 \\ & 0 \\ & \underset{\sim}{\circ} \end{aligned}$ | $\left\lvert\, \begin{aligned} & \stackrel{\sim}{\sim} \\ & \underset{\sim}{N} \\ & \underset{\sim}{\sim} \\ & \underset{\sim}{\sim} \\ & \underset{\sim}{m} \end{aligned}\right.$ | $\begin{aligned} & \infty \\ & \infty \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & \sim \end{aligned}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{7} \\ & \overrightarrow{0} \\ & 0 \\ & m \end{aligned}$ | $\begin{aligned} & \underset{\sim}{\underset{~}{n}} \\ & \underset{\sim}{\underset{\sim}{2}} \end{aligned}$ | $\begin{aligned} & n \\ & \infty \\ & \sim \\ & \underset{\sim}{1} \\ & \vdots \\ & o \\ & o \\ & \\ & \end{aligned}$ |  | $\begin{aligned} & \text { dy } \\ & \text { N } \\ & \infty \\ & \infty \\ & \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | M $\underset{N}{0}$ $\underset{N}{N}$ N | $\begin{aligned} & \infty \\ & \underset{\sim}{N} \\ & \infty \\ & \underset{\sim}{n} \\ & \underset{\sim}{n} \\ & \underset{\sim}{2} \end{aligned}$ | $\begin{aligned} & 7 \\ & y \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \underset{\sim}{\lambda} \\ & \underset{~}{7} \\ & \underset{\sim}{n} \end{aligned}$ |  |  | H O O O m | $\begin{aligned} & \underset{\sim}{~} \\ & \underset{\sim}{2} \\ & 0 \\ & i \\ & \sim \\ & \sim \end{aligned}$ |  |  | $\begin{array}{\|c} \text { n } \\ 0 \\ \infty \\ 0 \\ \\ \end{array}$ | $\begin{aligned} & \stackrel{n}{n} \\ & \underset{\sim}{n} \\ & \stackrel{n}{n} \\ & 0 \\ & \\ & \underset{\gamma}{2} \end{aligned}$ | $\underset{\sim}{\sim}$ | $\begin{aligned} & \infty \\ & 0 \\ & 0 \\ & \infty \\ & \infty \\ & 0 \\ & \sigma \end{aligned}$ | $\begin{array}{\|l} \underset{\infty}{\infty} \\ \infty \\ \infty \\ \underset{\sim}{\infty} \\ \underset{\sim}{n} \end{array}$ | $\circ$ $\stackrel{\circ}{\circ}$ $\stackrel{\circ}{~}$ N | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & \infty \\ & \infty \\ & \infty \\ & 0 \end{aligned}$ | $\begin{aligned} & n \\ & \sim \\ & \\ & \\ & \underset{\sim}{m} \\ & \hline \end{aligned}$ | $\stackrel{\circ}{N}$ N $\stackrel{ }{\circ}$ N |
|  | $\left\lvert\, \begin{gathered} \underset{\sim}{N} \\ \underset{\sim}{N} \\ \end{gathered}\right.$ | $\begin{aligned} & \infty \\ & \infty \\ & \underset{\sim}{\lambda} \\ & \infty \\ & \infty \end{aligned}$ | $\begin{aligned} & \infty \\ & \infty \\ & \vdots \\ & \infty \\ & \underset{\sim}{7} \end{aligned}$ | $\begin{aligned} & \underset{ }{r} \\ & \infty \\ & \infty \\ & \infty \\ & \end{aligned}$ | $\begin{aligned} & \text { N } \\ & \underset{Z}{0} \\ & \underset{\sim}{i} \end{aligned}$ | $\begin{gathered} n \\ \stackrel{n}{2} \\ \stackrel{2}{\infty} \\ \underset{\sim}{o} \end{gathered}$ | $\left\lvert\, \begin{gathered} \infty \\ \underset{\sim}{1} \\ \underset{\sim}{\infty} \\ \underset{\sim}{0} \end{gathered}\right.$ | $\begin{aligned} & \infty \\ & 0 \\ & 0 \\ & \underset{\sim}{N} \\ & \hline \end{aligned}$ | $\begin{aligned} & \infty \\ & \infty \\ & \infty \\ & \underset{\sim}{\overleftarrow{ }} \end{aligned}$ | $\begin{aligned} & -1 \\ & \infty \\ & \underset{\sim}{N} \\ & \underset{\sim}{2} \end{aligned}$ |  | $\begin{aligned} & 0 \\ & \infty \\ & \infty \\ & \underset{\sim}{\infty} \\ & \underset{\sim}{0} \\ & \underset{\sim}{2} \end{aligned}$ | $\begin{aligned} & 0 \\ & \underset{N}{N} \\ & \underset{Z}{J} \\ & \underset{\sim}{Z} \end{aligned}$ | $\begin{aligned} & \infty \\ & \infty \\ & \stackrel{\infty}{1} \\ & \underset{y}{\prime} \\ & \hline \end{aligned}$ | $\left\lvert\, \begin{gathered} 0 \\ \stackrel{\rightharpoonup}{\circ} \\ 0 \\ \underset{\sim}{2} \\ \underset{\sim}{N} \end{gathered}\right.$ |  | $\left\lvert\, \begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & \infty \\ & \infty \end{aligned}\right.$ | $\begin{aligned} & \stackrel{n}{\Omega} \\ & 0 \\ & \vdots \\ & \\ & \end{aligned}$ | $\begin{aligned} & -1 \\ & 0 \\ & 0 \\ & 0 \\ & \underset{\sim}{0} \end{aligned}$ | $\begin{aligned} & \overrightarrow{1} \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & \hline \end{aligned}$ |  | $\begin{aligned} & \text { m } \\ & \underset{y}{3} \\ & \vdots \\ & \underset{\sim}{2} \end{aligned}$ | $\begin{aligned} & \mathrm{I} \\ & \text { İ } \\ & \text { Jु } \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & \underset{\sim}{1} \\ & 7 \\ & 7 \end{aligned}$ |  | $\left\lvert\, \begin{gathered} \text { n } \\ \infty \\ \infty \\ \\ \infty \end{gathered}\right.$ |  |  | $\begin{aligned} & \stackrel{n}{n} \\ & i \\ & \infty \\ & 0 \\ & \stackrel{1}{\circ} \\ & \stackrel{\rightharpoonup}{n} \end{aligned}$ | $\begin{aligned} & \text { on } \\ & \infty \\ & \infty \\ & \text { y } \\ & \text { N } \end{aligned}$ | $\begin{aligned} & -1 \\ & 0 \\ & \underset{\sim}{N} \\ & \underset{\sim}{2} \end{aligned}$ | $\left.\begin{aligned} & \infty \\ & 0 \\ & 0 \\ & e \\ & 0 \\ & 0 \\ & \end{aligned} \right\rvert\,$ | $\begin{aligned} & \underset{ }{2} \\ & \underset{\sim}{n} \\ & \underset{0}{2} \end{aligned}$ | -1 0 0 $\sim$ $\sim$ $\sim$ $\sim$ $\sim$ | $\begin{aligned} & \text { n } \\ & \\ & \\ & 0 \\ & i \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & \\ & \tilde{n} \end{aligned}$ |  | $\begin{aligned} & 0 \\ & 0 \\ & \vdots \\ & \vdots \\ & \vdots \\ & 0 \\ & 0 \\ & \end{aligned}$ | $\begin{aligned} & \underset{~}{7} \\ & \underset{1}{2} \\ & \underset{~}{7} \\ & \underset{子}{6} \end{aligned}$ | $\underset{i}{\circ}$ | $\begin{aligned} & \underset{\sim}{n} \\ & \underset{\sim}{N} \\ & \underset{\sim}{n} \end{aligned}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{0} \\ & \vdots \\ & \vdots \\ & \text { or } \end{aligned}$ | $\stackrel{n}{\stackrel{n}{0}}$ | $\left\lvert\, \begin{array}{l\|l} \circ \\ \infty \\ \infty \\ \vdots \\ \vdots \\ \infty \end{array}\right.$ | $\begin{aligned} & \underset{\sim}{N} \\ & \underset{\sim}{N} \\ & \underset{\sim}{\sim} \end{aligned}$ | n Ñ N N N |
| $\left\lvert\, \begin{aligned} & \stackrel{\rightharpoonup}{4}_{4} \\ & a_{1} \\ & a_{1}^{2} \\ & \stackrel{r}{2} \end{aligned}\right.$ | $\stackrel{\stackrel{n}{7}}{\stackrel{\rightharpoonup}{\mathrm{~F}}}$ | $\begin{gathered} \underset{\sim}{9} \\ \stackrel{3}{n} \\ \underset{1}{2} \end{gathered}$ | $\underset{\sim}{n}$ | $\begin{gathered} 0 \\ 0 \\ 0 \\ 0 \end{gathered}$ | $\underset{\sim}{n} \underset{\sim}{n}$ | $\begin{gathered} \underset{\sim}{\infty} \\ \underset{\sim}{\infty} \end{gathered}$ | $\underset{\sim}{\underset{\sim}{\mathrm{N}}}$ | $\left\lvert\, \begin{aligned} & \text { n } \\ & 0 \\ & 0 \\ & \hline \end{aligned}\right.$ | $\underset{\sim}{\underset{\sim}{\sim}}$ | $\begin{aligned} & \text { N } \\ & \text { O} \\ & \text { n } \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \text { 几n } \\ & \infty \\ & \infty \\ & \infty \\ & \infty \\ & \hline \\ & \hline \end{aligned}$ |  | $\stackrel{n}{\AA}$ |  | $\begin{gathered} \underset{\sim}{n} \\ \underset{m}{2} \end{gathered}$ | $\underset{\substack{\mathrm{N} \\ \\ \text { in }}}{ }$ | $\stackrel{\bullet}{\circ}$ | $\begin{aligned} & \overrightarrow{0} \\ & \stackrel{\rightharpoonup}{0} \\ & \hline \end{aligned}$ | $\begin{aligned} & \underset{\sim}{\sim} \\ & 0 \\ & \underset{\sim}{7} \\ & \underset{\infty}{\infty} \end{aligned}$ | $\left\lvert\, \begin{aligned} & n \\ & \underset{\sim}{\infty} \\ & \underset{\sim}{2} \end{aligned}\right.$ | $\begin{aligned} & \text { H} \\ & \text { on } \\ & \underset{\sim}{n} \end{aligned}$ | $\begin{aligned} & \dot{\infty} \\ & \infty \end{aligned}$ | $\begin{aligned} & \underset{\infty}{\infty} \\ & \infty \\ & \infty \end{aligned}$ | $\begin{aligned} & \stackrel{N}{n} \\ & 0 \\ & \end{aligned}$ | $\underset{\underset{\sim}{\mathrm{N}}}{ }$ | $\begin{aligned} & \underset{\sim}{0} \\ & \text { ভ̀ } \end{aligned}$ | $\underset{\sim}{\infty} \underset{\substack{\infty \\ \underset{\sim}{2} \\ \hline}}{ }$ | $\begin{aligned} & \text { N} \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & \hline \end{aligned}$ | $\underset{\sim}{\stackrel{n}{\mathrm{~N}}} \underset{\sim}{\underset{\sim}{n}}$ | $\begin{gathered} \sim \\ \\ \underset{\sim}{2} \end{gathered}$ | $\stackrel{6}{\underset{N}{N}}$ | $\left\lvert\, \begin{aligned} & \infty \\ & 0 \\ & 0 \end{aligned}\right.$ | $\begin{aligned} & -\underset{\infty}{\infty} \\ & \infty \\ & \underset{\sim}{7} \\ & \underset{\sim}{\sim} \end{aligned}$ | $\begin{aligned} & \stackrel{\bullet}{N} \\ & \underset{\sim}{\sim} \end{aligned}$ | $\begin{aligned} & \text { y } \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & \vdots \end{aligned}$ | $\begin{aligned} & 0 \\ & \underset{y}{0} \\ & \infty \\ & \infty \\ & \underset{\sim}{\underset{O}{0}} \end{aligned}$ |  | $\begin{aligned} & \text { N } \\ & 0 \\ & \vdots \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & \end{aligned}$ | $\begin{aligned} & \infty \\ & \stackrel{\infty}{\sim} \\ & \hline \end{aligned}$ | N <br> O <br> © <br> O <br> U | $\underset{\sim}{\underset{\sim}{N}}$ | $\begin{aligned} & \underset{\sim}{2} \\ & \underset{\sim}{2} \end{aligned}$ |  | $\begin{aligned} & 2 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & \hline \end{aligned}$ |  |
| $\left\|\begin{array}{c} \underset{\sim}{x} \\ a_{1} \\ 0_{1} \end{array}\right\|$ |  | $\begin{aligned} & \text { ron } \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $$ | $\begin{aligned} & \text { ñ } \\ & \text { gí } \\ & \dot{0} \end{aligned}$ | $\begin{aligned} & \infty \\ & \infty \\ & 0 \\ & \vdots \\ & \dot{0} \end{aligned}$ | $\begin{aligned} & \text { n } \\ & \vdots \\ & 0 \\ & \dot{0} \end{aligned}$ | $\begin{aligned} & \text { N } \\ & \tilde{0} \\ & \vdots \\ & \dot{0} \end{aligned}$ | $\begin{aligned} & \infty \\ & 0 \\ & \vdots \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \text { n } \\ & \underset{\sim}{0} \\ & \stackrel{0}{0} \end{aligned}$ | $\begin{gathered} \text { n } \\ \text { m } \\ \vdots \\ \dot{0} \end{gathered}$ | $\begin{gathered} \lambda \\ \underset{H}{3} \\ 0 \\ \dot{0} \end{gathered}$ | $\begin{gathered} n \\ \underset{\sim}{2} \\ \vdots \\ \dot{0} \end{gathered}$ | $\begin{aligned} & \underset{\sim}{\tilde{N}} \\ & \text { N} \\ & \dot{0} \end{aligned}$ | $\begin{gathered} \underset{0}{0} \\ \stackrel{y}{N} \\ \vdots \end{gathered}$ |  | $\left\lvert\, \begin{aligned} & \stackrel{\rightharpoonup}{0} \\ & \stackrel{0}{0} \\ & 0 \\ & \dot{0} \end{aligned}\right.$ | $\begin{gathered} i \\ \infty \\ 0 \\ \vdots \\ \dot{0} \end{gathered}$ | $\begin{aligned} & 0 \\ & \stackrel{\rightharpoonup}{1} \\ & \vdots \\ & \vdots \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { n } \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{gathered} \tilde{Z} \\ \tilde{N} \\ \dot{0} \end{gathered}$ | $\begin{aligned} & \underset{Z}{7} \\ & \underset{y}{0} \\ & \dot{0} \end{aligned}$ | $\begin{gathered} n \\ \underset{\sim}{n} \\ \underset{\sim}{\circ} \end{gathered}$ | $\begin{aligned} & 0 \\ & \underset{y}{1} \\ & \vdots \\ & \vdots \\ & \dot{0} \end{aligned}$ | $\left\lvert\, \begin{aligned} & \underset{\sim}{7} \\ & \stackrel{y}{\infty} \\ & \dot{0} \end{aligned}\right.$ | $\begin{array}{r} \vec{\infty} \\ \vdots \\ \vdots \\ 0 \\ 0 \end{array}$ | $\stackrel{\circ}{\stackrel{\rightharpoonup}{N}}$ | $\left\lvert\, \begin{aligned} & \overrightarrow{0} \\ & \stackrel{0}{0} \\ & \stackrel{0}{0} \end{aligned}\right.$ | $\left\lvert\, \begin{aligned} & \infty \\ & 0 \\ & 0 \\ & \vdots \\ & \dot{\circ} \end{aligned}\right.$ | $\begin{aligned} & 0 \\ & \stackrel{0}{2} \\ & \vdots \\ & \dot{0} \end{aligned}$ | $\left\lvert\, \begin{aligned} & 0 \\ & 0 \\ & \vdots \\ & \vdots \\ & \dot{0} \end{aligned}\right.$ | $\begin{aligned} & \grave{+} \\ & \infty \\ & \infty \\ & \vdots \\ & \dot{0} \end{aligned}$ |  | $\begin{aligned} & \underset{\sim}{2} \\ & \tilde{m} \\ & \dot{0} \end{aligned}$ | $\begin{aligned} & \text { ry } \\ & \text { N } \\ & \vdots \\ & \dot{0} \end{aligned}$ | $\begin{aligned} & \infty \\ & \underset{\sim}{0} \\ & \dot{0} \\ & \dot{0} \end{aligned}$ |  | $\begin{aligned} & \underset{\sim}{g} \\ & \underset{y}{c} \\ & \dot{\circ} \end{aligned}$ | $\begin{gathered} 0 \\ \infty \\ 0 \\ \vdots \\ \dot{0} \end{gathered}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & \dot{0} \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & \vdots \\ & \vdots \\ & 0 \\ & \hline \end{aligned}$ | $\begin{aligned} & 0 \\ & y_{1} \\ & 0 \\ & \vdots \\ & \dot{0} \end{aligned}$ | $\left\lvert\, \begin{aligned} & 0 \\ & \\ & 0 \\ & 0 \\ & \dot{0} \end{aligned}\right.$ | $\begin{aligned} & \infty \\ & \infty \\ & 0 \\ & \vdots \\ & \dot{\circ} \end{aligned}$ | $\left\lvert\, \begin{aligned} & \text { n } \\ & \vdots \\ & \vdots \\ & \dot{\circ} \\ & \hline \end{aligned}\right.$ | $\begin{gathered} 0 \\ \sim \\ \underset{\sim}{n} \\ \vdots \end{gathered}$ | n ¢ 0 0 0 |
| $\left\|\begin{array}{l} \underset{\sim}{x} \\ \frac{\alpha}{2} \end{array}\right\|$ | $\left\lvert\, \begin{aligned} & \stackrel{\rightharpoonup}{\gamma} \\ & \stackrel{y}{\circ} \\ & \stackrel{\rightharpoonup}{\circ} \end{aligned}\right.$ |  | $\begin{aligned} & 0 \\ & 0 \\ & \vdots \\ & \vdots \\ & \vdots \end{aligned}$ | $\begin{aligned} & \hat{r} \\ & \infty \\ & 0 \\ & 0 \\ & i \\ & \hline \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & \vdots \\ & \vdots \\ & \vdots \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & \infty \\ & \infty \\ & - \\ & \hline \end{aligned}$ | $\begin{aligned} & \vec{\infty} \\ & 0 \\ & \cdots \\ & \cdots \end{aligned}$ | $\begin{aligned} & n \\ & \\ & 0 \\ & \vdots \\ & \vdots \end{aligned}$ | $\begin{gathered} \tilde{m} \\ \stackrel{y}{\grave{n}} \\ \vdots \end{gathered}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{\lambda} \\ & \underset{\sim}{n} \\ & \stackrel{r}{2} \end{aligned}$ | $\begin{aligned} & \infty \\ & \infty \\ & 0 \\ & \vdots \\ & \vdots \\ & i \end{aligned}$ | $\begin{aligned} & \stackrel{\aleph}{\infty} \\ & \underset{\sim}{n} \\ & \stackrel{m}{m} \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & i \end{aligned}$ | $\left\lvert\, \begin{aligned} & 0 \\ & 0 \\ & 0 \\ & \vdots \\ & 0 \\ & 0 \end{aligned}\right.$ | $\left\|\begin{array}{l} \circ \\ \stackrel{0}{\infty} \\ \infty \\ \dot{N} \end{array}\right\|$ | $\begin{gathered} \stackrel{\sim}{\infty} \\ \underset{\sim}{\sim} \\ \underset{\sim}{n} \end{gathered}$ | $\begin{gathered} \stackrel{\sim}{N} \\ \underset{\sim}{2} \\ \dot{\sim} \end{gathered}$ | $\begin{aligned} & \stackrel{n}{0} \\ & \stackrel{\circ}{\circ} \\ & \stackrel{\rightharpoonup}{\sim} \\ & \dot{N} \end{aligned}$ | $\begin{aligned} & n \\ & \underset{\sim}{n} \\ & 0 \\ & \dot{\sim} \end{aligned}$ | $\begin{aligned} & \stackrel{n}{n} \\ & \underset{\sim}{\infty} \\ & \infty \\ & 0 \\ & \hline \end{aligned}$ | $\begin{aligned} & \underset{\sim}{\underset{N}{2}} \\ & \infty \\ & \underset{\sim}{\sim} \end{aligned}$ | $\begin{aligned} & \stackrel{\sim}{n} \\ & \underset{\sim}{4} \\ & \underset{\sim}{2} \\ & \underset{\sim}{2} \end{aligned}$ | $\begin{aligned} & \infty \\ & 0 \\ & \vdots \\ & \underset{\sim}{2} \\ & \stackrel{1}{\sim} \end{aligned}$ |  | $\left\lvert\, \begin{aligned} & n \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}\right.$ |  | $\begin{aligned} & \infty \\ & \stackrel{\infty}{\lambda} \\ & 0 \\ & - \\ & - \end{aligned}$ | $\left\lvert\, \begin{gathered} 0 \\ \infty \\ \text { oun } \\ \dot{\sim} \end{gathered}\right.$ | $\begin{aligned} & \tilde{1} \\ & \tilde{n} \\ & \vdots \\ & \vdots \end{aligned}$ |  | $\begin{aligned} & \underset{\sim}{n} \\ & \underset{\sim}{7} \\ & \underset{\infty}{2} \end{aligned}$ |  | $\underset{\substack{\stackrel{n}{4} \\ \underset{\sim}{\varkappa} \\ \dot{m} \\ \hline}}{ }$ | $\left\lvert\,\right.$ |  | $\left\lvert\, \begin{aligned} & \infty \\ & \infty \\ & \infty \\ & \infty \\ & 0 \\ & 0 \end{aligned}\right.$ | $\begin{aligned} & \overrightarrow{\tilde{n}} \\ & \stackrel{y}{0} \\ & \cdots \\ & - \end{aligned}$ |  | $\begin{aligned} & \underset{0}{4} \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\left\lvert\, \begin{aligned} & n \\ & \infty \\ & \infty \\ & \infty \\ & \vdots \end{aligned}\right.$ |  | $\left\lvert\, \begin{aligned} & \text { n } \\ & \infty \\ & \infty \\ & 0 \\ & \sim \end{aligned}\right.$ | $\begin{gathered} \underset{\sim}{n} \\ \underset{\sim}{n} \\ \underset{\sim}{2} \end{gathered}$ | $\left\lvert\, \begin{aligned} & \infty \\ & \stackrel{\infty}{n} \\ & \stackrel{0}{\infty} \\ & \dot{\sim} \end{aligned}\right.$ | $\begin{aligned} & \underset{N}{N} \\ & \underset{\sim}{0} \\ & \vdots \end{aligned}$ | N N ¢ m． |
| $\left\|\begin{array}{c} { }_{x}^{x} \\ a_{1} \\ 0_{1} \end{array}\right\|$ |  | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & \vdots \end{aligned}$ | $\begin{gathered} \infty \\ \infty \\ 0 \\ \vdots \\ \dot{0} \end{gathered}$ | $\begin{gathered} \infty \\ \text { m } \\ 0 \\ \vdots \\ \dot{0} \end{gathered}$ | $\begin{gathered} \stackrel{\circ}{2} \\ \tilde{m} \\ \vdots \\ \dot{0} \end{gathered}$ | $\begin{aligned} & \text { d } \\ & \text { y } \\ & \text { di } \\ & \dot{0} \end{aligned}$ | $\begin{aligned} & \infty \\ & \text { o } \\ & \text { yु } \\ & \dot{0} \end{aligned}$ | $\begin{aligned} & \text { J } \\ & \vdots \\ & \vdots \\ & \vdots \\ & 0 \end{aligned}$ | $\begin{gathered} \stackrel{0}{n} \\ \text { n} \\ \vdots \\ \dot{0} \end{gathered}$ | $\begin{array}{\|c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}$ | $\begin{aligned} & \stackrel{n}{\sim} \\ & \underset{\sim}{0} \\ & \dot{0} \end{aligned}$ | $\begin{aligned} & \infty \\ & \infty \\ & 0 \\ & \vdots \\ & \dot{0} \end{aligned}$ | $\begin{aligned} & \underset{\sim}{2} \\ & \text { m } \\ & \vdots \\ & \dot{0} \end{aligned}$ | $\begin{aligned} & \underset{\sim}{0} \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \vec{r} \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \stackrel{\sim}{\mathrm{N}} \\ & \stackrel{0}{\circ} \end{aligned}$ | $\begin{aligned} & \stackrel{r}{\infty} \\ & \infty \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{gathered} 0 \\ \vdots \\ \vdots \\ 0 \\ 0 \\ 0 \end{gathered}$ | $\begin{aligned} & \ddot{0} \\ & 0 \\ & 0 \\ & \vdots \\ & 0 \end{aligned}$ | $\begin{aligned} & \underset{\sim}{n} \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \text { m } \\ & \underset{y}{0} \\ & 0 \\ & 0 \end{aligned}$ | $\begin{gathered} n \\ \stackrel{n}{n} \\ \underset{0}{0} \end{gathered}$ | $\begin{aligned} & \underset{y}{y} \\ & \text { dín } \\ & \dot{0} \end{aligned}$ | $\begin{aligned} & \underset{\sim}{n} \\ & \underset{y}{2} \\ & \vdots \\ & \dot{0} \end{aligned}$ | $\left\lvert\, \begin{aligned} & 1 \\ & \stackrel{n}{5} \\ & \vdots \\ & 0 \end{aligned}\right.$ | $\begin{aligned} & \stackrel{n}{0} \\ & \stackrel{0}{\circ} \\ & \vdots \\ & \dot{0} \end{aligned}$ | $\begin{aligned} & \vec{m} \\ & \underset{y}{0} \\ & \dot{0} \end{aligned}$ | $\begin{gathered} n \\ n \\ 0 \\ 0 \\ 0 \end{gathered}$ | $\begin{gathered} 0 \\ \text { M } \\ \text { on } \\ 0 \end{gathered}$ | $\begin{aligned} & \text { m } \\ & 0 \\ & \vdots \\ & \vdots \\ & \dot{0} \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \infty \\ & \infty \\ & 0 \\ & 0 \\ & \dot{0} \end{aligned}$ | $\left\lvert\, \begin{aligned} & \tilde{y} \\ & \vdots \\ & \vdots \\ & \dot{0} \end{aligned}\right.$ | $\begin{aligned} & 0 \\ & \stackrel{0}{0} \\ & \text { N} \\ & \dot{0} \\ & \hline \end{aligned}$ | $\begin{aligned} & \underset{\sim}{2} \\ & \\ & 0 \\ & \dot{0} \end{aligned}$ | $\begin{array}{\|c} \stackrel{n}{n} \\ \tilde{n} \\ \vdots \\ \dot{\circ} \end{array}$ | $\begin{aligned} & \substack{o \\ \dot{N} \\ 0 \\ 0 \\ \dot{0} \\ \hline} \end{aligned}$ | $\begin{gathered} \text { rín } \\ \text { m } \\ \vdots \\ \dot{\circ} \end{gathered}$ | $\begin{gathered} \underset{N}{N} \\ \\ \vdots \\ 0 \end{gathered}$ | $\begin{aligned} & \text { n } \\ & \stackrel{0}{0} \\ & \dot{0} \end{aligned}$ | $\begin{aligned} & \dot{8} \\ & \underset{\sim}{2} \\ & \dot{\circ} \\ & \hline \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\text { on } \begin{gathered} \infty \\ \infty \\ \\ \dot{0} \end{gathered}$ | $\left\lvert\, \begin{aligned} & 0 \\ & \infty \\ & \vdots \\ & \vdots \\ & \dot{0} \\ & \hline \end{aligned}\right.$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & \dot{0} \end{aligned}$ | － |
| $\left\|\begin{array}{l} \vec{x} \\ \vec{a} \end{array}\right\|$ | $\left\lvert\, \begin{aligned} & \vec{H} \\ & \stackrel{y}{\circ} \\ & 0 \\ & \dot{0} \end{aligned}\right.$ | $\begin{aligned} & \infty \\ & 0 \\ & 0 \\ & \vdots \\ & \vdots \end{aligned}$ | $\begin{gathered} \tilde{n} \\ \underset{\sim}{n} \\ \underset{\infty}{n} \end{gathered}$ | $\begin{aligned} & \stackrel{i}{\sim} \\ & \underset{\sim}{0} \\ & \underset{\sim}{n} \\ & \dot{n} \end{aligned}$ | $\begin{aligned} & \underset{\sim}{2} \\ & \tilde{m} \\ & \dot{\sim} \end{aligned}$ | $\left\lvert\, \begin{aligned} & \stackrel{\infty}{\infty} \\ & 0 \\ & \infty \\ & \vdots \\ & - \end{aligned}\right.$ | $\begin{aligned} & \circ \\ & \stackrel{\circ}{\infty} \\ & \underset{\sim}{+} \\ & \end{aligned}$ | $\left\lvert\, \begin{aligned} & \text { N } \\ & \infty \\ & \infty \\ & \infty \\ & \dot{n} \end{aligned}\right.$ | $\begin{aligned} & \stackrel{9}{0} \\ & 0 \\ & 0 \\ & \vdots \\ & \dot{0} \end{aligned}$ |  | $\begin{aligned} & \underset{\sim}{\infty} \\ & \underset{\sim}{\circ} \\ & \stackrel{1}{2} \end{aligned}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{\alpha} \\ & \infty \\ & \underset{\sim}{\infty} \\ & \dot{\sim} \end{aligned}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{0} \\ & 0 \\ & 0 \\ & \dot{\sigma} \end{aligned}$ | $\begin{aligned} & 6 \\ & \stackrel{0}{0} \\ & \\ & 0 \end{aligned}$ | $\left\lvert\, \begin{aligned} & 0 \\ & 0 \\ & 0 \\ & \vdots \\ & \dot{0} \end{aligned}\right.$ | $\begin{aligned} & \stackrel{\sim}{N} \\ & \underset{\sim}{\sim} \end{aligned}$ | $\begin{aligned} & 0 \\ & \stackrel{\sim}{N} \\ & \underset{m}{m} \end{aligned}$ | $\begin{aligned} & n \\ & \underset{n}{n} \\ & \dot{\sim} \\ & \dot{\sim} \end{aligned}$ | $\begin{aligned} & \vec{i} \\ & \hat{n} \\ & 0 \\ & \dot{N} \end{aligned}$ | $\begin{aligned} & \underset{\circ}{\circ} \\ & \stackrel{y}{\circ} \\ & \underset{-}{2} \end{aligned}$ | $\begin{aligned} & \infty \\ & 0 \\ & \stackrel{0}{n} \\ & \underset{\sim}{n} \end{aligned}$ | $\begin{array}{\|l\|l} \hline \\ \underset{\sim}{2} \\ \omega \\ 0 \\ \dot{0} \\ \hline \end{array}$ | $\begin{aligned} & \underset{\sim}{n} \\ & \underset{\sim}{n} \\ & \dot{\sim} \\ & \end{aligned}$ |  | $\left\lvert\, \begin{gathered} \infty \\ \stackrel{\Omega}{\grave{n}} \\ \stackrel{n}{0} \\ \hline \end{gathered}\right.$ |  | -1 $\stackrel{\rightharpoonup}{1}$ $\vdots$ - - | $\left\lvert\, \begin{aligned} & N \\ & \tilde{y} \\ & \underset{\sim}{4} \\ & \hline \end{aligned}\right.$ |  |  | $\begin{gathered} \circ \\ \infty \\ \infty \\ 0 \\ \infty \\ \infty \end{gathered}$ | $\begin{aligned} & \underset{\sim}{\ddot{0}} \\ & \dot{0} \\ & \dot{\sim} \\ & \dot{N} \end{aligned}$ | $\begin{aligned} & \underset{m}{m} \\ & \underset{y}{2} \\ & \vdots \\ & \dot{m} \end{aligned}$ | $\left\lvert\, \begin{aligned} & \underset{\sim}{n} \\ & \underset{\sim}{2} \\ & \dot{\sim} \end{aligned}\right.$ | $\begin{gathered} \underset{\sim}{N} \\ \underset{\sim}{m} \\ \underset{\sim}{2} \end{gathered}$ | $\begin{gathered} n \\ \underset{\sim}{n} \\ \vdots \\ - \end{gathered}$ | $\begin{gathered} \underset{N}{N} \\ \underset{\sim}{0} \\ 0 \end{gathered}$ | $\begin{aligned} & \sim \\ & \infty \\ & \infty \\ & \cdots \\ & - \\ & - \end{aligned}$ | $\begin{gathered} \underset{A}{1} \\ \infty \\ \vdots \\ \vdots \end{gathered}$ | $\stackrel{r}{\because}$ | $\begin{aligned} & -r \\ & 0 \\ & 0 \\ & 0 \\ & \dot{0} \end{aligned}$ | $\underset{\sim}{\underset{\sim}{\sim}} \underset{\sim}{\underset{\sim}{2}}$ | $\begin{gathered} \infty \\ \underset{\sim}{0} \\ \stackrel{y}{m} \\ \stackrel{m}{2} \end{gathered}$ | $\left\lvert\, \begin{gathered} -\underset{\infty}{\infty} \\ \stackrel{n}{\sim} \\ \dot{\sigma} \end{gathered}\right.$ | $\begin{aligned} & \underset{\sim}{O} \\ & \underset{\sim}{1} \\ & \infty \end{aligned}$ | $\stackrel{\circ}{\text { ¢ }}$ |
| $\left\|\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}\right\|$ | $1 \begin{aligned} & 1 \\ & \hline 0 \\ & 0 \\ & \vdots \\ & \vdots \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \circ \\ & \vdots \\ & \vdots \\ & \vdots \\ & \vdots \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { n} \\ & 0 \\ & \vdots \\ & \vdots \\ & \vdots \\ & 0 \end{aligned}$ | $\begin{aligned} & \text { ঃ} \\ & \stackrel{\rightharpoonup}{\circ} \\ & \stackrel{0}{0} \end{aligned}$ | $\begin{aligned} & n \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \grave{2} \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \circ \\ & \circ \\ & \vdots \\ & \vdots \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \hat{r} \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \stackrel{n}{0} \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \text { ry } \\ & \stackrel{0}{8} \\ & \vdots \\ & \vdots \\ & \hline \end{aligned}$ |  | $\begin{aligned} & \text { n} \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \text { di } \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \tilde{m} \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \text { N} \\ & \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & \vdots \\ & \vdots \\ & 0 \end{aligned}$ | $\infty$ <br> - <br> - <br> $\vdots$ <br> 0 | $\circ$ $\stackrel{\circ}{\circ}$ $\vdots$ $\vdots$ 0 | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & \vdots \\ & 0 \end{aligned}$ | $\begin{aligned} & \text { I } \\ & \hline 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\circ$ <br> $\circ$ <br>  <br> $\vdots$ <br> $\vdots$ | － $\stackrel{\circ}{8}$ $\stackrel{\rightharpoonup}{\circ}$ 0 | $\stackrel{\circ}{\circ}$ | $\infty$ <br> $\stackrel{\infty}{\circ}$ <br> $\stackrel{ }{\circ}$ <br> $\stackrel{\rightharpoonup}{\circ}$ | 0 <br>  <br>  <br>  <br> $\vdots$ <br> $\vdots$ <br> 0 | $\infty$ <br> - <br> - <br> $\vdots$ <br> 0 |  | $\stackrel{\curvearrowleft}{\circ}$ | $\stackrel{\text { n }}{\circ}$ | $\stackrel{\circ}{\circ}$ | $\begin{aligned} & 1 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \mathrm{N} \\ & \stackrel{\rightharpoonup}{\circ} \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\stackrel{\curvearrowleft}{\circ}$ | 흥 | J <br>  <br> $\circ$ <br> $\circ$ | 흥 | $\begin{aligned} & \text { do } \\ & 0 \\ & 0 \\ & 0 \\ & \dot{\circ} \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { IO } \\ & \hline 0 \\ & \vdots \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \circ \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \infty \\ & 0 \\ & 0 \\ & 0 \\ & \vdots \\ & \dot{0} \end{aligned}$ | $\begin{aligned} & \text { Io } \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \text { r} \\ & \hline 0 \\ & 0 \\ & 0 \\ & 0 \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { İ } \\ & \hline 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\circ$ <br> - <br> - <br> $\circ$ <br> - | $\begin{aligned} & \tilde{N} \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | \＃ |
| － |  | $\begin{aligned} & m \\ & \vdots \\ & \vdots \\ & \vdots \\ & \infty \\ & \\ & \end{aligned}$ | $\begin{aligned} & \bullet \\ & 0 \\ & \infty \\ & \infty \\ & \vdots \\ & \dot{\sim} \\ & \dot{\sim} \end{aligned}$ | $\begin{aligned} & \mathrm{I} \\ & \vec{a} \\ & \vec{~} \\ & \dot{m} \\ & \text { n } \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & \infty \\ & \infty \\ & \infty \\ & 0 \\ & \hline \end{aligned}$ | $\begin{aligned} & \infty \\ & \overrightarrow{0} \\ & \overrightarrow{0} \\ & 0 \\ & \cdots \\ & \dot{\omega} \end{aligned}$ | $\begin{gathered} \underset{\infty}{\underset{\infty}{\infty}} \\ \underset{\sim}{\sim} \\ \dot{\sim} \end{gathered}$ | $\begin{aligned} & \tilde{\sim} \\ & 0 \\ & 0 \\ & \vdots \\ & \vdots \end{aligned}$ | $\begin{aligned} & \text { No } \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & \infty \end{aligned}$ |  |  |  | $\begin{array}{\|c} \underset{\sim}{y} \\ \underset{\sim}{\sim} \\ \underset{\sim}{\sim} \\ \underset{\sim}{2} \end{array}$ | $\begin{aligned} & \text { N } \\ & \text { o } \\ & \infty \\ & \cdots \\ & \dot{\sim} \end{aligned}$ | $\begin{aligned} & \underset{N}{N} \\ & \underset{\sim}{2} \\ & \underset{\sim}{0} \\ & \dot{0} \end{aligned}$ | $\begin{aligned} & \underset{~}{~} \\ & \underset{y}{c} \\ & \underset{\sim}{c} \end{aligned}$ | $\begin{aligned} & \underset{n}{n} \\ & \tilde{n} \\ & \underset{\sim}{n} \\ & \underset{\sim}{m} \end{aligned}$ |  | $\begin{aligned} & n \\ & \tilde{n} \\ & \underset{\sim}{2} \\ & \underset{\sim}{0} \end{aligned}$ | $\begin{aligned} & \text { N } \\ & \infty \\ & \infty \\ & 0 \\ & \dot{\sim} \\ & \dot{\sim} \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \underset{0}{0} \\ & 0 \\ & 0 \\ & 0 \\ & \vdots \\ & \vdots \\ & \vdots \end{aligned}$ | $\begin{aligned} & 0 \\ & \stackrel{\rightharpoonup}{0} \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\left\lvert\, \begin{aligned} & \stackrel{\rightharpoonup}{\infty} \\ & \infty \\ & \vdots \\ & \vdots \\ & \dot{\sigma} \end{aligned}\right.$ | $\begin{aligned} & \stackrel{\rightharpoonup}{I} \\ & \underset{\sim}{I} \\ & \underset{\infty}{\infty} \end{aligned}$ | $\begin{aligned} & \dot{2} \\ & 0 \\ & 0 \\ & 0 \\ & \dot{0} \\ & \dot{\sim} \end{aligned}$ | $\begin{aligned} & \ddot{0} \\ & 0 \\ & 0 \\ & \infty \\ & \infty \\ & 0 \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & \tilde{N} \\ & 0 \\ & 0 \\ & \dot{0} \\ & \hline \end{aligned}$ |  | $\left\lvert\, \begin{aligned} & 0 \\ & \stackrel{\rightharpoonup}{n} \\ & n \\ & 0 \\ & n \\ & n \\ & n \end{aligned}\right.$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & \dot{0} \\ & \dot{-} \end{aligned}$ |  | $\begin{aligned} & \stackrel{\rightharpoonup}{0} \\ & \underset{n}{n} \\ & \underset{6}{2} \end{aligned}$ |  | $\begin{aligned} & \stackrel{\circ}{\circ} \\ & \stackrel{\circ}{7} \\ & \stackrel{\rightharpoonup}{i} \\ & \stackrel{2}{2} \end{aligned}$ | $\begin{aligned} & m \\ & \underset{\sim}{\infty} \\ & \infty \\ & \infty \\ & \infty \\ & \infty \end{aligned}$ | $\begin{aligned} & n \\ & \vdots \\ & \stackrel{n}{0} \\ & \stackrel{2}{n} \\ & \underset{\sim}{c} \end{aligned}$ |  | $\begin{aligned} & 7 \\ & 0 \\ & 0 \\ & \vdots \\ & 0 \\ & \\ & \end{aligned}$ |  |  |  | $\begin{aligned} & \tilde{m} \\ & \tilde{\sim} \\ & \vdots \\ & \dot{\sim} \\ & \dot{\sim} \end{aligned}$ | $\underset{\sim}{\underset{\sim}{n}} \underset{\sim}{\sim}$ | $\begin{aligned} & \stackrel{\imath}{\infty} \\ & \infty \\ & \infty \\ & \infty \\ & \vdots \\ & i \end{aligned}$ |  |
| $\left\|\begin{array}{c} \Phi_{1} \\ 0 \end{array}\right\|$ | , | $\begin{aligned} & \circ \\ & \stackrel{\circ}{\circ} \\ & \vdots \\ & \stackrel{0}{2} \end{aligned}$ | $\begin{aligned} & \circ \\ & \hline 0 \\ & \vdots \\ & \hline \end{aligned}$ | $\begin{aligned} & \circ \\ & \vdots \\ & \vdots \\ & \vdots \\ & \hline \end{aligned}$ | $\begin{aligned} & \therefore \\ & \vdots \\ & \vdots \\ & 0 \end{aligned}$ | $\begin{aligned} & \vec{g} \\ & \vdots \\ & \dot{0} \end{aligned}$ | $\begin{aligned} & -1 \\ & \stackrel{\rightharpoonup}{\circ} \\ & \dot{0} \end{aligned}$ | $\begin{aligned} & -1 \\ & \stackrel{\rightharpoonup}{\circ} \\ & \stackrel{0}{0} \end{aligned}$ | $\begin{aligned} & \stackrel{\circ}{\circ} \\ & \vdots \\ & 0 \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & \vdots \\ & 0 \end{aligned}$ | $\begin{aligned} & n \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \circ \\ & \therefore \\ & \vdots \\ & 0 \end{aligned}$ | $\begin{aligned} & \circ \\ & \vdots \\ & \vdots \\ & 0 \\ & \hline \end{aligned}$ | $\begin{array}{\|l} \hline \\ \vdots \\ \vdots \\ 0 \end{array}$ | $\begin{aligned} & \stackrel{n}{0} \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \circ \\ & \vdots \\ & \vdots \\ & \vdots \end{aligned}$ | $\begin{aligned} & 0 \\ & \vdots \\ & \vdots \\ & 0 \end{aligned}$ | $\begin{aligned} & \therefore \\ & \vdots \\ & \vdots \\ & \vdots \end{aligned}$ | $\begin{aligned} & -1 \\ & \vdots \\ & \dot{0} \end{aligned}$ | $\begin{aligned} & \circ \\ & \stackrel{\circ}{\circ} \\ & \vdots \\ & 0 \end{aligned}$ | $\begin{aligned} & \overrightarrow{0} \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & -1 \\ & 0 \\ & \vdots \\ & 0 \end{aligned}$ | $\begin{aligned} & \circ \\ & \stackrel{\circ}{\circ} \\ & \stackrel{0}{\circ} \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & -1 \\ & 0 \\ & 0 \\ & 0 \\ & \hline \end{aligned}$ | $\begin{aligned} & 0 \\ & \vdots \\ & \vdots \\ & 0 \end{aligned}$ | $\begin{array}{r} -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}$ | $\begin{aligned} & \therefore \\ & \stackrel{\circ}{\circ} \\ & \dot{\circ} \end{aligned}$ | $\begin{aligned} & \circ \\ & \vdots \\ & \vdots \\ & \vdots \end{aligned}$ | $\begin{aligned} & \therefore \\ & \stackrel{\circ}{\circ} \\ & \dot{0} \end{aligned}$ | $\begin{aligned} & \therefore \\ & \vdots \\ & \vdots \\ & \vdots \end{aligned}$ | $\begin{aligned} & \circ \\ & \stackrel{\circ}{\circ} \\ & \vdots \\ & \hline \end{aligned}$ | $\begin{aligned} & \therefore \\ & \therefore \\ & \vdots \\ & \vdots \end{aligned}$ | $\begin{aligned} & \therefore \\ & \stackrel{\circ}{\circ} \\ & \dot{0} \end{aligned}$ | $\begin{aligned} & \therefore \\ & \vdots \\ & \vdots \\ & 0 \end{aligned}$ | $\begin{aligned} & \therefore \\ & \stackrel{\circ}{\circ} \\ & \dot{\circ} \end{aligned}$ | $\begin{aligned} & \therefore \\ & \vdots \\ & \vdots \\ & 0 \end{aligned}$ | $\begin{aligned} & \circ \\ & \stackrel{\circ}{\circ} \\ & \dot{0} \end{aligned}$ | $\begin{aligned} & \therefore \\ & \vdots \\ & \vdots \\ & \vdots \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & \dot{\circ} \end{aligned}$ | $\begin{aligned} & \circ \\ & \hline \stackrel{\circ}{\circ} \\ & \dot{\circ} \end{aligned}$ | $\begin{aligned} & \stackrel{\circ}{\mathrm{O}} \\ & \dot{0} \end{aligned}$ | $\begin{aligned} & \circ \\ & \stackrel{\circ}{\circ} \\ & \dot{\circ} \end{aligned}$ | $\begin{aligned} & \therefore \\ & \vdots \\ & \vdots \\ & 0 \end{aligned}$ | N | O |
| A | $\begin{aligned} & \stackrel{\rightharpoonup}{\gamma} \\ & \underset{\sim}{\gamma} \\ & \dot{\infty} \\ & \underset{\sim}{\circ} \end{aligned}$ |  | $\begin{aligned} & n \\ & 0 \\ & 0 \\ & \stackrel{n}{0} \\ & \vdots \end{aligned}$ | $\left[\begin{array}{c} n \\ \underset{\sim}{n} \\ 0 \\ 0 \\ m \end{array}\right.$ | $\begin{aligned} & \underset{\sim}{2} \\ & \stackrel{\rightharpoonup}{\dot{N}} \\ & \underset{\sim}{2} \end{aligned}$ | $\begin{aligned} & \underset{\sim}{n} \\ & \underset{\sim}{6} \\ & \stackrel{1}{2} \end{aligned}$ | $\begin{aligned} & \circ \\ & \vdots \\ & \underset{\sim}{n} \\ & i \\ & m \end{aligned}$ | $\begin{gathered} \underset{J}{n} \\ \underset{\sim}{N} \\ \underset{\sim}{n} \end{gathered}$ | $\begin{aligned} & \underset{\sim}{r} \\ & \underset{\sim}{j} \\ & \dot{m} \end{aligned}$ | $\begin{aligned} & \underset{\sim}{n} \\ & \underset{\sim}{\sim} \\ & \underset{\sim}{\lambda} \end{aligned}$ | $\begin{aligned} & \underset{\sim}{\underset{~}{0}} \\ & \underset{\sim}{7} \\ & \underset{\sim}{2} \end{aligned}$ | $\begin{aligned} & \underset{\sim}{\underset{\sim}{2}} \\ & \underset{\sim}{n} \\ & \underset{\sim}{\sim} \end{aligned}$ | $\begin{aligned} & \underset{\sim}{n} \\ & \underset{\sim}{\dot{N}} \\ & \underset{\sim}{\sim} \end{aligned}$ | $\begin{gathered} \stackrel{i}{2} \\ \underset{1}{2} \\ \underset{\sim}{N} \end{gathered}$ | $\begin{aligned} & \stackrel{\bullet}{\bullet} \\ & \stackrel{0}{y} \\ & \underset{\sim}{\mathrm{~J}} \end{aligned}$ |  | $\begin{aligned} & \stackrel{n}{0} \\ & \infty \\ & \underset{\sim}{\sim} \\ & \underset{\sim}{\sim} \end{aligned}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{\circ} \\ & \stackrel{1}{m} \\ & \dot{n} \end{aligned}$ | $\begin{aligned} & \stackrel{0}{0} \\ & \tilde{0} \\ & \dot{6} \\ & \underset{\sim}{6} \end{aligned}$ | $\begin{aligned} & \stackrel{0}{\sim} \\ & \underset{\sim}{\sim} \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { m } \\ & \vdots \\ & 0 \\ & \dot{\sim} \\ & \vdots \end{aligned}$ | $\begin{aligned} & \underset{\sim}{n} \\ & \dot{6} \\ & \underset{\sim}{N} \end{aligned}$ | $\begin{gathered} \tilde{\sim} \\ \underset{\sim}{\alpha} \\ \underset{\sim}{n} \end{gathered}$ | $\begin{aligned} & \dot{o} \\ & \underset{y}{2} \\ & \dot{\rightharpoonup} \\ & \underset{\sim}{2} \end{aligned}$ | $\left\lvert\, \begin{aligned} & \underset{\sim}{n} \\ & \stackrel{n}{\sim} \\ & \underset{\sim}{4} \end{aligned}\right.$ | $\stackrel{\infty}{\infty} \begin{aligned} & \infty \\ & \infty \\ & \underset{\sim}{2} \end{aligned}$ | $\begin{array}{\|c} n \\ 0 \\ 0 \\ \stackrel{\rightharpoonup}{n} \\ \underset{\sim}{n} \\ \hline \end{array}$ | $\left\lvert\, \begin{aligned} & \stackrel{n}{2} \\ & \underset{\sim}{1} \\ & \text { gin } \end{aligned}\right.$ | $\begin{aligned} & \underset{\sim}{\lambda} \\ & \underset{\sim}{2} \\ & \underset{\sim}{\alpha} \\ & \underset{\sim}{2} \end{aligned}$ | $\begin{aligned} & n \\ & \infty \\ & 0 \\ & \vdots \\ & \underset{\sim}{n} \\ & \hline \end{aligned}$ |  | $\begin{aligned} & \underset{\sim}{0} \\ & \underset{\sim}{1} \\ & \underset{Z}{7} \end{aligned}$ | $\begin{aligned} & \infty \\ & \stackrel{\infty}{\square} \\ & \dot{\sim} \end{aligned}$ | $\begin{gathered} \underset{\sim}{7} \\ 0 \\ \dot{\sim} \\ \sim \\ \hline \end{gathered}$ | $\begin{gathered} \stackrel{0}{m} \\ \underset{\sim}{3} \\ \underset{\sim}{n} \end{gathered}$ |  | $\begin{gathered} \underset{\sim}{7} \\ \underset{\sim}{n} \\ \underset{\sim}{n} \end{gathered}$ | $\begin{aligned} & 0 \\ & 0 \\ & \infty \\ & 0 \\ & 0 \\ & \underset{\sim}{2} \end{aligned}$ |  | $\begin{aligned} & \mathrm{m} \\ & \underset{i}{\prime} \end{aligned}$ | $\left\lvert\, \begin{gathered} \infty \\ \tilde{m} \\ \dot{n} \\ \infty \\ \end{gathered}\right.$ | $\begin{aligned} & \infty \\ & \aleph \\ & \vdots \\ & \vdots \\ & \Omega \end{aligned}$ | $\left\lvert\, \begin{aligned} & 0 \\ & \vdots \\ & \vdots \\ & \dot{\sim} \\ & \hline \end{aligned}\right.$ | $\begin{aligned} & \tilde{y} \\ & \underset{~}{9} \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \stackrel{\circ}{n} \\ & \mathrm{y} \\ & \underset{\sim}{\mathrm{~m}} \end{aligned}$ | n n m m m |
| $\left\|\begin{array}{c} Q_{6}^{\mid} \\ \underset{O}{\mid} \end{array}\right\|$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 呙 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\left\|\begin{array}{l} \stackrel{\rightharpoonup}{0} \\ 0 \\ \stackrel{0}{0} \\ 0 \end{array}\right\|$ | $\begin{array}{\|c\|c} \stackrel{\circ}{N} \\ \underset{\sim}{N} \\ \text { 号 } \end{array}$ | $$ | $\begin{aligned} & m \\ & \Omega \\ & \infty \\ & \end{aligned}$ | $\begin{aligned} & -\vec{m} \\ & \overrightarrow{1} \\ & \underset{\sim}{c} \\ & \end{aligned}$ | $\begin{aligned} & \tilde{N} \\ & \\ & \underset{\sim}{\alpha} \\ & \underset{\sim}{2} \end{aligned}$ | $\begin{aligned} & \text { on } \\ & 0 \\ & \\ & \text { E } \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & \underset{\sim}{0} \\ & 0 \\ & \end{aligned}$ | $\begin{aligned} & \hat{y} \\ & \underset{\sim}{n} \\ & 0 \\ & y \end{aligned}$ | $\begin{aligned} & \underset{Z}{7} \\ & \underset{\sim}{n} \\ & y_{m} \end{aligned}$ |  |  | $\begin{gathered} \text { J } \\ \underset{0}{2} \\ \text { N } \\ \text { S} \end{gathered}$ |  | $\begin{gathered} n \\ \\ \underset{O}{n} \\ \vdots \\ b \end{gathered}$ | $\begin{gathered} \underset{\sim}{N} \\ \underset{\sim}{n} \\ b \end{gathered}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & \infty \\ & 0 \\ & 0 \\ & \end{aligned}$ | $\begin{array}{\|c} 0 \\ \stackrel{0}{2} \\ \\ \stackrel{2}{z} \end{array}$ |  |  | $$ | $\begin{aligned} & \infty \\ & 0 \\ & 0 \\ & \vdots \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \left.\begin{array}{l} 7 \\ 0 \\ n \\ 0 \\ 0 \\ 0 \end{array} \right\rvert\, \end{aligned}$ | 앙 <br> 号 |  |  | $\begin{gathered} n \\ 0 \\ \tilde{n} \\ 0 \\ \vdots \\ 0 \end{gathered}$ |  | $\left\lvert\, \begin{gathered} 0 \\ 0 \\ 0 \\ 0 \\ u \\ \vdots \\ 0 \end{gathered}\right.$ | $\begin{array}{\|c} \infty \\ 0 \\ 0 \\ 0 \\ \vdots \\ \vdots \\ 0 \end{array}$ |  | $\begin{array}{\|c} 0 \\ 0 \\ 0 \\ 0 \\ u \\ u \\ c \end{array}$ | $\begin{array}{\|c} \infty \\ o \\ \\ \vdots \\ u \\ 0 \end{array}$ | $\left\lvert\, \begin{gathered} 0 \\ 0 \\ \\ \vdots \\ u \\ 0 \\ 0 \end{gathered}\right.$ |  | $\begin{aligned} & -7 \\ & \vec{\sigma} \\ & \text { 回 } \\ & \text { d } \end{aligned}$ | $\begin{aligned} & \text { d } \\ & \sigma \\ & \sigma \\ & \text { 四 } \\ & \vdots \end{aligned}$ |  | $\begin{aligned} & \stackrel{\Gamma}{1} \\ & \text { 先 } \end{aligned}$ | $\begin{aligned} & \infty \\ & \stackrel{\infty}{1} \\ & \underset{\sim}{\pi} \\ & \stackrel{\pi}{6} \end{aligned}$ | $\begin{gathered} \infty \\ \infty \\ \underset{\sim}{\sim} \\ 0 \\ 0 \end{gathered}$ | $\left\lvert\, \begin{aligned} & 0 \\ & 0 \\ & \underset{\sim}{0} \\ & 0 \\ & 0 \end{aligned}\right.$ | $\begin{aligned} & 0 \\ & \tilde{0} \\ & \underset{\sim}{2} \\ & 0 \\ & 0 \end{aligned}$ | $\left\lvert\, \begin{aligned} & \vec{a} \\ & \underset{\sim}{x} \\ & 0 \\ & 0 \end{aligned}\right.$ | $\begin{aligned} & \underset{\sim}{N} \\ & \underset{\sim}{N} \\ & u \end{aligned}$ | $\stackrel{ }{\square}$ | m |

## Double Star Photometry - April 2019

(Continued from page 23)
the following structure:

- Object $=$ Discoverer ID
- $\quad$ Comp $=$ Components $(\mathrm{AB}$ if blank)
- $\quad \mathrm{PA}=$ Position angle in degrees
- e_PA = Error position angle
- $\quad$ Sep $=$ Separation in arcseconds
- $\quad$ e_Sep $=$ Error separation
- Plx1 = Parallax primary in mas
- e_Plx1 = Error parallax primary
- Plx2 = Parallax secondary in mas
- e_Plx2 = Error parallax secondary
- $\quad$ Min_D_AU $=$ Minimum spatial distance in AU between components (see Appendix)
- Med_D_AU = Median spatial distance in AU between components (see Appendix)
- Max_D_AU = Maximum spatial distance in AU between components (see Appendix)
- $\quad$ LPGR $=$ Likelihood of potential gravitational relationship (see Appendix)
- M1_50 = DR2 StarHorse mass50 value primary
- M2_50 = DR2 StarHorse mass50 value secondary

All objects in Table 2 were already cross-matched with Gaia data in other reports, so the values given here on separation and position angle are referenced as input for assessing the likelihood of potential gravitational relationship but are not intended for updating the WDS catalog. For the 4 objects with LPGR > 50 WDS code " T " is suggested for likely physical by common parallaxes. For the objects with LPGR $<10$ WDS code "U" for likely optical is suggested.

For objects with LPGR $>50$ the minimum and median period of a potential orbit is calculated using the smallest and median spatial distance between the components as estimation for the semi-major axis assuming zero inclination (this assumption ignores the influence of eccentricity, so it is most likely that the observed separation for high eccentricity pairs is near apastron which means that the "real" semi-major axis might in most cases somewhat different) and using the mass50 values from the GAIA DR2 StarHorse catalog sharing the caveats of Anders et al. 2019 for using these data. The results are listed in Table 3 with the following structure:

Object $=$ Discoverer ID
P_min_yr = Minimum period of a potential orbit
$\mathrm{P}_{-}^{-}$med $\_$yr $=$Maximum period of a potential orbit

| Object | P_min_yr |  | P_med_yr |  |  |
| :---: | ---: | :---: | :---: | :---: | :---: |
| BVD 226 | 147496 | 62 | 267 | 755 |  |
| CRB | 93 | 156 | 266 | 35 | 961 |
| GRV | 864 | 143 | 495 | 44 | 856 |
| NI | 30 | 18 | 206 | 11 | 215 |

Table 3: Potential orbit periods in years

In all these cases the potential orbit period is far too long to detect any changes in separation and position angle by visual observation over a reasonable time frame.

## Summary

A good part of the 46 measured objects shows the expected magnitude difference larger than 0.5 compared with the WDS catalog data especially for the secondary but for many objects the given WDS magnitudes were simply confirmed within the given error range. 4 objects have parallaxes and angular separations allowing for a higher than $50 \%$ likelihood for a spatial distance between the components of less than 200,000 AU ( $\sim 1$ parsec) suggesting potential gravitational relationship and 33 objects are most likely opticals.

## Acknowledgements

The following tools and resources have been used for this research:

- Washington Double Star Catalog
- GAIA DR2 catalog
- DSS2 images
- Aladin Sky Atlas v10.0
- iTelescope
- iT24: 610 mm CDK with 3962 mm focal length. Resolution 0.625 arcsec/pixel. Vfilter. No transformation coefficients available. Located in Auberry, California. Elevation 1405 m
- AAVSO VPhot
- Astrometrica v4.10.0.427
- URAT1 catalog
- AstroPlanner v2.2
- MaxIm DL6 v6.08
- GAIA DR2 StarHorse catalog available through the Gaia@AIP services hosted by the LeibnizInstitute for Astrophysics Potsdam using the ADQL query interface at gaia.aip.de


## Double Star Photometry - April 2019

## References

Anders, F., Khalatyan, Arman, et al., 2019, "Photoastrometric distances, extinctions, and astrophysical parameters for Gaia DR2 stars brighter than $\mathrm{G}=18$ ", $A s$ tronomy \& Astrophysics, DOI 10.1051/00046361/201935765

Knapp, Wilfried R. A., 2018, "A New Concept for Counter-Checking of Assumed Binaries", Journal of Double Star Observations, 14 (3), 487-491.

## Appendix

## Description of the PGR assessment procedure

GAIA DR2 data for RA/Dec and Plx are used for a Monte Carlo simulation assuming a normal distribution for these parameters with the given error range as standard deviation. The spatial distance between the components is (according to Knapp 2018) calculated from the inverted simulated parallax data and the simulated angular separation using the law of cosine

$$
\text { sep }=\sqrt{a^{2}-2 a b \cos \gamma+b^{2}}
$$

with $a$ and $b=$ distance vectors for the stars A and B in lightyears calculated as $(1000 / \mathrm{Plx}) * 3.261631$ and $\gamma=$ angular separation in degrees calculated as

$$
\gamma=\arccos [\sin (D E 1) \sin (D E 2)+\cos (D E 1) \cos (D E 2) \cos (|R A 1-R A 2|)]
$$

The likelihood for potential gravitational relationship (LPGR) is the percentage of simulation results $<200,000 \mathrm{AU}(\sim 1$ parsec) out of the simulation sample with a size of 120,000

The given smallest, median and largest spatial distance between the components is the smallest, median and largest result out of the simulation sample.

# Orbit Determination of Close Binary Systems 

F. M. Rica<br>Editor of News Section, El Observador de Estrellas Dobles (OED) Magazine, Spain<br>frica0@gmail.com<br>H. Zirm<br>Markt Schwaben, Germany<br>binary71@gmx.de


#### Abstract

Several years ago we published in IAU circulars the orbital parameters for 6 close binary stars (A 957, HEI 35, A 2833, STF1554, BU 606, I 952). These were the first orbits ever published for those close binaries. The Docobo's method of orbital calculation was used to determine the orbital solutions and it was then improved by using a least square process. A weighting scheme was followed for all astrometric measures (individually for theta and rho). All these orbital solutions are still the most recent published orbits for these binaries.

In this article we publish for the first time, the astrophysical study, the orbit plots and all the astrometric measures with their residuals and ephemerides for these objects.


## 1. Introduction

In the circular number 174 (June 2011) of IAU G1 Commission, we published the orbital solutions for six close visual binary stars. These orbits were the first calculated for these objects (the literature search produced no evidence of previously computed orbits). These orbital solutions are still the most recent published.

That circular only published the orbital parameters. The orbital plots, the ephemerids for the next few years, the tables with measures, the astrophysical data, etc. are published in this article for the first time. The calculated periods range from 29 to 708 years. The measurements were mostly obtained from the Washington Double Star Catalogue by request from Brian Mason \& colleagues.

All the orbits are calculated with the algorithm described in Docobo (1985). The three base points have been chosen carefully from the observational data that seemed most reliable with respect to instrumentation, data density, or critical arc coverage. We also tried to cover as much of the observed arc as possible. Often, we use a parabolic or cubic fit to $\theta(t)$ and $\rho(t)$ to obtain the base points. This may let the area around a single observation represent a base point without additional observational coverage.

Finally, we applied least squares refinement to the preliminary elements by using the formula for differential corrections in polar coordinates (Heintz 1967). The
uncertainties of the elements were obtained by the covariance matrix of the normal equations and the sum of the residuals in position angle and separation.

The weights for speckle measures follow the scheme described by Mason (1999). For initial weightings on visual measurements, we choose the algorithm described by Docobo \& Ling (2003). The proposal by Heintz $(1967,1978)$ to reduce the weights for the distances from visual measurements by a factor of $1: 4$ to 1:5 was used in most cases. Position angles are corrected for precession, Equinox 2000.

The organization of this paper is as follows. In Section 2, we present the general methods for our astrophysical procedures and the data table and figures are published. In Section 3, we include general information for each of the binaries studied.

## 2. General Information

### 2.1. Astrophysical procedure

To obtain astrophysical data (spectral type, masses, etc.) of the individual component of close binaries, we used a tool designed by one of us (called Binary_Deblending_Tool) that deblends the combined observed multi-band photometry (Hipparcos, Tycho-2, 2MASS, etc.) into two separate entries corresponding to both stellar components. To create the input table for this tool, we used the CMD 2.7 isochronal to the related photo-metric bands and colors with astrophysical prop-

## Orbit Determination of Close Binary Systems

erties. The tool searches in this table for two entries that matches with observational data (multiband photometry, Dmag, reddening and parallax) and selects those which give minimum $\chi^{2}$.

GAIA-DR2 does not list useful data for these bright and close binary stars.

The reddening in the line of sight was estimated using the maps of Schlegel, Finkbeiner, and Davis (1998) and the more recent of Schlafly and Finkbeiner (2011). The resulting values were scaled to the initial distance using the formula published by AnthonyTwarog, and Twarog (1994).

### 2.2 Table data and Figures

First, general information about the systems is discussed, followed by important identifications and the calculated orbital elements in Table 3. A comparison of the recalculated Hipparcos parallaxes (van Leeuwen 2007) with the dynamical parallaxes, the resulting masses, absolute magnitudes, and determined spectral types for the individual components can be found in Table 4. The calculation of the dynamical parallaxes and masses follows the Baize-Romani algorithm described by Heintz in 1978. The calculated ephemerids for 2020, 2021, 2022, 2023 and 2024 are available at Table 5. At the appendix we list the astrometric measurements compared with calculated residuals. Finally we show the orbital plots. Tables 6-11 lists the observations for each binary star. The columns are from left to right: the date of the observations, the position angle and angular separation, the number of nights observed, the WDS reference of the publication, the aperture of the telescope (in meters), WDS codes. All these columns are taken directly from WDS. And finally, we list the $\mathrm{O}-\mathrm{C}$ for $\theta$ and $\rho$. The plots in Figures 1-6 came from USNO catalog Sixth Catalog of Orbits of Visual Binary Stars

## 3. Description of the Close Binary Stars

WDS 02048+6030 = A 957 = ADS 1632
Mag. 8.52 and 10.48. Spectral type F8V.
First measurement: 1905.82
Last measurement: 2011.8569
Used observations: 21
The components are cataloged as HD 12529 and were first resolved by R. G. Aitken in 1905. Since then it has shown relatively consistent visual measurements showing a clear curved arc of about 100 degrees. But the typical scatter in visual measures in regard to the average distance of 0.5 arc seconds makes the calculation of an orbit very difficult. However, our derived orbit is marked as preliminary solution. After the publication of our orbit, two new astrometric points were

|  | Observed <br> Photometry | Source | Syntetic <br> Photometry | Difference |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{U}$ | $\ldots$ | $\ldots$ | $\ldots$ |  |
| $\mathbf{B}$ | $8.83 \pm 0.01$ | Hipparcos | 8.78 | 0.05 |
| $\mathbf{V}$ | $8.22 \pm 0.01$ | Hipparcos | 8.21 | 0.01 |
| $\mathbf{I}$ | $7.54 \pm 0.01$ | Hipparcos | 7.57 | -0.03 |
| $\mathbf{J}$ | $7.12 \pm 0.02$ | 2MASS | 7.15 | -0.03 |
| $\mathbf{H}$ | $6.84 \pm 0.02$ | 2MASS | 6.94 | -0.10 |
| $\mathbf{K}$ | $6.80 \pm 0.02$ | 2MASS | 6.87 | -0.06 |
| $\mathbf{B - V}$ | $0.61 \pm 0.02$ | Hipparcos | 0.57 | 0.04 |
| $\mathbf{V - I}$ | $0.68 \pm 0.02$ | Hipparcos | 0.64 | 0.04 |
| $\mathbf{V - K}$ | $1.42 \pm 0.02$ |  | 1.35 | 0.07 |
| $\mathbf{J - H}$ | $0.28 \pm 0.03$ |  | 0.21 | 0.07 |
| $\mathbf{H - K}$ | $0.04 \pm 0.02$ |  | 0.07 | -0.03 |
| J-K | $0.28 \pm 0.03$ |  | 0.21 | 0.07 |

Table 1. Comparison of observed and synthetic data for A 957

|  | Comp. A | Bomp. B |
| :---: | :---: | :---: |
| V | 8.30 | 10.20 |
| (B-V) O | 0.48 | 0.70 |
| Mv | 3.40 | 5.30 |
| Mass [M ${ }_{\odot}$ ] | 1.37 | 1.08 |
| Teff [ ${ }^{\circ} \mathrm{K}$ ] | 6488 | 5793 |
| log g | 0.6 | 0.8 |
| SpT | F5V | G8V |
| Distance [pc] | 97 |  |
| Age [Gyr] | 1.4 |  |

Table 2. Astrophysical Data for the Binary A 957
included in WDS catalog. The residuals for these recent observations suggest that the orbit is opening and that the orbital period is greater than the calculated value (220 years).

This is a system rich in metals (Nordstrom et al. 2004; Casagrande et al. 2011) with an age of 3-4 Gyr and located at 97 pc . The literature shows combined spectral types that ranges from F8V to G2V. Our astrophysical study concludes that this stellar system is composed of F5V and G8V (masses of 1.3 and $0.9 \mathrm{M}_{\odot}$ ) stars. The astrophysical parameters were corrected by a reddening of $\mathrm{E}(\mathrm{B}-\mathrm{V}) \approx 0.05$. Table 1 lists observed photometric data and compares them with the synthetic photometry determined by using our excel tool Binary_Deblending_Tool. Table 2 lists the individual astrophysical properties for both stellar components deter-

## Orbit Determination of Close Binary Systems

| WDS | HD | ADS | $\mathrm{P}^{\mathrm{yr}}$ | T | e | a" | $i^{\circ}$ | $\omega^{\circ}$ | $\Omega^{\circ}{ }_{2000}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| D.D. | HIP | other | $\pm$ | $\pm$ | $\pm$ | $\pm$ | $\pm$ | $\pm$ | $\pm$ |
| 02048+6030 | 12529 | 1632 | 219.97 | 1870.07 | 0.462 | 0.471 | 125.3 | 250.1 | 107.7 |
| A 957 | 9704 |  | - | - | - | - | - | - | - |
| 04102+1722 | 285465 | - | 28.91 | 2000.53 | 0.832 | 0.282 | 115.7 | 35.9 | 165.9 |
| HEI 35 | 19472 |  | $\pm 1.50$ | $\pm 0.01$ | $\pm 0.007$ | $\pm 0.008$ | $\pm 0.7$ | $\pm 1.5$ | $\pm 0.9$ |
| $06549+1158$ | 50722 | 5571 | 209.09 | 1987.13 | 0.927 | 0.429 | 43.1 | 240.7 | 107.2 |
| A 2833 | 33240 |  | $\pm 35.98$ | $\pm 0.19$ | $\pm 0.010$ | $\pm 0.040$ | $\pm 4.6$ | $\pm 8.3$ | $\pm 8.9$ |
| $11361+1251$ | 100797 | 8230 | 478.88 | 1962.44 | 0.902 | 0.868 | 74.8 | 308.6 | 84.6 |
| STF1554 | 56589 |  | $\pm 72.49$ | $\pm 0.47$ | $\pm 0.011$ | $\pm 0.066$ | $\pm 0.8$ | $\pm 1.9$ | $\pm 0.9$ |
| 12260-1457 | 108215 | 8547 | 707.61 | 2009.44 | 0.383 | 1.441 | 96.9 | 123.6 | 100.1 |
| BU 606 | 60665 |  | $\pm 36.19$ | $\pm 6.63$ | $\pm 0.052$ | $\pm 0.188$ | $\pm 3.9$ | $\pm 4.5$ | $\pm 6.3$ |
| 14531-4638 | 131078 | - | 99.13 | 1981.39 | 0.659 | 0.521 | 156.1 | 268.1 | 27.1 |
| I 952 | 72821 |  | $\pm 0.82$ | $\pm 0.37$ | $\pm 0.015$ | $\pm 0.014$ | $\pm 4.5$ | $\pm 7.3$ | $\pm 7.8$ |

Table 3. Identification and orbital elements

| WDS D. ${ }^{\text {d }}$. | $\begin{aligned} & \mathrm{V}_{\mathrm{A}} \\ & \mathrm{~V}_{\mathrm{B}} \end{aligned}$ | Sp. | B-V | $\pi{ }^{\text {mas }}{ }_{\text {trig }}{ }^{1}$ | $\Sigma \mathrm{M} / \mathrm{M}^{\text {sol }}$ | $\pi^{\text {mas }}{ }_{\text {dyn }}$ | $\begin{aligned} & \mathrm{M} / \mathrm{M}_{\text {sol }} \mathrm{A} \\ & \mathrm{M} / \mathrm{M}_{\text {sol }} \end{aligned}$ | $\begin{aligned} & \mathbf{M}_{\mathrm{vis} A} \\ & \mathbf{M}_{\mathrm{vis} B} \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 02048+6030 | 8.5 | F5V+G8V | 0.61 | $\begin{gathered} 10.28 \\ \pm 1.19 \end{gathered}$ | 2.0 | 9.8 | 1.4 | 3.5 |
| A 957 | 10.5 |  |  |  |  |  | 0.9 | 5.4 |
| $04102+1722$ | 9.5 | K2V+K6V | 1.07 | 26.98 | 1.37 | 28.1 | 0.72 | 6.7 |
| HEI 35 | 10.9 |  |  | $\pm 2.27$ | $\pm 0.37$ |  | 0.50 | 8.2 |
| 06549+1158 | 8.3 | F2IV+F2IV-V | 0.39 | 7.68 | 4.0 | 8.6 | 1.6 | 3.0 |
| A 2833 | 8.8 |  |  | $\pm 1.24$ | $\pm 2.6$ |  | 1.3 | 3.9 |
| $11361+1251$ | 9.4 | G0V+G0 / 1V | 0.58 | 10.25 | 2.7 | 11.1 | 1.1 | 4.7 |
| STF1554 | 9.5 |  |  | $\pm 1.16$ | $\pm 1.3$ |  | 1.0 | 4.9 |
| 12260-1457 | 7.4 | F4V+G5V | 0.51 | 14.07 | 2.1 | 13.3 | 1.6 | 3.1 |
| BU 606 | 9.4 |  |  | $\pm 0.69$ | $\pm 0.9$ |  | 1.0 | 5.0 |
| 14531-4638 | 8.5 | G5V+K1V | 0.70 | 20.96 | 1.6 | 20.6 | 1.0 | 5.1 |
| I 952 | 10.1 |  |  | $\pm 1.06$ | $\pm 0.3$ |  | 0.7 | 6.7 |

Table 4. Astrophysical Data
Remarks: The individual visual magnitudes for each component are calculated from combined visual magnitudes adopted from Hipparcos catalogue and the visual magnitude difference adopted from WDS catalogue. Note 1 is the parallax adopted from Van Leeuwen 2007. Note 2 marks the estimation as main sequence stars.

| WDS | 2020 |  | 2021 |  | 2022 |  | 2023 |  | 2024 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\vartheta_{2000}\left({ }^{\circ}\right.$ ) | p (") | $\vartheta_{2000}\left({ }^{\circ}\right)$ | p (") | $\vartheta_{2000}\left({ }^{\circ}\right.$ ) | م (") | $\vartheta_{2000}\left({ }^{\circ}\right.$ ) | p (") | $\vartheta_{2000}\left({ }^{\circ}\right.$ ) | م (") |
| 02048+6030 | 2.1 | 0.368 | 0.8 | 0.368 | 359.4 | 0.367 | 358.0 | 0.367 | 356.6 | 0.367 |
| 04102+1722 | 321.1 | 0.357 | 319.1 | 0.332 | 316.7 | 0.303 | 313.8 | 0.271 | 310.1 | 0.235 |
| 06549+1158 | 142.8 | 0.481 | 143.1 | 0.487 | 143.5 | 0.494 | 143.8 | 0.500 | 144.2 | 0.507 |
| $11361+1251$ | 220.2 | 0.338 | 220.8 | 0.344 | 221.3 | 0.351 | 221.8 | 0.357 | 222.3 | 0.363 |
| 12260-1457 | 286.6 | 0.655 | 286.3 | 0.668 | 286.1 | 0.681 | 285.8 | 0.695 | 285.6 | 0.708 |
| 14531-4638 | 311.3 | 0.766 | 310.2 | 0.771 | 309.0 | 0.774 | 307.9 | 0.778 | 306.8 | 0.781 |

Table 5. Ephemerides

## Orbit Determination of Close Binary Systems



Figure 1. WDS 02048+6030
mined by us using the Excel tool.
The dynamic parallax and dynamic masses are in good agreement with the Hipparcos trigonometric parallax and the expected masses.

The 2012 measure was performed by Francisco Rica using the $1.5-\mathrm{m}$ Carlos Sánchez Telescope and the Fastcam lucky Imaging Camera. It is published here for the first time. For more detail about the instrument used, see Rica et al. (2012).

## WDS 04102+1722 = HEI 35

Mag. 9.46 and 10.93 ; spectral type K5
First measurement: 1979.00
Last measurement: 2016.1338
Used observations: 22
This is a faint system (HD 285465) with high proper motion located in the Hyades open cluster. It was discovered in 1979 by W. D. Heintz and is composed of two stars with a visual brightness difference of 1.5 magnitudes and a medium- K spectrum. It has been measured in 22 occasions, many of them by Elliot Horch. The astrometric measures used for the orbital calculation (up to 2008.69) cover more than one revolution. The work of Y. Y. Balega since 1999 shows a rapid movement nearing a periastron phase. Observations by E. Horch up to now consistently show increasing distances from similar positions nearly 30 years old, observed by Heintz. Since our publication of the orbit in 2011, several astrometric points were published


Figure 2. WDS 04102+1722
showing small residuals with respect our orbit.
Literature lists combined spectral types of K3 (Bidelman 1985) and K4V (Pickles \& Depagne 2010) which is in agreement with our combined spectral type. Our astrophysical study concludes that this stellar system is composed of K2V and K6V stars. The astrophysical parameters were corrected by a reddening of $\mathrm{E}(\mathrm{B}-$ $\mathrm{V}) \approx 0.05$. The dynamic parallax and dynamic mass are in good agreement with the Hipparcos trigonometric parallax and the expected masses.

## WDS 06549+1158 = A $2833=$ ADS 5571

Mag. 8.3 and 9.2; spectral types F5
First measurement: 1914.51
Last measurement: 2008.111
Used observations: 18
This binary system (HD 50722) was discovered by Aitken in 1914, and up to the present day 18 observations have been made. One measurement, obtained by Argue in 1987 via CCD technique, was not used for the calculation. This measurement is listed in the WDS catalog with the comment "... Identification error, position error, or misprint in publication, NOT corrected..." and the calculated theoretical position at this epoch seems to confirm the fact that cannot be the same component. Several tests with different basis points (using Docobo orbital method) showed that the Hipparcos measurement must be assigned a significantly reduced weight (compared to the last speckle measurement).

## Orbit Determination of Close Binary Systems



Figure 3. WDS 06549+1158

This binary is listed in the literature as a F5 ( ppN catalog) and as a F5IV star by Pickles, Depagne (2010). Fehrenbach, Burnage \& Figuiere (1992) catalogued it as a low metallicity star, $[\mathrm{Fe} / \mathrm{H}]=-0.29$, with an age of 2.3 Gyr. In this work, we determined the absolute magnitudes from apparent individual components (8.28 and 8.75) and Hipparcos trigonometric parallax. The differential magnitude of Hipparcos has a very significant error and so we don't use its individual photometry. Instead, we use the Hipparcos combined V magnitude and the mean differential magnitude from the WDS historical data. Using evolutionary isochrones for the metallicity listed in the literature and different ages, we determine a stellar age of about 2.2 Gyr (in excellent agreement with the literature) and spectral types of F2IV and F2IV-V. The astrophysical parameters were corrected by a reddening of $\mathrm{E}(\mathrm{B}-\mathrm{V}) \approx 0.02$.

Our calculations produced a very eccentric orbit (e $=0.93$ ). The dynamic parallax and dynamic mass are in moderate agreement with the Hipparcos trigonometric parallax and the expected masses.

## WDS 11361+1251 = STF1554 = ADS 8230:

Mag. 9.4 and 9.6 ; spectral types G5
First measurement: 1829.29
Last measurement: 2010.3510
Used observations: 57
This pair (HD 100797) was discovered by Struve in


Figure 4. WDS 11361:1251
1829. Nearly 120 years later, there are 57 astrometric measures of its long period orbit. The period is about 480 years and the periastron passage of this eccentric orbit occurred in 1962.

This object has been very poorly studied and no radial velocity and spectral types (except those of Cannon \& Pickering 1918) were obtained. Hipparcos determined a distance of 98 pc (GAIA-DR2 does not list this star). Astronomical literature shows combined spectral types of G5 (Cannon \& Pickering 1918) and a photometrically determined spectral type of F8V (Pickles \& Depagne 2010), in excellent agreement with our results using BVIJHK combined photometry. Therefore, the G5 spectral type listed in Hipparcos catalog must be in error.

Our astrophysical study concludes that this stellar system is composed of G0V and G0/1V stars. The CMD 2.7 isochrone gives two stars with an age of about 3 Gyr . The astrophysical parameters were corrected by a reddening of $\mathrm{E}(\mathrm{B}-\mathrm{V}) \approx 0.01$. The dynamic parallax and dynamic mass are in good agreement with the Hipparcos trigonometric parallax and the expected masses.

WDS 12260-1457 = BU $606=$ ADS 8547:
Mag. 7.4 and 9.4. Spectral type F6V
First measurement: 1878.3
Last measurement: 2016.3699
Used observations: 24

## Orbit Determination of Close Binary Systems



Figure 5. WDS 12260-1457

The components of this binary star (HD 108215) were first resolved by Burnham in 1878. Currently it has 24 measures which are all micrometric except that of Hipparcos satellite. In 138 years, the measures have covered a large, nearly rectilinear arc of about 167 degrees. The older visual measures have significant residuals.

This pair is composed of stars with V magnitudes of 7.32 and 9.26 (Hipparcos magnitudes converted to V band). The Tycho-2 catalog lists a proper motion of $118.3 \pm 1.6 \mathrm{mas} \mathrm{yr}^{-1}$ in RA and $-25.1 \pm 1.5 \mathrm{mas} \mathrm{yr}^{-1}$ in DEC. The re-reduced (Leeuwen 2007) Hipparcos trigonometric parallax corresponds to a distance of pc. For this distance we calculated a reddening of $\mathrm{E}(\mathrm{B}-\mathrm{V})=$ 0.02 which was used for our study.

In astronomical literature, BU 606 has been classified as a F6V (Houk \& Smith-Moore 1988) or F7V (Abt 1981) star. From combined and differential photometry, we estimated individual spectral types F4V and G5V. Absolute magnitudes +3.1 and +5.1 were calculated using Hipparcos data, which matches the spectral types estimated.

The large linear motion invites the investigation of the nature of the pair of stars. The observed velocity was calculated using the historical measures, with nonzero weights, which covers a time baseline of nearly 128 years (using astrometric data up to 2006). Our result was an apparent motion of $-15.42 \pm 0.40$ mas $\mathrm{yr}^{-1}$ in

E-W direction and $+3.17 \pm 0.16{\text { mas } \mathrm{yr}^{-1} \text { in N-S direc- }}^{2}$ tion. So, the secondary is moving to the WNW $15.74 \pm$ 0.43 mas $_{\mathrm{yr}}{ }^{-1}$ and at the distance of 71.1 pc corresponds to a projected and relative velocity of $5.30 \pm 0.15 \mathrm{~km} \mathrm{~s}^{-}$ ${ }^{1}$. A Monte Carlo simulation shows that the projected observed velocity was less than the escape velocity in $100 \%$ of the simulations. We conclude that BU 606 stellar components are gravitationally bound.

Holmberg, Nordström \& Andersen (2009) calculated a galactocentric velocity of $(\mathrm{U}, \mathrm{V}, \mathrm{W})=(-33,-20,-$ 16) $\mathrm{km} \mathrm{s}-1$ and an age of about 2.6 Gyr. According to this kinematic, this system is a member of the young galactic disk. The calculated $\mathrm{fG}=0.18$ (Grenon parameter), corresponding to young-middle age thin disk stars of 3-4 Gyr old. This binary is an X-ray emitter whose luminosity in this band suggests that has an age of about 0.1-0.6 Gyr while the projected rotation velocity of $13.5 \mathrm{~km} / \mathrm{s}$ calculated by Glebocki \& Gnacinski (2005) and $15.0 \mathrm{~km} / \mathrm{s}$ calculated by Nordström (2004) also suggest an age between those of Pleiades and Hyades open clusters. This age is in contradiction with the kinematical age. One possible explanation is that one of the stars is an unresolved and very close binary with synchronized orbital periods.

The derived orbit is very inclined and passed periastron in 2009. The orbital solution presented here is very preliminary. The dynamic parallax and total stellar masses are consistent with Hipparcos trigonometric parallax and with the stellar masses obtained using their spectral types.

A total mass of $2.2 \pm 0.4 \mathrm{M}_{\odot}$ was obtained using our orbital parameters and the Hipparcos trigonometric parallax. This value is in good agreement with that calculated using Baize-Romaní method $(1.56+0.98=$ $2.54 \mathrm{M}_{\text {. }}$ ) and with that calculated from spectral types ( $2.27 \mathrm{M}_{\circ}^{\circ}$ ).

## WDS 14531-4638 = I 952:

Mag. 8.5 and 10.1. Spectral type G5V.
First measurement: 1910.6
Last measurement 2013.1276
Used observations: 14
The components of this system were first resolved by Innes in 1910. The last three measures (since 1996) were taken (by Mason and Tokovinin) using speckle technique. The observations span about 103 years, similar to the orbital period, and therefore they cover a complete revolution. The dynamic parallax and total stellar masses are consistent with Hipparcos trigonometric parallax and with the stellar masses obtained using their spectral types. From combined photometric and spec-

## Orbit Determination of Close Binary Systems



Figure 6. WDS 14531-4638
troscopic data in addition of the differential V magnitude, the individual spectral types of G5V and K1V were determined.

## Acknowledgements

This research has made use of the Washington Double Star Catalogue maintained at the U.S. Naval Observatory, the NASA Astrophysics Data System Bibliographic Services and the SIMBAD database (operated at CDS, Strasbourg, France). The authors thank Frank Smith for their respective reviews of this work.

## References

Abt, H. A. 1981, ApJS, 45, 437.
Anthony-Twarog B. J., Twarog B. A. 1994, $A J, 107$, 1577.

Bidelman, W. P., 1985, ApJS, 59, 197.
Cannon A.J., Pickering E.C. <Harv. Ann. 91-100 (1918-1924)>
Casagrande L. et al., 2011, $A \& A, \mathbf{5 3 0}, 138$.
Docobo, J.A., 1985, CeMec, 36, 143.
Docobo, J. A.; Ling, J. F., 2003, $A \& A, 409,989$.
Fehrenbach, Ch.; Burnage, R.; Figuiere, J., 1992, $A \& A S, 95,541$.
Glebocki R., Gnacinski P., 2005, csss, 13, 571.

Heintz, W. D., 1978, Double Stars (revised edition), Dordrecht, D. Reidel Publishing Co. (Geophysics and Astrophysics Monographs. Volume 15), (1978GAM....15.....H).
Heintz, W.D, 1967, ActaAstr., 17, 311.
Houk N., Smith-Moore M., 1988, Ann Arbor, Dept. of Astronomy, Univ. of Michigan.

Mason, B. D., Douglass, G.G., Hartkopf, W. I., 1999, $A J, 117,1023$.
Nordstrom B. et al., 2004, $A \& A, 418,989$.
Pickles A., Depagne E., 2010, PASP, 122, 1437.
Rica, F. M., Barrena, R., Vázquez. G., Henríquez, J. A., Hernández, F. et al. 2012, MNRAS, 419, 197R.

Schlafly \& Finkbeiner, 2011, ApJ, 737, 103.
Schlegel D.J., Finkbeiner D.P., \& Davis M., 1998, ApJ, 500, 525.
van Leeuwen, F., 2007, Hipparcos, the New Reduction of the Raw Data, Springer, New York (data obtained from Simbad data base: I/311)

## Orbit Determination of Close Binary Systems

## Appendix 1. Observational Data and Residuals

The values in parentheses are the calculated theoretical positions for a given epoch.

| Date | $\vartheta\left({ }^{\circ}\right)$ | $\rho$ (") | N | Ref. | $A p$ | Tec | Cod | $\begin{gathered} 0-\mathrm{C} \text { ( }) \end{gathered}$ | $\begin{gathered} \hline \text { O-C p } \\ \text { (") } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1905.8200 | 107.2 | 0.43 | 3 | A__1906a | 0.9 | Ma |  | 1.8 | -0.02 |
| 1915.6400 | 100.4 | 0.57 | 1 | A__1929a | 0.9 | Ma |  | 3.0 | 0.07 |
| 1917.8200 | 94.2 | 0.58 | 2 | A 1929a | 0.9 | Ma |  | -1.6 | 0.08 |
| 1921.6300 | 93.9 | 0.54 | 2 | A__1929a | 0.9 | Ma |  | 0.9 | 0.03 |
| 1925.8200 | 89.8 | 0.48 | 1 | A__1929a | 0.9 | Ma |  | -0.3 | -0.04 |
| 1929.7900 | 88.8 | 0.51 | 2 | A__1933d | 0.9 | Ma |  | 1.3 | -0.01 |
| 1933.7700 | 84.6 | 0.58 | 2 | Kui1961b | 0.3 | Ma |  | -0.2 | 0.05 |
| 1935.2600 | 78.9 | 0.49 | 2 | VBs 1954 | 1.0 | Ma |  | -4.9 | -0.04 |
| 1943.7900 | 74.9 | 0.52 | 1 | VBs 1954 | 2.1 | Mb |  | -3.2 | 0.00 |
| 1952.4000 | 74.6 | 0.58 | 2 | Mrz1956 | 0.7 | Ma |  | 2.5 | 0.07 |
| 1960.0200 | 63.8 | 0.50 | 1 | Cou1962a | 0.4 | Ma |  | -2.7 | 0.01 |
| 1962.7500 | 64.1 | 0.45 | 4 | B___1963b | 0.9 | Ma |  | -0.3 | -0.04 |
| 1965.1500 | 66.2 | 0.61 | 2 | Baz1967 | 0.4 | Ma |  | 3.7 | 0.13 |
| 1966.7120 | 60.0 | 0.42 | 4 | Wor1971 | 0.7 | Ma |  | -1.3 | -0.06 |
| 1975.8800 | 51.4 | 0.41 | 3 | Hei1978b | 0.6 | Ma |  | -2.1 | -0.04 |
| 1978.8900 | 47.7 | 0.42 | 3 | Heil980a | 0.6 | Ma |  | -3.0 | -0.02 |
| 1985.7400 | 46.6 | 0.36 | 2 | Heil987a | 0.6 | Ma |  | 2.6 | -0.07 |
| 1991.2500 | 28.0 | 0.468 | 1 | HIP1997a | 0.3 | Ht |  | -10.2 | 0.056 |
| 1997.0400 | 37.2 | 0.34 | 2 | Heil998 | 0.6 | Ma |  | 5.5 | -0.06 |
| 2008.764 | 2.0 | 0.514 | 1 | Gii2012 | 0.7 | S |  | -15.2 | 0.136 |
| 2011.8569 | 39.4 | 0.3 | 1 | Gur2018 | 2.1 | S |  | 26.2 | -0.074 |
| 2012.7263 | 356.5 | 0.447 | 1 | Fmr9999 | 1.5 | Cl |  | 15.5 | 0.074 |

Table 6. Measurements of WDS 02048+6030

## Orbit Determination of Close Binary Systems

| Date | $\vartheta\left({ }^{\circ}\right)$ | $\rho$ (") | N | Ref. | Ap | Tec | Cod | $\begin{gathered} \mathrm{O}-\mathrm{C} \text { Э } \\ \left({ }^{\circ}\right) \end{gathered}$ | $\begin{gathered} \hline \text { O-C } \rho \\ (") \\ \hline \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1979.00 | 336.7 | 0.35 | 3 | Hei1980a | 0.6 | Ma |  | -0.4 | -0.06 |
| 1986.99 | 340.0 | 0.34 | 3 | Hei1987a | 0.6 | Ma |  | 12.7 | -0.09 |
| 1991.25 | 322.0 | 0.363 | 1 | HIP1997a | 0.3 | Hh |  | 1.2 | 0.010 |
| 1999.8185 | 205.6 | 0.060 | 1 | Bag2004 | 6.0 | S |  | 0.1 | 0.001 |
| 2000.8760 | 47.6 | 0.031 | 1 | Bag2006b | 6.0 | S |  | 0.1 | 0.000 |
| 2001.7585 | 358.4 | 0.131 | 1 | Bag2006b | 6.0 | S | q | 0.7 | -0.001 |
| 2004.1126 | 340.7 | 0.273 | 1 | Hor2008 | 3.5 | S |  | -3.7 | -0.018 |
| 2004.1126 | 341.4 | 0.291 | 1 | Hor2008 | 3.5 | S |  | -3.0 | 0.000 |
| 2004.1126 | 339.6 | 0.280 | 1 | Hor2008 | 3.5 | S |  | -4.8 | -0.011 |
| 2004.8241 | 343.4 | 0.323 | 1 | Bag2007b | 6.0 | S |  | 0.8 | 0.001 |
| 2004.9726 | 345.4 | 0.334 | 1 | Hor2008 | 3.5 | S |  | 3.1 | 0.006 |
| 2004.9726 | 343.9 | 0.321 | 1 | Hor2008 | 3.5 | S |  | 1.6 | -0.007 |
| 2007.0041 | 337.2 | 0.396 | 1 | Hor2010 | 3.5 | S |  | -1.4 | 0.006 |
| 2007.8178 | 338.0 | 0.419 | 1 | Hor2010 | 3.5 | S |  | 0.6 | 0.011 |
| 2007.8204 | 338.1 | 0.415 | 1 | Hor2010 | 3.5 | S |  | 0.7 | 0.007 |
| 2008.6900 | 335.6 | 0.428 | 1 | Hor2009 | 3.5 | S |  | -0.6 | 0.005 |
| 2011.6906 | 332.1 | 0.4607 | 1 | Hor2017 | 3.5 | S |  | 0.0 | 0.012 |
| 2011.6906 | 332.3 | 0.4636 | 1 | Hor2017 | 3.5 | S |  | 0.2 | 0.015 |
| 2011.8543 | 333.1 | 0.46 | 1 | Gur2018 | 2.1 | S |  | 0.9 | 0.011 |
| 2012.0949 | 332.6 | 0.4557 | 1 | Hor2017 | 3.5 | S |  | 0.7 | 0.007 |
| 2012.0949 | 332.8 | 0.4565 | 1 | Hor2017 | 3.5 | S |  | 0.9 | 0.008 |
| 2016.1338 | 327.3 | 0.4529 | 2 | Tok2018c | 4.1 | St |  | 0.3 | 0.026 |

Table 7. Measurements of WDS 04102+1722

| Date | $\vartheta\left({ }^{\circ}\right)$ | $\rho$ (") | N | Ref. | Ap | Tec | Cod | $\begin{gathered} \hline \mathrm{O}-\mathrm{C} \vartheta \\ \left({ }^{\circ}\right) \end{gathered}$ | $\begin{gathered} \hline \text { O-C } \mathrm{P} \\ \text { (") } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1914.51 | 166.0 | 0.56 | 2 | A__1914c | 0.9 | Ma |  | 0.0 | -0.05 |
| 1921.89 | 167.8 | 0.58 | 2 | A__1932a | 0.9 | Ma |  | 0.0 | 0.00 |
| 1929.20 | 166.8 | 0.59 | 1 | Fur1932 | 0.7 | Ma |  | -3.0 | 0.04 |
| 1933.22 | 168.4 | 0.61 | 2 | Fur1932 | 0.7 | Ma |  | -2.7 | 0.08 |
| 1938.21 | 172.4 | 0.57 | 3 | Baz1942b | 0.3 | Ma |  | -0.3 | 0.06 |
| 1942.88 | 171.2 | 0.58 | 3 | Vou1955 | 0.4 | Ma |  | -3.2 | 0.10 |
| 1961.15 | 184.5 | 0.51 | 3 | Cou1962a | 0.4 | Ma |  | 1.0 | 0.15 |
| 1961.85 | 179.4 | 0.42 | 2 | Cou1962b | 0.9 | Mb |  | -4.6 | 0.07 |
| 1962.25 | 195.9 | 0.40 | 4 | Hei1963b | 0.3 | Ma |  | 11.6 | 0.05 |
| 1963.017 | 188.5 | 0.32 | 4 | Wor1967b | 0.7 | Ma |  | 3.6 | -0.02 |
| 1963.10 | 184.9 | 0.31 | 4 | B__1963b | 0.9 | Ma |  | -0.1 | -0.03 |
| 1965.25 | 188.3 | 0.42 | 4 | Hei1967b | 0.3 | Ma |  | 1.6 | 0.10 |
| 1973.12 | 197.0 | 0.23 | 3 | Hei1975a | 0.6 | Ma |  | 1.5 | -0.01 |
| 1978.12 | 213.6 | 0.15 | 3 | Hei1980a | 0.6 | Ma |  | 8.2 | -0.03 |
| 1985.8408 | 262.6 | 0.058 | 1 | McA1987b | 4.0 | Sc | P | -0.4 | 0.000 |
| 1987.03 | 100.0 | 0.74 | 1 | Aru1992 | 1.0 | C | T | (327.1) | (0.03) |
| 1991.2500 | 128.0 | 0.132 | 1 | HIP1997a | 1.4 | Hh |  | 13.9 | -0.010 |
| 2004.2034 | 134.1 | 0.341 | 2 | Hrt2008 | 1.5 | Su |  | -0.7 | -0.002 |
| 2008.111 | 135.9 | 0.396 | 1 | Gii2012 | 0.7 | S |  | -1.3 | 0.012 |

Table 8. Measurements of WDS 06549+1158

## Orbit Determination of Close Binary Systems

| Date | $\vartheta\left({ }^{\circ}\right)$ | $\rho$ (") | N | Ref. | Ap | Tec | Cod | $\begin{gathered} \hline \text { O-C } \\ \left({ }^{\circ}\right) \end{gathered}$ | $\begin{gathered} \text { O-C } \mathrm{P} \\ (\mathrm{l} \text { ) } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1829.29 | 255.4 | 1.01 | 3 | StF1837 | 0.3 | Ma |  | 3.8 | -0.10 |
| 1844.30 | 252.2 | 1.02 | 1 | Mad1845 | 0.3 | Ma | Q | -0.2 | -0.06 |
| 1848.35 | 259.5 | 0.92 | 1 | Mad1856 | 0.3 | Ma | Q | 6.9 | -0.15 |
| 1856.28 | 255.2 | 0.90 | 1 | Se_1860b | 0.3 | Ma | Q | 2.1 | -0.15 |
| 1866.30 | 252.2 | 0.86 | 3 | D__1884 | 0.2 | Ma |  | -1.5 | -0.16 |
| 1892.81 | 254.8 | 0.85 | 4 | Sp_1909 | 0.5 | Ma |  | -0.7 | -0.05 |
| 1896.50 | 252.0 | 0.94 | 4 | A__1914d | 0.3 | Ma |  | -3.8 | 0.06 |
| 1898.32 | 260.2 | 0.99 | 2 | Glp1899 | 0.2 | Ma | Q | 4.2 | 0.12 |
| 1899.28 | 251.5 | 0.69 | 1 | Bry1899 | 0.7 | Ma |  | -4.6 | -0.18 |
| 1899.29 | 258.6 | 0.72 | 1 | L__1899 | 0.7 | Ma |  | 2.5 | -0.15 |
| 1900.33 | 259.9 | 0.86 | 2 | Bow1900 | 0.7 | Ma |  | 3.7 | 0.00 |
| 1901.32 | 250.4 | 0.78 | 1 | Bry1901 | 0.7 | Ma |  | -5.9 | -0.08 |
| 1901.38 | 251.9 | 0.96 | 1 | L__1901 | 0.7 | Ma |  | -4.4 | 0.10 |
| 1903.28 | 260.0 | 0.84 | 1 | L__1903 | 0.7 | Ma |  | 3.6 | 0.00 |
| 1903.29 | 257.0 | 0.91 | 3 | Bow1903a | 0.7 | Ma |  | 0.6 | 0.07 |
| 1904.34 | 256.3 | 0.93 | 1 | Bow1904a | 0.7 | Ma |  | -0.2 | 0.09 |
| 1906.28 | 253.6 | 1.01 | 1 | Frm1907 | 0.2 | Ma |  | -3.1 | 0.19 |
| 1908.32 | 257.7 | 0.87 | 3 | Bow1908 | 0.7 | Ma |  | 0.8 | 0.06 |
| 1909.35 | 257.2 | 1.04 | 1 | Bow1909 | 0.7 | Ma | Q | 0.2 | 0.24 |
| 1909.35 | 262.0 | 0.77 | 1 | L__1909 | 0.7 | Ma | Q | 5.0 | -0.03 |
| 1911.12 | 252.6 | 0.90 | 3 | Doo1915b | 0.5 | Ma | Q | -4.6 | 0.11 |
| 1911.57 | 259.1 | 0.76 | 3 | Wz_1923 | 0.5 | Ma |  | 1.9 | -0.03 |
| 1912.25 | 262.4 | 0.68 | 3 | Bow1921 | 0.7 | Ma | Q | 5.1 | -0.10 |
| 1914.30 | 260.1 | 0.76 | 2 | Rab1923 | 0.2 | Ma |  | 2.6 | -0.01 |
| 1916.34 | 255.2 | 0.70 | 2 | J__1918 | 0.7 | Ma |  | -2.6 | -0.05 |
| 1924.19 | 256.2 | 0.73 | 4 | B__1925a | 0.3 | Ma |  | -2.5 | 0.05 |
| 1925.71 | 261.9 | 0.67 | 3 | Fat1928 | 0.3 | Ma |  | 2.9 | 0.00 |
| 1934.37 | 256.9 | 0.57 | 4 | Baz1936b | 0.3 | Ma |  | -3.5 | 0.00 |
| 1941.97 | 257.4 | 0.51 | 5 | Rab1953 | 0.3 | Ma |  | -4.8 | 0.04 |
| 1943.13 | 259.3 | 0.57 | 3 | Vou1955 | 0.4 | Ma |  | -3.2 | 0.11 |
| 1950.58 | 261.3 | 0.29 | 4 | Baz1952b | 0.4 | Ma | XQ | -4.4 | -0.04 |
| 1953.39 | 233.6 | 0.25 | 4 | Baz1954a | 0.4 | Ma | Q | (267.8) | (0.26) |
| 1954.24 | 273.6 | 0.20 | 2 | Cou1955c | 0.4 | Ma | X | 5.0 | -0.04 |
| 1954.40 | 253.1 | 0.26 | 1 | Mlr1955c | 0.5 | Mc | Q | -15.7 | 0.02 |
| 1955.83 | 253.1 | 0.22 | 2 | Baz1957b | 0.4 | Ma |  | -17.6 | 0.02 |
| 1956.18 | 273.9 | 0.19 | 3 | Mlr1956a | 0.6 | Mc |  | 2.6 | 0.00 |
| 1956.27 | 278.8 | 0.20 | 1 | Cou1958a | 0.4 | Ma |  | 7.3 | 0.01 |
| 1957.31 | 278.0 | 0.20 | 1 | Cou1958a | 0.4 | Ma |  | 4.2 | 0.04 |
| 1958.05 | 277.2 | 0.13 | 2 | VBs1960 | 2.1 | Mb |  | 1.1 | 0.00 |
| 1960.25 | 267.0 | 0.19 | 1 | Cou1962a | 0.4 | Ma |  | (297.1) | (0.05) |
| 1960.25 |  |  | 1 | Cou1962a | 0.4 | Ma | S | (297.1) | (0.05) |
| 1960.32 |  | 0.25 | 3 | Hei1961 | 0.3 | Ma | S | (298.9) | (0.05) |
| 1961.27 |  |  | 1 | Cou1962a | 0.4 | Ma | X | (0.5) | (0.02) |
| 1962.23 |  |  | 1 | B__1963b | 0.9 | Ma | F | (61.4) | (0.05) |
| 1965.140 | 43.3 | 0.13 | 1 | VBs1975 | 2.1 | Mb | Q | (89.7) | (0.12) |
| 1967.32 | 94.4 | 0.15 | 2 | Cou1968b | 0.5 | Ma | Q | -4.1 | 0.02 |
| 1969.231 | 104.8 | 0.17 | 1 | Wak1972 | 1.5 | Mb | Q | -0.8 | 0.04 |
| 1971.24 |  | 0.16 | 1 | M1r1976 | 0.8 | Mc | U | (113.0) | (0.13) |

Table 9. Measurements of WDS 11361+1251. Continues on the next page.

## Orbit Determination of Close Binary Systems

| Date | $\vartheta\left({ }^{\circ}\right)$ | م (") | N | Ref. | Ap | Tec | Cod | $\begin{gathered} \mathrm{O}-\mathrm{C} \text { ( }{ }^{\circ} \text { Э } \end{gathered}$ | $0-C \rho$ <br> (") |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1976.38 |  | 0.18 | 3 | M1r1978b | 0.5 | Ma | D | (133.2) | (0.13) |
| 1981.65 | 158.2 | 0.14 | 3 | Mrl1983 | 0.7 | Ma |  | 3.6 | 0.01 |
| 1982.30 | 177.0 | 0.10 | 2 | Heil983a | 0.6 | Ma | U | 19.8 | -0.03 |
| 1984.24 | 152.8 | 0.14 | 3 | Cou1985a | 0.7 | Ma |  | -11.6 | 0.01 |
| 1986.24 | 144.6 | 0.13 | 1 | Cou1987b | 0.7 | Ma |  | (171.3) | (0.14) |
| 1988.32 | 163.9 | 0.15 | 1 | LBu1989 | 0.5 | Ma |  | -13.8 | 0.00 |
| 1989.36 | 196.8 | 0.15 | 5 | LBu1990 | 0.5 | Ma |  | 16.1 | 0.00 |
| 1990.320 | 191.9 | 0.15 | 1 | LBu1991 | 0.5 | Ma |  | 8.7 | -0.01 |
| 1991.2500 | 187.0 | 0.163 | 1 | HIP1997a | 1.4 | Ma |  | 1.4 | 0.001 |
| 1993.21 | 195.8 | 0.15 | 1 | LBu1994 | 0.5 | Ma |  | 5.7 | -0.02 |
| 1994.40 | 194.8 | 0.15 | 1 | LBu1996 | 0.5 | Ma |  | 2.2 | -0.03 |
| 2004.2068 | 202.8 | 0.228 | 1 | Hrt2008 | 1.5 | S |  | -4.7 | -0.010 |
| 2008.251 | 204.9 | 0.269 | 1 | Gii2012 | 0.7 | S |  | -6.9 | 0.007 |
| 2008.281 | 204.8 | 0.237 | 1 | Gii2012 | 0.7 | S |  | -7.0 | 0.026 |
| 2010.3525 | 208.7 | 0.26 | 1 | Orl2015 | 2.1 | S |  | -4.9 | -0.016 |

Table 9 (conclusion). Measurements of WDS 11361+1251

| Date | $\vartheta\left({ }^{\circ}\right)$ | $\rho(1)$ | N | Ref. | Ap | Tec | Cod | $\begin{gathered} 0-\mathrm{C} \\ \left({ }^{\circ}\right) \end{gathered}$ | $\text { O-C } \rho$ (") |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1878.30 | 97.9 | 1.38 | 2 | Bu_1879 | 0.5 | Ma |  | -0.8 | -0.02 |
| 1882.41 | 134.5 | 0.40 | 2 | Sp_1888 | 0.2 | Ma | U | (98.3) | (1.36) |
| 1890.70 | 97.8 | 1.20 | 4 | Sp_1909 | 0.5 | Ma |  | -0.1 | -0.10 |
| 1891.26 | 99.1 | 1.25 | 3 | Bu_1894 | 0.9 | Ma |  | 1.3 | -0.04 |
| 1898.917 | 96.8 | 1.40 | 3 | Doo1901 | 0.5 | Ma |  | -0.4 | 0.18 |
| 1899.431 | 101.0 | 1.19 | 1 | Brs1900b | 0.7 | Ma |  | 3.8 | -0.02 |
| 1899.431 | 100.9 | 1.45 | 1 | See1900e | 0.7 | Ma |  | 3.7 | 0.24 |
| 1899.432 | 101.0 | 1.32 | 1 | Brs1911 | 0.7 | Ma |  | 3.8 | 0.11 |
| 1900.324 | 92.0 | 0.88 | 2 | See1911 | 0.7 | Ma |  | -5.1 | -0.33 |
| 1902.423 | 94.9 | 1.34 | 3 | Doo1905 | 0.5 | Ma |  | -2.0 | 0.16 |
| 1909.32 | 95.2 | 1.10 | 1 | Ol_1909 | 0.7 | Ma |  | -1.1 | -0.01 |
| 1915.648 | 97.7 | 1.16 | 5 | Fox1925 | 0.5 | Ma |  | 2.0 | 0.12 |
| 1917.20 | 96.2 | 1.10 | 3 | O1_1920c | 0.7 | Ma |  | 0.7 | 0.08 |
| 1921.82 | 103.6 | 1.16 | 2 | Gcb1934 | 0.4 | Ma |  | 8.7 | 0.20 |
| 1938.10 | 95.4 | 0.80 | 3 | Fin1951a | 0.7 | Ma |  | 3.1 | 0.06 |
| 1949.45 | 85.6 | 0.49 | 4 | B__1950c | 0.7 | Ma |  | -3.5 | -0.08 |
| 1952.83 | 83.5 | 0.42 | 4 | B__1953a | 0.7 | Ma |  | -4.2 | -0.10 |
| 1955.50 |  |  | 5 | Hei1956a | 0.2 | Ma |  | (86.4) | (0.47) |
| 1959.17 | 78.2 | 0.29 | 2 | B__1959d | 0.7 | Ma |  | -6.0 | -0.12 |
| 1962.30 | 78.3 | 0.33 | 4 | B__1962d | 0.9 | Ma |  | -3.4 | -0.03 |
| 1989.39 | 325.0 | 0.10 | 2 | Hei1990b | 1.0 | Mb | Q | 8.3 | -0.08 |
| 1991.25 | 326.0 | 0.199 | 1 | HIP1997a | 1.4 | Ht |  | 15.1 | -0.011 |
| 2006.1997 | 292.1 | 0.456 | 2 | Msn2009 | 4.0 | S |  | 0.0 | 0.007 |
| 2014.244 | 289.3 | 0.54 | 1 | Ant2015 | 0.5 | Cl |  | 0.9 | -0.033 |
| 2016.3699 | 289.3 | 0.646 | 5 | Hsw2017d | 0.3 | Cv |  | 1.6 | 0.042 |

Table 10. Measurements of WDS 06549+1158

## Orbit Determination of Close Binary Systems

| Date | $\vartheta\left({ }^{\circ}\right)$ | $\rho$ (") | N | Ref. | Ap | Tec | Cod |  | $\begin{gathered} \hline \text { O-C } \rho \\ (1) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1910.60 | 300.0 | 0.65 | 2 | I__9999 | 9 | A |  | -24.7 | -0.05 |
| 1913.53 | 319.8 | 0.78 | 2 | I__1914 | 9 | A |  | -0.9 | 0.06 |
| 1928.52 | 301.9 | 0.74 | 2 | B__1928d | 26 | A |  | -1.2 | -0.05 |
| 1928.95 | 301.7 | 0.77 | 3 | Rst1955 | 27 | A |  | -0.9 | -0.02 |
| 1930.33 | 301.9 | 0.80 | 4 | Vou1932 | 24 | A |  | 0.8 | 0.01 |
| 1934.55 | 298.1 | 0.75 | 4 | B__1937b | 26 | A |  | 1.6 | -0.04 |
| 1956.41 | 270.4 | 0.71 | 3 | B__1957b | 26 | A |  | 1.2 | 0.05 |
| 1960.50 | 260.2 | 0.60 | 3 | B__1961a | 26 | A |  | -2.0 | -0.01 |
| 1985.36 | 42.5 | 0.23 | 2 | Heil987a | 36 | B | Q | -1.7 | -0.02 |
| 1991.18 | 4.6 | 0.37 | 2 | Heil992a | 40 | B | Q | -0.8 | -0.04 |
| 1991.25 | 5.0 | 0.404 | 1 | HIP1997a | 54 | T |  | -0.1 | -0.003 |
| 1996.1815 | 349.6 | 0.512 | 1 | Msn1998b | 158 | S | Q | 0.4 | 0.001 |
| 2009.2602 | 324.7 | 0.692 | 1 | Tok2009 | 160 | S | Q | -0.1 | 0.001 |
| 2013.1276 | 319.6 | 0.7284 | 2 | Tok2014a | 4.1 | St |  | -0.1 | 0.004 |

Table 11. Measurements of WDS 14531-4638

# The Number of Binaries in the Sky Compared to a Random Distribution of Similar Stars 

T. V. Bryant III<br>Little Tycho Observatory<br>703 McNeill Road, Silver Spring, Md 20910<br>NGC7492@gmail.com


#### Abstract

The likelihood of a pair of stars orbiting one another is investigated using statistics generated from the Gaia DR2 data[1]. Stars brighter than 11, 15, and 18 G band magnitude (Gmag) were searched for pairs with separations less than 60 ". The Gaia DR2 stars' positions were then randomized and again searched. The differences between pairs found in the real sky vs the randomized sky indicate that the likelihood that a pair is physical is inversely proportional to the pairs' separation.


When two stars are close together in the sky, they are often listed as double stars. Whether a particular pair actually orbit one another or is simply in the same line of sight as seen from the earth has been debated for at least a century. Below is an excerpt from R. Aitken's 1918 book The Binary Stars[2].

The data consist of all visual double stars as bright as 0.9 B. D. magnitude which fall within the distance limits set by the following 'working' definition of a double star proposed by me in 1911:
(1) Two stars shall be considered to constitute a double star when the apparent distance between them falls within the following limits:
$1 "$ if the combined magnitude of the components is fainter than 11.0
$3 "$ if the combined magnitude of the components is fainter than 9.0 B. D.

5 " if the combined magnitude of the components lies between 6.0 and 9.0 B. D.
$10^{\prime \prime}$ if the combined magnitude of the components lies between 4.0 and 6.0 B. D.

20" if the combined magnitude of the components lies between 2.0 and 4.0 B. D.
$40^{\prime \prime}$ if the combined magnitude of the components is brighter than 2.0 B. D.
(2) Pairs which exceed these limits shall be entitled to the name double star only when it has been shown
(a) that orbital motion exists;
(b) that the two components have a well defined common proper motion, or proper motions of the 61 Cygni type;
(c) that the parallax is decidedly greater than the average for stars of corresponding magnitude.

Aitken's time as the numbers and accuracies of proper motion, parallax, and radial velocity studies have increased significantly. There are, however, other ways to try to analyze this question.

In this paper, a statistical treatment of all of the Gaia DR2 stars brighter than 11.0 Gmag (1,240,319 stars), 15.0 Gmag ( $35,399,780$ stars), and 18.0 Gmag ( $299,758,720$ stars) was done using the programs that can be obtained from the SourceForge site(4). Each of these stars was searched in this way:

The area within a radius of $60^{\prime \prime}$ of the given star was searched for the star that was closest to that star and would be selected as the other member of the pair.

The pair found was tested for its commonality of distance, spatial velocity, and orbital velocity. If the parameters were within certain limits (see below), the pair was counted as a binary.

Once a star had been identified as a member of a pair, it was flagged as such and removed from the database of stars being searched.

These searches were then repeated using the same stars, with their positions randomized. Randomization of a star's position consisted of moving the star an arc minute from its J2000 position in a random direction. The results of the 6 runs that searched for the closest star are shown here, with the number of pairs or binaries found on the vertical axis, and their separation in arcseconds on the horizontal axis. Note that the magenta line represents pairs and binaries found in the real

The Number of Binaries in the Sky Compared to a Random Distribution of Similar Stars


Figure 1


Figure 3


Figure 5


Figure 2


Figure 4


Figure 6

## The Number of Binaries in the Sky Compared to a Random Distribution of Similar Stars

(Continued from page 42)
sky, and the green line represents pairs and binaries found in the randomized sky. Comparing the real sky with the randomized sky gives an idea as to the number of actual binaries vs the number of optical pairs in the sky.

## Pairs within 2.0 Gmag of One Another.

A majority of the pairs in the WDS were discovered visually. Pairs with a large brightness difference were either ignored or not seen, as the difficulty of resolving a close pair is a function of the difference in brightness of that pair. The Figures 1-6 do not consider this fact. Would the results be different if the stars were required to be close in brightness as well as the other criteria? The programs were revised so as to exclude pairs that were more than 2.0 Gmag different in brightness. Figures 7-12 are the results for the 11.0 Gmag, 15.0 Gmag, and 18.0 Gmag runs with pairs required to be within 2.0 Gmag of one another. These six plots show the results for the closest pairs found within an arcminute of a given star.

The number of pairs and binaries is much reduced by limiting the brightness differential to 2 Gmag , but the overall shape of the curves is unchanged.

## Average Distances of Binaries with Respect to Their Separation.

When the program found that a pair was a possible binary, the distance to the pair, that is the average distance (in light years) of both members from the sun, was computed and saved in its arc second separation bin. When the run ended, the average distance of all binaries found in that separation bin was computed. Please note that Figures 13-18 do not show the number of binaries found in each separation bin. Refer to Figures 7-12 for that data, although in most cases, the spikes on Figures 13-18 only represent one or two stars, so making any statistical trends based on this data inaccurate, with the possible exception of the 18 Gmag data (Figures 17 and 18).

## Discussion

The number of pairs less than 1 to 2 arc seconds apart is under counted as it is close to the resolution on the Gaia telescope, especially in a crowded star field.

In order for a pair to be considered for binary membership, both stars needed to have a Gaia DR2 parallax recorded for them. Over $98 \%$ of all stars brighter than 18.0 Gmag did.

The chance of a pair of stars being close to one another in the sky is a function of the number of stars in the sky. Note that there are about 30 times more stars brighter than 15 Gmag than 11 Gmag, and about 240
times as many 18 Gmag stars than 11 Gmag. This is reflected in the above plots, which show the ratio of the number of pairs and binaries found in the actual sky compared to those found in the randomized sky decreases as the number of stars searched increases. Note that there are so many pairs found in the 18 Gmag data that most stars are found to be part of a pair, and only when they are checked for binary membership do we see a difference between the real and randomized sky.

The criteria for determining if a pair was mutually orbiting one another were as follows. The ratio of the star's spatial velocities over their estimated orbital velocities needed to be within a factor of 10 of each other and their parallaxes needed to indicate that the stars' distances were within a light year of one another. This distance was chosen as the average distance of stars in the Milky Way is about 5 light years [3]. Two solar mass stars in a circular orbit at this distance take about 6 million years, and after a few orbits, there is a significant chance that the pair will encounter other stars that will disrupt their orbit, so a light year was thought to be a good limit on a stable binaries' orbital diameter.

The stars' orbital velocities were calculated as described by Rica[4]:

$$
\begin{aligned}
& M=L^{(1 / 3.5)} \\
& m=l^{(1 / 3.5)} \\
& V=\sqrt{\frac{1.9891 \times 10^{30} G(M+m)}{d r}}
\end{aligned}
$$

Where:

- $\quad d r$ is the average distance to the stars multiplied by their apparent separation in radians.
- $\quad L$ is the first star's luminosity, in solar units.
- $\quad l$ is the second star's luminosity, in solar units.
- $\quad M$ is the mass of the first star.
- $\quad m$ is the mass of the second star.
- $\quad V$ is the orbital velocity, in $\mathrm{m} / \mathrm{s}$.
- $G$ is the gravitational constant, $6.67408 \times 10^{-11}$.
- The mass of the sun is $1.989 \times 10^{30} \mathrm{~kg}$.

The stars are assumed to be main sequence stars, making the relationship approximately valid.

Note that this simple calculation makes the unlikely assumptions that the orbits are circular, perpendicular to the plane of the sky, with their velocity vectors pointing at the sun. The generous limits of a light year's separation and an order of magnitude between this orbital velocity and the observed spatial velocity are used to
(Text continues on page 47)

The Number of Binaries in the Sky Compared to a Random Distribution of Similar Stars


Figure 7.


Figure 9.


Figure 11.


Figure 8.


Figure 10.


Figure 12.

The Number of Binaries in the Sky Compared to a Random Distribution of Similar Stars


Figure 13.


Figure 15.


Figure 17.


Figure 14.


Figure 16.


Figure 18.

The Number of Binaries in the Sky Compared to a Random Distribution of Similar Stars
(Continued from page 44)
offset the assumptions made about the orbits. Even with these wide margins, the number of binaries found drops markedly when compared to the number of pairs found.

Initially, it was thought that if the parallax uncertainty limits of the two stars in a pair overlapped, then the pair would be considered to be at approximately the same distance from the sun. This method was abandoned when it was found that over $99 \%$ of pairs met this criterion. It was replaced by the requirement that the parallaxes of the two stars needed to indicate that they were within a light year of one another.

The results for the 18.0 Gmag runs simply indicate that virtually all stars have another star 18 Gmag or brighter within an arc minute of them. This was somewhat true of the 15.0 Gmag runs as well. These often optical pairs were minimized in the 11.0 Gmag run. In all runs, however, the above results indicate that the number of binaries found favors the nearer pairs.

## Conclusion

Aitken's 1911 guesses were perhaps a bit too conservative, but the basic idea, the closer a given pair, the more likely it is to be a binary, is sound.

While careful measurements over time of the position angle, separation, radial velocities, and parallaxes of a pair are required to determine if the pair is indeed a binary, an initial guess can be made simply from the pair's separation. While using these graphs to determine if a pair is a binary is only a rough estimate of a given pair's binary nature, they might well be useful in selecting which pairs should be chosen for further study, much like Aitken's 1918 criteria were.

## Acknowledgements

The grammar and clarity of this paper were greatly enhanced by Tom Corbin's and Kathie Bryant's edits.

This work has made use of data from the European Space Agency (ESA) mission Gaia (https:// www.cosmos.esa.int/gaia), processed by the Gaia Data Processing and Analysis Consortium (DPAC, https:// www.cosmos.esa.int/web/gaia/dpac/consortium). Funding for the DPAC has been provided by national institutions, in particular the institutions participating in the Gaia Multilateral Agreement.

## References

[1]The Gaia DR2 data can be downloaded here: http:// cdn.gea.esac.esa.int/Gaia/gdr2/gaia_source/csv/
[2] R. G. Aitken, The Binary Stars, 1918, The University of California.
[3] http://boojum.as.arizona.edu/~jill/EPO/Stars/ galaxy.html
[3] F. M. Rica, "Determining the Nature of a Double Star: The Law of Conservation of Energy and the Orbital Velocity", Journal of Double Star Observations, 7 (4), 254, 2011.
[4] The C programs used to create, search and randomize the Gaia RD2 data can be downloaded here: http://sourceforge.net/p/realvsrandomdoublestarcount. The plots were created using gnuplot: http:// www.gnuplot.info/

# Counter-Check of Reported Common Origin Pairs 

Wilfried R.A. Knapp<br>Vienna, Austria<br>wilfried.knapp@gmail.com


#### Abstract

All stars are born in molecular clouds most likely together with other stars nearby in the same cloud but most such systems are separated over time by the tidal forces of the galaxy. Kamdar et al. 2019 report the detection of 111 pairs of co-moving stars with similar metallicity assumed to be born together but separated later on.

This report counter-checks this proposition by cross-matching the listed objects with the GAIA DR2 catalog and using the found data to calculate the spatial distance between the components as well as spatial velocity speed and direction. The results confirm with some caveats the data given in the Kamdar et al. 2019 paper but do not necessarily confirm the conclusion that all reported pairs have to be indeed of common origin.

Finally all WDS pairs listed as common proper motion pairs (note code "V") but with spatial separation likely too large for gravitational relationship are checked for common origin.


## 1. Introduction

Kamdar et al. 2019 report 111 co-moving pairs in the solar neighborhood (which means up to 1 kpc distance from the Sun) with distances between the components too large to allow for gravitational relationship but assumed to be of common origin. This report coun-ter-checks this proposition using astrometric data from GAIA DR2 and metallicity data from the GAIA DR2 StarHorse catalog.

## 2. Cross-Match of WDS FAR Objects with Gaia DR2

The number of KMD objects (for objects reported in Kamdar et al. 2019) is small enough to access the GAIA DR2 data for the counter-check manually by entering the positions of the components directly into Aladin and load the GAIA DR2 data over the default DSS images. The GAIA DR2 data is then copied into a spreadsheet checking for common proper motion and potential gravitational relationship based on Monte Carlo simulation for the distance between the components with a sample size of 120,000 which means a margin of error of $0.37 \%$ at $99 \%$ confidence. The resulting data is then copied again in another spreadsheet created specifically for this purpose calculating spatial velocity speed and direction. The results are given in table 1 below and confirm the values given in the Kamdar et al. 2019 pa-
per for all objects with a few minor exceptions. The additional information from the LAMOST DR4 catalog suggesting similar metallicities giving additional support for the proposition that these pairs are indeed most likely of common origin are counter-checked by comparison with GAIA DR2 StarHorse metallicity data (Anders et al. 2019).

Table 1 lists the cross-matching results with the following structure:

- $\quad \mathrm{Obj}=$ Running object number
- $\quad$ Disc $=$ WDS discoverer code in case of components (mostly A) overlapping with existing WDS objects
- $\quad$ No $=$ Number of additional GAIA DR2 objects with similar values for proper motion, parallax and radial velocity but mostly with spatial velocity not similar enough to be considered also comoving
- $\quad$ CPMS $=$ Common proper motion score (see Appendix)
- Plx1 = Parallax 1 in mas
- e_Plx1 = Error parallax 1
- $\quad \operatorname{Plx} 2=$ Parallax 2 in mas
- e_Plx2 = Error parallax 2
- Min_D_AU $=$ Minimum spatial distance in AU


## Counter-Check of Reported Common Origin Pairs

between components (see Appendix)

- Med_D_AU = Median spatial distance in AU between components (see Appendix)
- Max_D_AU = Maximum spatial distance in AU between components (see Appendix)
- $\quad$ LPGR $=$ Likelihood of potential gravitational relationship (see Appendix)
- $\quad \mathrm{V} 1=$ Spatial velocity $1 \mathrm{in} \mathrm{km} / \mathrm{s}$
- $\mathrm{V} 2=$ Spatial velocity 2 in $\mathrm{km} / \mathrm{s}$
- $\quad$ DV1 $=$ Direction of spatial velocity 1 in degrees
- $\quad$ DV2 $=$ Direction of spatial velocity 2 in degrees
- $\quad \mathrm{AV} 1=$ Angle between spatial and radial velocity 1 in degrees $\left(<0-45^{\circ}\right.$ more radial, $>45-90^{\circ}$ more tangential)
- $\quad$ AV2 $=$ Angle between spatial and radial velocity 2 in degrees $\left(<0-45^{\circ}\right.$ more radial, $>45-90^{\circ}$ more tangential)

The proper motion vector direction and length for 31 of the listed objects is similar enough to consider these objects as common proper motion pairs.

15 objects have parallax values similar enough to give a likelihood larger than $5 \%$ for a spatial distance between the components smaller than 200,000 AU suggesting potential gravitational relationship. The postulated non-existence of a gravitational relationship is therefore not completely ensured for all listed objects.

Four objects have a difference in spatial velocity larger than $10 \%$ of the average speed of the components (values given in red type), speaking against common movement. Six objects have differences in the direction of the spatial velocity larger than $10^{\circ}$ (values given in red type), also speaking against common movement. Seven objects have a difference in the angle between radial and spatial velocity larger than $5^{\circ}$ (values given in red type), also speaking against common movement.

Combining these factors (with the exception of common proper motion) results in 28 objects showing rather not common movement or with a small likelihood potential gravitational relationship.

## 3. Comparison of LAMOST Effective Temperature and Metallicity Values with data from the Gaia DR2 StarHorse Catalog

As additional information for the listed objects, I selected the median mass values from the Gaia DR2 StarHorse catalog (Anders et al. 2019) as well as the median effective temperature data given there listed in Table 2 below.

- $\quad \mathrm{Obj}=$ Running KMD object number
- Source_ID1 = GAIA DR2 source ID
- mass50_1 = Median GAIA DR2 StarHorse Sun mass for the primary
- teff50_1 = Median GAIA DR2 StarHorse effective temperature for the primary
- dTeff_1 = Difference effective temperature between LAMOST and GAIA DR2 StarHorse cata$\log$ for the primary
- X-out_1 = LAMOST effective temperature outside percentile 16 to 84 GAIA DR2 StarHorse values for the primary
- $\quad$ Source_ID2 $=$ GAIA DR2 source ID
- mass50_2 = Median GAIA DR2 StarHorse Sun mass for the secondary
- $\quad$ teff50_2 $=$ Median GAIA DR2 StarHorse effective temperature for the secondary
- dTeff_2 = Difference effective temperature between LAMOST and GAIA DR2 StarHorse cata$\log$ for the secondary
- X-out_2 = LAMOST effective temperature outside percentile 16 to 84 GAIA DR2 StarHorse values for the secondary
- dmass $=$ Difference in GAIA DR2 StarHorse mass between primary and secondary

Most interesting are the differences between the LAMOST effective temperature values (as given in the Kamdar et al. 2019 report) and the corresponding values in the Gaia DR2 StarHorse catalog with a mean value of 183.512 (which means that the Gaia DR2 StarHorse values are generally somewhat higher) and a standard deviation of 328.036 with a few outliers as for example object 20 and 58 . The error range of the given LAMOST values is below 40 while the spread between the 16 and the 84 percentile values is close to 700 . Yet about $30 \%$ of the LAMOST values are outside of the corresponding 16 and 84 percentile values, but this does not allow for any conclusions as such a percentage is by definition to expect from such percentile values.

On average the median Gaia DR2 StarHorse masses for the 111 pairs are quite similar with an average difference of 0.094 with a standard deviation of 0.117 with a few outliers, especially objects 20,70 and 85. Same origin should mean same age and same composition, so different mass should account for different effective temperature - this conclusion is not fully confirmed by the listed data as two of the pairs with the largest differences in mass are listed with rather similar effective temperatures.

The comparison of differences in mass with differences in Teff shows. with the exception of a few outliers, a good relationship between the values of the Gaia DR2 StarHorse catalog (see Figure 1).

Completely different impression when compar-

Counter-Check of Reported Common Origin Pairs

| Obj | Disc | No | CPMS | P1x1 | e_Plx1 | P1x2 | e_Plx2 | Min_D_AU | Med_D_AU | Max_D_AU | LPGR | v1 | v2 | DV1 | DV2 | AV1 | AV2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  | 2 | 4 | 6.1510 | 0.0420 | 6.2039 | 0.0423 | 2490518 | 2562572 | 3084989 | 0.00 | 26.72617 | 25.95566 | 119.28 | 117.61 | 64.64 | 63.52 |
| 2 |  | 2 | 0 | 5.9359 | 0.0418 | 5.6563 | 0.0351 | 1340812 | 2168992 | 3584779 | 0.00 | 6.41619 | 7.57869 | 327.67 | 301.54 | 14.17 | 15.31 |
| 3 |  | 2 | 59 | 6.5862 | 0.0586 | 6.6346 | 0.0483 | 2255418 | 2327511 | 2898098 | 0.00 | 34.62374 | 34.44556 | 132.68 | 134.04 | 69.89 | 67.52 |
| 4 |  | 3 | 5 | 2.9011 | 0.0248 | 2.8558 | 0.1259 | 1819541 | 2950889 | 18609962 | 0.00 | 27.10355 | 26.32522 | 170.73 | 170.68 | 46.27 | 43.75 |
| 5 |  | 5 | 5 | 8.7257 | 0.0486 | 10.0437 | 0.0617 | 3344882 | 3994724 | 4599410 | 0.00 | 52.92963 | 52.32685 | 140.99 | 140.55 | 30.02 | 36.21 |
| 6 |  | 2 | 15 | 7.9693 | 0.0409 | 8.0168 | 0.0565 | 3174971 | 3239318 | 3423317 | 0.00 | 21.55138 | 21.05706 | -227.22 | -229.32 | 89.37 | 87.08 |
| 7 |  | 2 | 0 | 4.7087 | 0.0529 | 4.5288 | 0.0494 | 2588252 | 3170787 | 5566692 | 0.00 | 20.12437 | 19.62902 | 143.27 | 149.69 | 60.49 | 60.98 |
| 8 |  | 2 | 95 | 2.5524 | 0.0369 | 2.5066 | 0.0351 | 25185 | 1601611 | 8772892 | 6.40 | 38.75502 | 39.67870 | 101.69 | 101.41 | 89.03 | 87.86 |
| 9 |  | 2 | 0 | 8.0198 | 0.0834 | 8.3657 | 0.0426 | 3364319 | 3603404 | 4173810 | 0.00 | 12.45912 | 12.26464 | -248.76 | -244.17 | 40.13 | 52.69 |
| 10 | SMA 39 | 2 | 62 | 2.4601 | 0.0428 | 2.4956 | 0.0371 | 4441 | 1552725 | 9550162 | 6.87 | 44.63466 | 45.33590 | 153.93 | 154.95 | 56.28 | 55.42 |
| 11 |  | 2 | 1 | 2.6953 | 0.0408 | 2.6380 | 0.0395 | 794142 | 1932572 | 8960889 | 0.00 | 29.54426 | 27.46444 | 141.54 | 144.34 | 22.62 | 23.53 |
| 12 |  | 2 | 18 | 2.5369 | 0.0431 | 2.5306 | 0.0361 | 2297731 | 2696252 | 8745762 | 0.00 | 19.56407 | 20.59840 | 143.63 | 140.70 | 33.78 | 31.90 |
| 13 |  | 2 | 95 | 2.7248 | 0.0393 | 2.7821 | 0.0411 | 12630 | 1637446 | 8280270 | 6.12 | 37.57576 | 36.72981 | -255.92 | -255.52 | 41.68 | 41.87 |
| 14 |  | 3 | 0 | 1.5758 | 0.0412 | 1.5849 | 0.0435 | 3242973 | 4819195 | 23286169 | 0.00 | 21.00581 | 21.58346 | 183.80 | 188.76 | 55.20 | 54.38 |
| 15 |  | 2 | 0 | 4.3440 | 0.0453 | 4.5413 | 0.0479 | 2504468 | 3286001 | 5916375 | 0.00 | 23.62195 | 23.87626 | 236.29 | 232.53 | 59.37 | 56.27 |
| 16 | BVD 36 | 2 | 78 | 5.9611 | 0.0352 | 6.0729 | 0.0365 | 6448 | 635697 | 1973643 | 6.46 | 35.66566 | 35.09343 | 175.97 | 177.70 | 61.63 | 61.84 |
| 17 |  | 3 | 0 | 2.5724 | 0.0429 | 2.6205 | 0.0334 | 3255361 | 3738475 | 10439137 | 0.00 | 30.23736 | 30.38125 | 114.67 | 123.03 | 4.55 | 2.86 |
| 18 |  | 2 | 7 | 5.6230 | 0.0319 | 5.7776 | 0.0389 | 1216456 | 1569028 | 2653799 | 0.00 | 39.06484 | 38.14419 | 137.89 | 139.98 | 47.38 | 49.07 |
| 19 |  | 6 | 1 | 2.8574 | 0.0566 | 2.7725 | 0.0382 | 2902826 | 3749319 | 9867425 | 0.00 | 40.64690 | 41.62320 | 136.10 | 131.98 | 73.31 | 75.83 |
| 20 |  | 2 | 0 | 1.1505 | 0.1346 | 1.1415 | 0.0349 | 3582010 | 15419254 | 197262875 | 0.00 | 29.79526 | 30.02603 | -328.22 | -323.20 | 15.32 | 16.44 |
| 21 | DAM1028 | 20 | 78 | 3.1956 | 0.0528 | 3.1507 | 0.0440 | 3974 | 1161257 | 6686862 | 9.03 | 42.50810 | 42.12051 | 174.20 | 174.72 | 56.49 | 57.66 |
| 22 |  | 2 | 74 | 1.5443 | 0.0391 | 1.5444 | 0.0438 | 2773325 | 4510029 | 23695340 | 0.00 | 76.70089 | 77.43085 | -186.76 | -185.11 | 30.32 | 30.27 |
| 23 |  | 47 | 92 | 2.1530 | 0.0408 | 2.0931 | 0.0421 | 2595031 | 3968481 | 14661876 | 0.00 | 17.42199 | 16.29870 | -173.16 | -172.21 | 39.08 | 43.95 |
| 24 |  | 16 | 4 | 1.9251 | 0.0360 | 1.8903 | 0.0645 | 2872334 | 4389400 | 23917932 | 0.00 | 23.26423 | 24.04781 | -280.27 | -281.29 | 46.48 | 50.14 |
| 25 |  | 7 | 4 | 4.7728 | 0.0444 | 5.0092 | 0.0419 | 425854 | 2081281 | 4429465 | 0.00 | 35.88259 | 36.73337 | 101.41 | 100.24 | 78.36 | 77.79 |
| 26 |  | 8 | 0 | 3.9337 | 0.0366 | 3.8958 | 0.0434 | 2917160 | 3067307 | 4700177 | 0.00 | 14.59396 | 15.31586 | 282.87 | 286.58 | 82.61 | 84.24 |
| 27 |  | 6 | 92 | 1.6805 | 0.0407 | 1.6772 | 0.0516 | 763969 | 3359673 | 22136607 | 0.00 | 24.44049 | 23.99614 | -232.58 | -231.97 | 69.52 | 72.19 |
| 28 |  | 2 | 5 | 1.8477 | 0.0363 | 1.8721 | 0.0356 | 2512153 | 3504031 | 14852686 | 0.00 | 16.18745 | 16.92244 | -110.79 | -109.88 | 54.06 | 56.86 |
| 29 |  | 5 | 0 | 1.4043 | 0.0401 | 1.3897 | 0.0460 | 3233182 | 5676292 | 33276142 | 0.00 | 14.87956 | 14.03716 | -250.25 | -254.90 | 60.84 | 60.52 |
| 30 |  | 2 | 74 | 3.7426 | 0.0423 | 3.8179 | 0.0347 | 3577907 | 3843701 | 5766522 | 0.00 | 29.32221 | 29.10828 | 131.96 | 132.51 | 81.29 | 81.25 |
| 31 |  | 3 | 1 | 3.0996 | 0.2180 | 3.1114 | 0.0203 | 2800834 | 4263671 | 30062404 | 0.00 | 36.81569 | 37.07512 | -153.39 | -149.85 | 29.95 | 28.99 |
| 32 |  | 2 | 1 | 1.9149 | 0.0276 | 1.9350 | 0.0169 | 609605 | 1590269 | 10711428 | 0.00 | 32.97147 | 33.13897 | -175.47 | -177.58 | 37.84 | 39.34 |
| 33 |  | 2 | 56 | 2.6906 | 0.0313 | 2.5880 | 0.1371 | 2148113 | 4220370 | 26774866 | 0.00 | 17.98388 | 18.26202 | -267.91 | -266.38 | 32.48 | 33.77 |
| 34 |  | 2 | 37 | 3.0828 | 0.0377 | 3.2584 | 0.0371 | 511230 | 3635068 | 8412399 | 0.00 | 51.92573 | 50.85835 | 271.59 | 270.87 | 60.32 | 59.44 |
| 35 | A 2135 | 2 | 0 | 3.3147 | 0.0649 | 3.4052 | 0.0498 | 26470 | 1698536 | 8182503 | 5.73 | 21.61542 | 20.54833 | 202.09 | 196.13 | 28.16 | 28.20 |
| 36 |  | 2 | 95 | 4.4173 | 0.0470 | 4.3372 | 0.0447 | 18051 | 877172 | 3988092 | 10.65 | 59.31021 | 59.97372 | -191.38 | -191.44 | 46.55 | 47.32 |
| 37 | HJ 91 | 2 | 95 | 1.7665 | 0.0758 | 1.7476 | 0.0529 | 11063 | 4263607 | 31987814 | 2.49 | 17.62868 | 17.59778 | 223.21 | 223.01 | 79.67 | 82.52 |
| 38 |  | 2 | 74 | 2.6756 | 0.0425 | 2.7783 | 0.0675 | 1385082 | 3223549 | 12327833 | 0.00 | 36.31997 | 35.53158 | 205.63 | 205.63 | 41.36 | 41.36 |
| 39 |  | 2 | 37 | 3.8017 | 0.0408 | 3.6788 | 0.0509 | 282998 | 1842958 | 6083579 | 0.00 | 36.79865 | 36.55879 | 209.33 | 208.92 | 41.27 | 42.19 |
| 40 |  | 2 | 5 | 5.4762 | 0.1673 | 5.2636 | 0.0748 | 1255648 | 2027303 | 6789692 | 0.00 | 47.99150 | 47.43376 | 245.19 | 244.57 | 42.07 | 41.42 |
| 41 |  | 2 | 0 | 8.6380 | 0.0704 | 7.6073 | 0.0402 | 2624528 | 3588373 | 4504860 | 0.00 | 65.33412 | 64.62761 | 216.99 | 212.96 | 52.83 | 52.28 |

Counter-Check of Reported Common Origin Pairs

| Obj | Disc | No | CPMS | Plx1 | e_Plx1 | Plx2 | e_Plx2 | Min_D_AU | Med_D_AU | Max_D_AU | LPGR | v1 | v2 | DV1 | DV2 | AV1 | AV2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 42 | GRV 805 | 2 | 76 | 3.0532 | 0.0484 | 2.9928 | 0.0509 | 11974 | 1498745 | 7820905 | 6.95 | 34.49595 | 35.09479 | -143.82 | -142.72 | 61.45 | 61.00 |
| 43 |  | 2 | 5 | 3.1851 | 0.0616 | 3.3693 | 0.0514 | 1518923 | 3880802 | 10761995 | 0.00 | 34.53613 | 35.11742 | 247.81 | 248.33 | 75.29 | 72.82 |
| 44 |  | 3 | 4 | 3.8092 | 0.0552 | 3.9904 | 0.0487 | 2895833 | 3883863 | 7367667 | 0.00 | 37.80677 | 37.62482 | 251.15 | 249.57 | 88.44 | 84.68 |
| 45 |  | 2 | 0 | 3.4292 | 0.0547 | 3.5960 | 0.0387 | 271357 | 2801455 | 8364659 | 0.00 | 18.94977 | 17.73630 | -293.96 | -298.45 | 3.21 | 3.12 |
| 46 |  | 3 | 1 | 6.5159 | 0.0476 | 6.8929 | 0.0385 | 3354390 | 3784177 | 4575766 | 0.00 | 11.79206 | 11.51175 | -223.40 | -229.30 | 78.87 | 74.10 |
| 47 |  | 2 | 0 | 9.2591 | 0.0402 | 9.3571 | 0.0501 | 1497854 | 1534555 | 1747949 | 0.00 | 33.43232 | 33.09230 | 219.67 | 228.38 | 37.82 | 40.18 |
| 48 |  | 3 | 1 | 2.1520 | 0.0513 | 2.1954 | 0.0456 | 1836787 | 3122694 | 16885450 | 0.00 | 42.30440 | 42.43229 | 197.44 | 199.52 | 33.36 | 35.53 |
| 49 |  | 2 | 1 | 4.9194 | 0.0448 | 4.6273 | 0.0445 | 2933304 | 3957858 | 6049743 | 0.00 | 41.58095 | 42.57121 | 208.50 | 211.16 | 29.29 | 26.39 |
| 50 |  | 2 | 92 | 3.0131 | 0.0343 | 3.0486 | 0.0372 | 2673196 | 2921597 | 6322768 | 0.00 | 28.71449 | 28.43545 | 131.91 | 131.14 | 77.03 | 75.26 |
| 51 | KPP2169 | 2 | 1 | 6.2609 | 0.0383 | 5.9648 | 0.0369 | 3151730 | 3543851 | 4392069 | 0.00 | 56.74280 | 56.35038 | -158.12 | -154.71 | 39.31 | 41.46 |
| 52 |  | 2 | 1 | 6.2478 | 0.0457 | 6.5193 | 0.0388 | 369384 | 1419190 | 2757076 | 0.00 | 20.92264 | 21.85981 | -253.24 | -255.50 | 28.62 | 28.42 |
| 53 |  | 2 | 1 | 3.0357 | 0.0499 | 2.9417 | 0.0486 | 2062801 | 3062672 | 9082849 | 0.00 | 26.00852 | 25.86391 | -211.88 | -208.90 | 43.82 | 43.48 |
| 54 |  | 2 | 0 | 2.8376 | 0.0363 | 2.9499 | 0.0653 | 2853212 | 4031268 | 10739274 | 0.00 | 12.75264 | 11.63919 | 209.79 | 215.13 | 54.53 | 51.31 |
| 55 |  | 2 | 29 | 2.2583 | 0.0286 | 2.2160 | 0.0579 | 2945075 | 3775205 | 15069757 | 0.00 | 17.11587 | 16.97311 | 216.91 | 215.29 | 77.76 | 79.82 |
| 56 |  | 2 | 0 | 1.7065 | 0.0427 | 1.6929 | 0.0422 | 1162845 | 3212767 | 19918679 | 0.00 | 20.39381 | 19.79175 | -190.34 | -199.33 | 15.80 | 17.84 |
| 57 |  | 2 | 1 | 3.9322 | 0.1212 | 3.9957 | 0.0508 | 1346499 | 1915504 | 9381293 | 0.00 | 14.60067 | 14.12095 | -279.77 | -282.79 | 66.59 | 67.99 |
| 58 | DJU 3 | 2 | 0 | 9.1826 | 0.0468 | 8.5682 | 0.0454 | 3125445 | 3396053 | 3861949 | 0.00 | 34.57088 | 33.82232 | -301.57 | -292.73 | 39.11 | 39.94 |
| 59 |  | 2 | 4 | 8.7676 | 0.0238 | 8.5874 | 0.0353 | 3162403 | 3226525 | 3355315 | 0.00 | 50.80591 | 51.12598 | -306.43 | -307.56 | 63.37 | 56.79 |
| 60 |  | 2 | 15 | 4.1098 | 0.0463 | 4.2631 | 0.0322 | 2776957 | 3365251 | 5786600 | 0.00 | 65.35614 | 65.16584 | -262.13 | -264.33 | 62.91 | 60.93 |
| 61 |  | 2 | 0 | 7.4659 | 0.0503 | 6.9625 | 0.0483 | 2268995 | 2901133 | 3879576 | 0.00 | 16.74321 | 16.95336 | -154.37 | -158.97 | 34.05 | 28.93 |
| 62 | SEI 537 | 2 | 97 | 4.8101 | 0.0368 | 4.9042 | 0.0440 | 1463 | 823442 | 3496196 | 8.71 | 68.29955 | 67.16688 | -135.71 | -135.45 | 79.21 | 79.45 |
| 63 |  | 2 | 95 | 3.0739 | 0.0272 | 3.0919 | 0.0273 | 29364 | 630767 | 4739515 | 16.83 | 50.02468 | 49.87565 | -221.63 | -221.77 | 87.78 | 88.08 |
| 64 |  | 4 | 0 | 5.8600 | 1.1521 | 6.2855 | 0.0270 | 2038330 | 4903603 | 158568938 | 0.00 | 32.15929 | 31.13647 | -42.08 | -52.11 | 17.53 | 21.58 |
| 65 |  | 2 | 0 | 4.0196 | 0.0231 | 3.7978 | 0.0193 | 1959842 | 3388930 | 5018626 | 0.00 | 22.54966 | 23.53348 | -261.74 | -257.88 | 35.17 | 37.25 |
| 66 |  | 2 | 5 | 7.7009 | 0.0343 | 7.2244 | 0.0327 | 1095145 | 1817946 | 2587273 | 0.00 | 32.23475 | 32.78033 | -250.40 | -251.16 | 72.16 | 71.06 |
| 67 |  | 2 | 1 | 1.9542 | 0.0360 | 1.9227 | 0.0295 | 17560 | 2147937 | 13629208 | 4.98 | 30.94525 | 30.40673 | 254.26 | 256.45 | 59.48 | 58.31 |
| 68 |  | 2 | 76 | 4.3955 | 0.0246 | 4.4926 | 0.0335 | 18329 | 1014558 | 2938270 | 2.77 | 21.12998 | 21.20677 | -242.61 | -244.25 | 52.22 | 50.35 |
| 69 |  | 2 | 0 | 7.4806 | 0.0259 | 7.1392 | 0.0211 | 2815019 | 3000490 | 3293342 | 0.00 | 16.84239 | 16.50907 | -347.91 | -352.21 | 79.09 | 83.37 |
| 70 |  | 2 | 37 | 1.8922 | 0.0549 | 1.9337 | 0.0199 | 524939 | 2858863 | 18321698 | 0.00 | 44.42350 | 45.00879 | -343.69 | -344.43 | 67.75 | 66.31 |
| 71 |  | 2 | 59 | 2.3846 | 0.0218 | 2.4707 | 0.0225 | 192086 | 3016066 | 7528636 | 0.06 | 15.52448 | 15.16066 | -346.09 | -345.06 | 39.69 | 38.34 |
| 72 | GRV 503 | 2 | 97 | 2.7852 | 0.0315 | 2.7492 | 0.0322 | 7272 | 1108984 | 6511569 | 9.46 | 57.20110 | 58.01430 | -224.14 | -224.35 | 72.19 | 73.36 |
| 73 |  | 3 | 0 | 2.9543 | 0.0464 | 2.9732 | 0.0289 | 3943691 | 4197092 | 8320601 | 0.00 | 11.67337 | 10.50429 | -99.51 | -103.64 | 53.38 | 53.85 |
| 74 |  | 2 | 0 | 3.9800 | 0.0257 | 3.7682 | 0.0307 | 1270802 | 3123738 | 5511708 | 0.00 | 15.44167 | 15.89318 | -207.86 | -203.25 | 69.03 | 67.20 |
| 75 |  | 2 | 1 | 2.7516 | 0.0334 | 2.8026 | 0.0296 | 1830712 | 2342581 | 7312536 | 0.00 | 30.88683 | 31.02633 | -59.62 | -54.89 | 70.52 | 69.83 |
| 76 |  | 2 | 59 | 3.9413 | 0.0423 | 3.9043 | 0.0312 | 200176 | 636255 | 3556104 | 0.00 | 38.84212 | 38.95838 | -179.53 | -178.14 | 17.90 | 18.26 |
| 77 |  | 2 | 0 | 3.2180 | 0.0206 | 3.3151 | 0.0233 | 2442620 | 3101555 | 4986031 | 0.00 | 29.51920 | 29.07355 | -39.67 | -43.87 | 9.71 | 9.06 |
| 78 |  | 2 | 0 | 5.6869 | 0.0286 | 5.4580 | 0.0234 | 3636326 | 3915398 | 4394465 | 0.00 | 30.63439 | 29.64318 | -335.49 | -2.21 | 15.62 | 18.74 |
| Table 1 (continued): List of Kamdar et al. 2019 objects with spatial movement values derived from GAIA DR2 data |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

Counter－Check of Reported Common Origin Pairs

| N | $\stackrel{\rightharpoonup}{\sim}$ | $\begin{aligned} & \underset{\sim}{\dot{\sim}} \\ & \hline \end{aligned}$ | $\begin{aligned} & \underset{\sim}{\underset{\sim}{n}} \\ & \underset{\sim}{2} \end{aligned}$ | $\begin{gathered} m \\ \underset{n}{m} \\ \underset{n}{2} \end{gathered}$ | $\begin{aligned} & \stackrel{0}{0} \\ & \dot{0} \end{aligned}$ | $\begin{aligned} & ⿱ 丷 ⿱ 一 廾 刂 \\ & \underset{\sim}{2} \end{aligned}$ | $\begin{aligned} & \underset{-}{7} \\ & \underset{6}{2} \end{aligned}$ | $\begin{aligned} & 0 \\ & \underset{\sim}{0} \\ & \stackrel{n}{2} \end{aligned}$ | $\begin{gathered} \stackrel{0}{2} \\ \underset{\sim}{n} \\ \stackrel{n}{2} \end{gathered}$ | $\begin{aligned} & \infty \\ & \stackrel{\infty}{n} \\ & \dot{n} \end{aligned}$ | $\begin{aligned} & \underset{\sim}{r} \\ & \underset{子}{2} \end{aligned}$ | $\begin{aligned} & \underset{N}{N} \\ & \underset{\sim}{j} \\ & \dot{m} \end{aligned}$ | $\begin{aligned} & m \\ & \underset{\sim}{n} \\ & \underset{\sim}{2} \end{aligned}$ | $\begin{aligned} & \bullet \\ & \underset{\sim}{\infty} \\ & \underset{\sim}{\infty} \end{aligned}$ | $\begin{gathered} \infty \\ \stackrel{0}{+} \\ \stackrel{\rightharpoonup}{2} \end{gathered}$ | $\begin{aligned} & \underset{\sim}{7} \\ & \underset{\sim}{n} \end{aligned}$ | $$ | $\begin{aligned} & \stackrel{\rightharpoonup}{\sim} \\ & \underset{\sim}{4} \end{aligned}$ | $\begin{aligned} & \infty \\ & \underset{m}{m} \\ & \underset{\sim}{n} \end{aligned}$ | $\begin{aligned} & \infty \\ & \stackrel{\infty}{\dot{0}} \\ & \stackrel{1}{n} \end{aligned}$ |  | $\left\lvert\, \begin{aligned} & 0 \\ & 0 \\ & \dot{\circ} \end{aligned}\right.$ | $\begin{gathered} \stackrel{0}{\tilde{L}} \\ \dot{U} \end{gathered}$ | $\begin{aligned} & 0 \\ & \infty \\ & \infty \\ & \infty \end{aligned}$ | $\begin{gathered} \text { n } \\ \stackrel{y}{\dot{n}} \\ \underset{\sim}{n} \end{gathered}$ | $\begin{aligned} & \infty \\ & \underset{\sim}{\infty} \\ & \dot{m} \end{aligned}$ | $\begin{gathered} \dot{y} \\ \stackrel{\rightharpoonup}{m} \\ \dot{m} \end{gathered}$ | $\begin{aligned} & \overrightarrow{0} \\ & \dot{W} \\ & \dot{\sim} \end{aligned}$ | $\begin{array}{r} -1 \\ 0 \\ \dot{0} \\ \dot{6} \end{array}$ | $\underset{\sim}{\underset{\sim}{\sim}} \underset{\underset{\sim}{2}}{ }$ | $\begin{aligned} & \infty \\ & \stackrel{\infty}{\circ} \\ & \stackrel{1}{\infty} \\ & \infty \end{aligned}$ | $\begin{aligned} & 0 \\ & \vdots \\ & \vdots \\ & 0 \end{aligned}$ | $\stackrel{\bullet}{\stackrel{0}{n}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 㝐 |  | $\begin{aligned} & \stackrel{L}{4} \\ & \stackrel{i}{0} \end{aligned}$ | $\begin{aligned} & \vec{\rightharpoonup} \\ & \dot{\sim} \\ & \hline \end{aligned}$ | $\begin{aligned} & \stackrel{n}{0} \\ & \dot{0} \\ & \dot{6} \end{aligned}$ | $\begin{aligned} & \underset{\sim}{\prime} \\ & \underset{\sim}{8} \end{aligned}$ | $\begin{aligned} & \vec{m} \\ & \dot{m} \end{aligned}$ | $\begin{aligned} & \text { O} \\ & \dot{C} \\ & \underset{6}{2} \end{aligned}$ | $\begin{gathered} 0 \\ \stackrel{\rightharpoonup}{i} \\ \stackrel{i}{2} \end{gathered}$ | $\begin{aligned} & \stackrel{0}{0} \\ & 0 \\ & \dot{0} \end{aligned}$ | $\begin{aligned} & \stackrel{n}{\sim} \\ & \stackrel{\sim}{\sim} \end{aligned}$ | $\begin{gathered} \infty \\ \substack{m \\ \infty \\ \sim_{0}} \end{gathered}$ |  | $\begin{aligned} & \infty \\ & \underset{\sim}{\perp} \\ & \underset{\sim}{2} \end{aligned}$ | $\begin{aligned} & \underset{\sim}{\underset{\sim}{2}} \\ & \dot{\sim} \\ & \underset{\sim}{2} \end{aligned}$ | $\stackrel{\sim}{\sim}$ | $\begin{aligned} & 0 \\ & \infty \\ & 0 \\ & \vdots \\ & \hline \end{aligned}$ | $\begin{aligned} & \underset{\sim}{\underset{~}{~}} \\ & \underset{\sim}{2} \end{aligned}$ | $\begin{aligned} & \vec{\infty} \\ & \dot{y} \\ & \underset{\sim}{2} \end{aligned}$ | $\begin{aligned} & \underset{~}{7} \\ & \underset{\sim}{1} \end{aligned}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{N} \\ & \stackrel{\rightharpoonup}{c} \end{aligned}$ | $\begin{aligned} & \stackrel{\circ}{\dot{\gamma}} \\ & \underset{\gamma}{2} \end{aligned}$ | $\begin{aligned} & n \\ & \underset{0}{m} \\ & \dot{O} \end{aligned}$ | $\begin{gathered} \stackrel{0}{0} \\ 0 \\ \stackrel{\rightharpoonup}{6} \end{gathered}$ | $\begin{aligned} & \circ \\ & \dot{\circ} \\ & \infty \end{aligned}$ | $\begin{aligned} & \underset{\sim}{\dot{N}} \\ & \underset{\sim}{2} \end{aligned}$ | $\begin{aligned} & \dot{-} \\ & \dot{\alpha} \\ & \dot{\sim} \end{aligned}$ | $\begin{aligned} & \underset{y}{y} \\ & \dot{\sim} \\ & \text { n } \end{aligned}$ | $\begin{aligned} & \underset{\sim}{N} \\ & \dot{\sim} \end{aligned}$ |  | $\begin{gathered} \underset{\sim}{\circ} \\ \dot{\infty} \\ \underset{\sim}{2} \end{gathered}$ | $\begin{aligned} & \dot{\alpha} \\ & \dot{\infty} \\ & \infty \\ & \infty \end{aligned}$ | $\begin{gathered} n \\ \underset{y}{n} \\ \underset{\sim}{2} \end{gathered}$ | di $\stackrel{1}{6}$ $m$ |
| ～ | $\begin{aligned} & \stackrel{\circ}{0} \\ & \dot{\sim} \\ & \underset{\sim}{\mathrm{~N}} \end{aligned}$ | $\begin{gathered} \underset{\sim}{m} \\ \stackrel{\rightharpoonup}{\sim} \end{gathered}$ | $\begin{aligned} & 0 \\ & \stackrel{0}{0} \\ & \stackrel{6}{1} \end{aligned}$ | $\begin{aligned} & 0 \\ & \tilde{0} \\ & \vdots \\ & \vdots \\ & \underset{1}{1} \end{aligned}$ | $\begin{aligned} & \text { N } \\ & \infty \\ & \underset{\sim}{N} \\ & \text { in } \end{aligned}$ | $\begin{aligned} & \underset{\sim}{0} \\ & \stackrel{0}{0} \\ & \underset{\sim}{2} \end{aligned}$ | $\underset{\underset{\sim}{\sim}}{\underset{\sim}{\sim}}$ | $\begin{gathered} \underset{\sim}{N} \\ \underset{\sim}{\sim} \\ \underset{\sim}{n} \end{gathered}$ | $\begin{aligned} & 0 \\ & 0 \\ & 7 \\ & 7 \\ & 7 \\ & \hline \end{aligned}$ | $\begin{aligned} & \circ \\ & \vdots \\ & \underset{\sim}{7} \\ & \underset{1}{2} \end{aligned}$ | $\begin{aligned} & \stackrel{\bullet}{\sim} \\ & \underset{\sim}{n} \\ & \end{aligned}$ | $\begin{aligned} & \stackrel{\infty}{0} \\ & \dot{\sim} \\ & \underset{\sim}{\omega} \end{aligned}$ | $\begin{aligned} & \infty \\ & \stackrel{\infty}{\dot{\sim}} \\ & \stackrel{\sim}{\sim} \end{aligned}$ | $\begin{gathered} \underset{H}{H} \\ \dot{\omega} \\ \underset{\sim}{n} \end{gathered}$ | $\begin{aligned} & 0 \\ & 0 \\ & i \\ & i \\ & 0 \end{aligned}$ | $\begin{aligned} & \underset{\sim}{2} \\ & \underset{\sim}{\sim} \\ & \underset{\sim}{2} \end{aligned}$ | $\begin{aligned} & \mathfrak{m} \\ & \dot{0} \\ & \dot{0} \\ & \stackrel{1}{2} \end{aligned}$ | $\begin{gathered} \sim \\ \sim \\ \underset{\sim}{\sim} \\ \underset{\sim}{2} \end{gathered}$ | $\begin{aligned} & \sim \\ & 0 \\ & \stackrel{n}{n} \\ & \underset{\sim}{n} \end{aligned}$ | $\begin{gathered} 0 \\ \underset{\sim}{0} \\ \infty \\ 0 \\ 1 \end{gathered}$ | $\begin{aligned} & \text { 广. } \\ & \dot{\circ} \\ & \text { N} \end{aligned}$ | $\begin{array}{\|c} 0 \\ \tilde{n} \\ \dot{0} \\ \underset{\sim}{n} \end{array}$ | $\begin{aligned} & \infty \\ & \underset{m}{m} \\ & \underset{m}{2} \end{aligned}$ | $\begin{aligned} & \underset{\sim}{7} \\ & \dot{\sim} \\ & \underset{\sim}{2} \end{aligned}$ | $\begin{aligned} & \infty \\ & \infty \\ & \stackrel{\rightharpoonup}{N} \\ & \underset{\sim}{n} \end{aligned}$ | $\begin{gathered} \stackrel{r}{c} \\ \underset{\sim}{n} \\ \underset{\sim}{2} \end{gathered}$ | $\begin{aligned} & \text { d} \\ & \text { n } \\ & 0 \\ & 0 \\ & 1 \\ & i \end{aligned}$ | $\begin{gathered} n \\ 0 \\ \\ \\ 1 \end{gathered}$ | $\begin{aligned} & \infty \\ & \infty \\ & \underset{\sim}{N} \\ & \underset{\sim}{2} \end{aligned}$ | $\begin{array}{\|c} \stackrel{\circ}{n} \\ \stackrel{n}{4} \\ \underset{\sim}{7} \\ \hline \end{array}$ | $\underset{\underset{\sim}{\underset{\sim}{c}}}{\underset{\sim}{2}}$ |  | $\stackrel{\sim}{n}$ $\sim$ $\sim$ $\sim$ 1 |
| 5 | $\left\|\begin{array}{l} \stackrel{\sim}{0} \\ \dot{\sim} \\ \underset{\sim}{N} \end{array}\right\|$ | $\begin{aligned} & \text { no } \\ & \dot{m} \\ & \underset{\sim}{2} \end{aligned}$ | $\begin{aligned} & \bullet \\ & \stackrel{0}{n} \\ & \stackrel{n}{1} \\ & \vdots \end{aligned}$ | $\begin{aligned} & n \\ & m \\ & n \\ & \underset{n}{n} \end{aligned}$ | $\begin{aligned} & 6 \\ & \\ & \underset{\sim}{n} \\ & \underset{\sim}{n} \end{aligned}$ | $\begin{aligned} & \stackrel{n}{n} \\ & \dot{0} \\ & \dot{\circ} \end{aligned}$ | $\begin{gathered} m \\ \underset{\sim}{\beth} \\ \underset{\sim}{Z} \end{gathered}$ | $\begin{aligned} & \infty \\ & \infty \\ & \dot{\infty} \\ & \underset{\sim}{n} \\ & \end{aligned}$ | $\begin{gathered} \circ \\ \stackrel{~}{2} \\ \underset{\sim}{1} \\ \vdots \end{gathered}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{n} \\ & \underset{\sim}{m} \\ & \underset{\sim}{m} \end{aligned}$ | $\begin{aligned} & \text { e } \\ & \substack{2 \\ \vdots \\ \vdots} \end{aligned}$ | $\begin{aligned} & \infty \\ & \stackrel{\infty}{\circ} \\ & \stackrel{1}{\sim} \\ & \underset{\sim}{2} \end{aligned}$ | $\begin{aligned} & \widehat{\infty} \\ & \dot{0} \\ & \stackrel{\rightharpoonup}{0} \end{aligned}$ | $\begin{aligned} & \tilde{m} \\ & 0 \\ & \dot{0} \\ & \end{aligned}$ | $\begin{aligned} & \underset{\sim}{\pi} \\ & \dot{m} \\ & \underset{\sim}{2} \end{aligned}$ | $\begin{aligned} & 8 \\ & \vdots \\ & 0 \\ & \underset{\sim}{n} \end{aligned}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{\Omega} \\ & \dot{0} \\ & \grave{N} \end{aligned}$ | $\begin{aligned} & \underset{\sim}{\underset{\sim}{n}} \\ & \underset{\sim}{n} \end{aligned}$ | $\left\lvert\, \begin{aligned} & 0 \\ & \stackrel{\sim}{n} \\ & \stackrel{0}{\circ} \\ & \stackrel{n}{2} \end{aligned}\right.$ | $\begin{aligned} & \circ \\ & \\ & 0 \\ & 0 \\ & 1 \end{aligned}$ | $\begin{gathered} \stackrel{\sim}{\mu} \\ \underset{\sim}{0} \\ \underset{\sim}{2} \end{gathered}$ | $\begin{aligned} & \infty \\ & \underset{\sim}{\infty} \\ & \underset{\sim}{0} \\ & \underset{\sim}{n} \end{aligned}$ | $\begin{aligned} & \dot{0} \\ & \underset{\sim}{m} \\ & \dot{m} \\ & \hline \end{aligned}$ | $\begin{aligned} & 0 \\ & \infty \\ & \dot{\sim} \\ & \underset{\sim}{7} \end{aligned}$ | $\begin{gathered} \text { N } \\ \dot{\sim} \\ \underset{\sim}{N} \end{gathered}$ | $\begin{aligned} & \underset{\sim}{0} \\ & \dot{0} \\ & \stackrel{\sim}{n} \end{aligned}$ | $\begin{aligned} & \dot{8} \\ & \dot{0} \\ & \dot{\sim} \\ & \underset{1}{2} \end{aligned}$ | $\begin{array}{\|c} \underset{\sim}{2} \\ \underset{\sim}{\lambda} \\ \underset{\sim}{1} \end{array}$ | $\stackrel{\infty}{\underset{\sim}{\underset{\sim}{\underset{\sim}{n}}}}$ | $\begin{aligned} & \underset{\sim}{7} \\ & \dot{n} \\ & \underset{1}{2} \end{aligned}$ | $\begin{aligned} & \infty \\ & \infty \\ & \underset{\sim}{\infty} \\ & \end{aligned}$ | $\underset{\substack{\underset{ }{n} \\ \underset{\sim}{n} \\ \underset{N}{n}}}{ }$ | $\stackrel{\sim}{\infty}$ |
| ） |  | $\begin{aligned} & \overrightarrow{7} \\ & \stackrel{1}{0} \\ & \underset{\sim}{7} \end{aligned}$ | $\begin{aligned} & \text { o} \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & \hline \end{aligned}$ |  | $\begin{aligned} & \text { g} \\ & 0 \\ & \tilde{\sim} \\ & \tilde{m} \\ & \sigma \end{aligned}$ | $\begin{aligned} & \infty \\ & \infty \\ & 0 \\ & 0 \\ & \vdots \\ & \vdots \\ & \stackrel{\sim}{0} \end{aligned}$ |  | $\begin{aligned} & 0 \\ & \infty \\ & \underset{\omega}{\infty} \\ & \infty \\ & \vdots \\ & \vdots \end{aligned}$ |  |  |  | $\begin{aligned} & \underset{\sim}{2} \\ & \stackrel{\rightharpoonup}{2} \\ & \dot{\sim} \\ & \dot{\sim} \end{aligned}$ | $\begin{aligned} & \infty \\ & \infty \\ & \infty \\ & 0 \\ & \infty \\ & \dot{\infty} \\ & \dot{\sim} \end{aligned}$ | $\begin{aligned} & \infty \\ & \infty \\ & \infty \\ & \infty \\ & \vdots \\ & \infty \\ & \cdots \end{aligned}$ | $\begin{aligned} & \infty \\ & \infty \\ & \infty \\ & \infty \\ & \\ & \dot{\sim} \\ & \dot{\sim} \end{aligned}$ | $\begin{aligned} & \text { I } \\ & \underset{\sim}{N} \\ & \underset{\sim}{\sim} \\ & \underset{\sim}{m} \end{aligned}$ | $\begin{aligned} & 0 \\ & \infty \\ & \infty \\ & \stackrel{0}{n} \\ & \stackrel{i}{2} \end{aligned}$ | $\begin{aligned} & 0 \\ & \underset{\sim}{0} \\ & \underset{\sim}{c} \\ & \underset{\sim}{\lambda} \end{aligned}$ | $\begin{gathered} \infty \\ \infty \\ \\ \underset{\sim}{1} \\ \vdots \\ \vdots \end{gathered}$ | $\begin{aligned} & \infty \\ & \infty \\ & \infty \\ & \infty \\ & \cdots \\ & \cdots \\ & \cdots \\ & \end{aligned}$ | $\begin{aligned} & \tilde{0} \\ & 0 \\ & 0 \\ & 0 \\ & \dot{0} \\ & \dot{o} \end{aligned}$ | $\begin{aligned} & \tilde{N} \\ & 0 \\ & \vdots \\ & \vdots \\ & \dot{n} \\ & \end{aligned}$ |  | $\begin{aligned} & \underset{y}{t} \\ & 0 \\ & 0 \\ & \vdots \\ & \vdots \\ & - \\ & - \end{aligned}$ | $\begin{gathered} \infty \\ \infty \\ \stackrel{\infty}{n} \\ \stackrel{n}{\wedge} \\ \underset{\sim}{\infty} \end{gathered}$ | $\begin{aligned} & \underset{\sim}{2} \\ & \underset{\sim}{\sim} \\ & \underset{\sim}{n} \end{aligned}$ | $\stackrel{N}{\infty}$ $\underset{\infty}{\infty}$ $\underset{\sim}{n}$ $\underset{\sim}{n}$ |  |  | $\begin{aligned} & \vec{y} \\ & \infty \\ & \underset{\sim}{n} \\ & \dot{\sim} \end{aligned}$ | $\left\lvert\, \begin{gathered} \infty \\ \infty \\ \infty \\ \vdots \\ \vdots \\ \dot{n} \end{gathered}\right.$ | $\begin{aligned} & \text { on } \\ & \text { w } \\ & \infty \\ & m \\ & \dot{m} \end{aligned}$ |  |
| 5 | $\begin{aligned} & \overrightarrow{\tilde{H}} \\ & \underset{\sim}{0} \\ & \infty \\ & \dot{\sim} \\ & \dot{y} \end{aligned}$ |  | $\begin{aligned} & n \\ & 0 \\ & 0 \\ & \vdots \\ & \vdots \\ & \vdots \\ & \end{aligned}$ | $\begin{aligned} & \underset{\sim}{n} \\ & \underset{\sim}{n} \\ & \underset{\sim}{n} \\ & \underset{\sim}{n} \end{aligned}$ | $\begin{aligned} & \text { m } \\ & 0 \\ & 0 \\ & m \\ & \cdots \\ & \dot{o} \end{aligned}$ |  | $\begin{aligned} & \stackrel{\rightharpoonup}{6} \\ & \stackrel{1}{\circ} \\ & \underset{\sim}{\infty} \\ & \dot{\sim} \end{aligned}$ | $\begin{aligned} & \text { d } \\ & \\ & \tilde{0} \\ & \dot{\sim} \\ & \text { in } \end{aligned}$ |  | $\begin{aligned} & \infty \\ & \infty \\ & 0 \\ & 0 \\ & \sim \\ & \sim \\ & \sim \end{aligned}$ |  | $\infty$ <br> $\stackrel{\infty}{0}$ <br> $\vdots$ <br> $\vdots$ <br> $\underset{\sim}{~}$ | $\begin{aligned} & 0 \\ & \infty \\ & 0 \\ & 0 \\ & 0 \\ & \dot{0} \\ & \dot{m} \end{aligned}$ |  | $\begin{aligned} & \infty \\ & \stackrel{\infty}{\stackrel{1}{2}} \\ & \underset{\sim}{4} \\ & \dot{子} \end{aligned}$ | $\begin{aligned} & \text { N} \\ & \underset{\sim}{\lambda} \\ & \text { N} \\ & \dot{\sim} \end{aligned}$ | $\begin{gathered} m \\ \text { m } \\ \text { on } \\ \vdots \\ \vdots \\ \vdots \end{gathered}$ |  | $\begin{aligned} & \underset{\sim}{n} \\ & \underset{\sim}{n} \\ & \underset{\sim}{j} \\ & i \end{aligned}$ | $\begin{aligned} & \underset{A}{\lambda} \\ & \underset{A}{\alpha} \\ & \infty \\ & \underset{\sim}{n} \end{aligned}$ | $\begin{aligned} & \sim \\ & \tilde{n} \\ & \infty \\ & \infty \\ & \dot{\infty} \\ & \dot{m} \end{aligned}$ |  | $\begin{aligned} & 0 \\ & 0 \\ & \tilde{n} \\ & \vdots \\ & \vdots \\ & \vdots \\ & i \end{aligned}$ | $\begin{aligned} & \underset{\sim}{n} \\ & \underset{\sim}{\infty} \\ & \infty \\ & \dot{m} \\ & \hline \end{aligned}$ |  | $\begin{aligned} & 2 \\ & \underset{\sim}{2} \\ & \underset{m}{2} \\ & \dot{n} \\ & \text { n } \end{aligned}$ |  | $\left\lvert\, \begin{aligned} & \underset{\infty}{\infty} \\ & \infty \\ & 0 \\ & \vdots \\ & \dot{\infty} \\ & \dot{\sim} \end{aligned}\right.$ |  | $\begin{aligned} & \stackrel{r}{\infty} \\ & \infty \\ & \infty \\ & \infty \\ & \infty \\ & \dot{m} \\ & \dot{N} \end{aligned}$ | $\begin{aligned} & \text { or } \\ & \text { Ì } \\ & \text { on } \\ & \stackrel{0}{2} \end{aligned}$ |  |  |
| $\begin{array}{\|l} \substack{\begin{subarray}{c}{0 \\ 0 \\ \hline} }} \\ {\hline} \end{array}$ | $\begin{aligned} & \underset{\sim}{\mathrm{O}} \\ & \dot{\sim} \end{aligned}$ | $\stackrel{\stackrel{\circ}{\infty}}{\infty}$ | $\stackrel{\circ}{\circ}$ | $\stackrel{\circ}{\circ}$ | $\begin{aligned} & \circ \\ & \hline 0 \\ & \hline \end{aligned}$ | $\begin{aligned} & \substack{m \\ \vdots \\ \dot{n} \\ \hline} \end{aligned}$ | $\stackrel{\circ}{\circ}$ | $\stackrel{\circ}{\circ}$ | $\begin{aligned} & \circ \\ & \vdots \\ & \dot{0} \end{aligned}$ | $\stackrel{\circ}{\circ}$ | $\begin{aligned} & \circ \\ & \vdots \\ & 0 \end{aligned}$ | $\stackrel{\circ}{\circ}$ | $\stackrel{\circ}{\circ}$ | $\stackrel{\circ}{0}$ | $\stackrel{\circ}{0}$ | $\begin{aligned} & \circ \\ & \vdots \\ & \hline \end{aligned}$ | $\stackrel{\circ}{\circ}$ | $\stackrel{\circ}{\circ}$ | $\stackrel{\circ}{\circ}$ | $\begin{aligned} & i \\ & \vdots \\ & i \end{aligned}$ | $\begin{aligned} & \infty \\ & \infty \\ & \dot{\sim} \end{aligned}$ | $\stackrel{N}{\stackrel{\rightharpoonup}{\circ}}$ | $\begin{aligned} & \underset{\sim}{\underset{\sim}{2}} \\ & \dot{\sim} \end{aligned}$ | $\begin{aligned} & \circ \\ & \vdots \\ & \vdots \end{aligned}$ | $\stackrel{\circ}{\circ}$ | $\begin{aligned} & \circ \\ & \vdots \\ & \hline \end{aligned}$ | $\begin{aligned} & \tilde{N} \\ & \sim \\ & i \end{aligned}$ | $\stackrel{\circ}{\circ}$ | $\stackrel{\circ}{\circ}$ | $\stackrel{\circ}{\circ}$ | $\begin{aligned} & \circ \\ & \vdots \\ & \vdots \end{aligned}$ | $\begin{aligned} & \circ \\ & \vdots \\ & 0 \end{aligned}$ | $\stackrel{\circ}{\circ}$ |
| $\left\lvert\, \begin{aligned} & D_{4} \\ & a_{1} \\ & x_{1} \\ & \underset{\sim}{2} \end{aligned}\right.$ | $\begin{aligned} & \text { N } \\ & \text { i} \\ & 0 \\ & \text { in } \\ & \text { N } \end{aligned}$ |  |  | $\begin{aligned} & \overrightarrow{0} \\ & \text { on } \\ & \vec{n} \\ & \overrightarrow{0} \end{aligned}$ | $\begin{aligned} & \underset{\infty}{\infty} \\ & \mathbf{N}_{1} \\ & \infty \\ & \hline \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \text { o } \\ & \underset{~}{n} \\ & \underset{\sim}{I} \end{aligned}$ | $\begin{aligned} & \text { of } \\ & 0 \\ & \infty \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ |  | $\begin{aligned} & \text { O} \\ & \underset{A}{I} \\ & \text { I } \\ & \text { I } \end{aligned}$ |  | $\begin{aligned} & \vec{N} \\ & \tilde{\sim} \\ & \tilde{N} \\ & \underset{\sim}{7} \end{aligned}$ | $\begin{aligned} & \text { N } \\ & \text { on } \\ & \text { No } \\ & \infty \\ & \end{aligned}$ | $\begin{aligned} & \infty \\ & \stackrel{\infty}{1} \\ & \stackrel{\rightharpoonup}{0} \\ & 0 \\ & \stackrel{\rightharpoonup}{0} \end{aligned}$ |  | $\begin{aligned} & 6 \\ & \underset{6}{1} \\ & \stackrel{0}{0} \\ & \underset{\sim}{\sigma} \end{aligned}$ | $\begin{aligned} & \underset{\sim}{\underset{N}{N}} \\ & \underset{\sim}{N} \\ & \underset{\sim}{2} \end{aligned}$ |  | $\left\lvert\, \begin{gathered} 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ \\ \end{gathered}\right.$ | $\begin{aligned} & \hline \\ & \hline \\ & \vdots \\ & \vdots \\ & 0 \\ & 0 \\ & \end{aligned}$ |  | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & \vdots \\ & \vdots \\ & \hline \end{aligned}$ | $\begin{array}{\|c} 0 \\ \stackrel{0}{2} \\ \stackrel{3}{2} \\ 0 \\ - \end{array}$ | $\circ$ Ö in in $i$ | $\begin{aligned} & \underset{\sim}{7} \\ & \underset{\sim}{0} \\ & \underset{0}{0} \end{aligned}$ |  | $\begin{array}{\|c} \underset{\sim}{\infty} \\ \infty \\ \infty \\ \\ \\ \hline \end{array}$ | $\begin{aligned} & \text { n } \\ & \infty \\ & \\ & \\ & \text { on } \\ & \text { on } \end{aligned}$ | $\begin{aligned} & 0 \\ & \underset{\sim}{n} \\ & \underset{N}{N} \\ & \underset{\sim}{2} \end{aligned}$ |  | $\begin{aligned} & \stackrel{0}{0} \\ & \stackrel{0}{0} \\ & \stackrel{1}{\mathrm{~N}} \end{aligned}$ | $\begin{aligned} & 0 \\ & \stackrel{0}{2} \\ & 0 \\ & 0 \\ & 0 \\ & \end{aligned}$ |  |
| $\left\lvert\, \begin{aligned} & \text { 呆 } \\ & \text { a } \\ & 0 \\ & 0 \\ & 0 \\ & \end{aligned}\right.$ | $\begin{aligned} & 0 \\ & \\ & - \\ & \underset{\sim}{n} \\ & \end{aligned}$ | $\begin{aligned} & \infty \\ & \stackrel{0}{0} \\ & \stackrel{0}{0} \\ & \underset{\sim}{1} \end{aligned}$ | $\begin{aligned} & \underset{N}{N} \\ & \underset{\sim}{\infty} \\ & \infty \\ & \end{aligned}$ | $\begin{aligned} & \infty \\ & \underset{\sim}{\infty} \\ & \infty \\ & \infty \\ & \stackrel{\infty}{m} \end{aligned}$ |  | $\begin{aligned} & \underset{ }{7} \\ & \infty \\ & \infty \\ & \end{aligned}$ | $\begin{aligned} & \text { n } \\ & \stackrel{\rightharpoonup}{g} \\ & \underset{0}{0} \\ & 0 \\ & \hline \end{aligned}$ | $\begin{aligned} & n \\ & \underset{\sim}{n} \\ & \underset{\sim}{0} \\ & 0 \end{aligned}$ | $\begin{aligned} & 0 \\ & \\ & \\ & \underset{7}{2} \end{aligned}$ | $\begin{aligned} & \infty \\ & 0 \\ & 0 \\ & 0 \\ & \vdots \\ & \vdots \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { N } \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & \text { of } \end{aligned}$ |  | $\begin{aligned} & 0 \\ & 0 \\ & \infty \\ & \infty \\ & \\ & \end{aligned}$ | $\circ$ $\infty$ $\infty$ 0 O n | $\begin{aligned} & \underset{\sim}{0} \\ & \stackrel{0}{N} \\ & \underset{\sim}{n} \\ & \hline \end{aligned}$ | $\begin{aligned} & \infty \\ & \underset{\sim}{\infty} \\ & \infty \\ & \underset{\sim}{\omega} \\ & \underset{\sim}{2} \end{aligned}$ | $\begin{aligned} & \text { N} \\ & \text { N} \\ & \text { N} \\ & \text { N } \end{aligned}$ |  | $\left\lvert\, \begin{aligned} & \sim \\ & \tilde{n} \\ & \infty \\ & \\ & \end{aligned}\right.$ |  | $\begin{aligned} & \tilde{N} \\ & \underset{\sim}{\tilde{\omega}} \\ & \underset{\sim}{n} \end{aligned}$ | $\begin{aligned} & \stackrel{n}{7} \\ & \underset{\sim}{\sim} \\ & \infty \end{aligned}$ | $\begin{aligned} & \underset{\sim}{\underset{\infty}{2}} \\ & \infty \\ & \underset{\sim}{\circ} \end{aligned}$ | $\begin{aligned} & N \\ & N \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{\infty} \\ & \stackrel{\infty}{\infty} \\ & \stackrel{\rightharpoonup}{\mathrm{N}} \end{aligned}$ | $\left\lvert\, \begin{aligned} & o \\ & 0 \\ & \underset{\sim}{0} \\ & \underset{\sim}{e} \\ & \underset{m}{2} \end{aligned}\right.$ | $\left\lvert\, \begin{aligned} & \infty \\ & \infty \\ & \infty \\ & 0 \\ & \infty \\ & \sim \end{aligned}\right.$ | $\begin{aligned} & \vec{Z} \\ & 0 \\ & \underset{\sim}{n} \\ & \underset{m}{2} \end{aligned}$ |  | $\begin{gathered} \underset{\sim}{N} \\ \underset{\sim}{N} \\ \underset{\sim}{0} \end{gathered}$ | $\begin{aligned} & \underset{n}{n} \\ & \underset{\sim}{1} \\ & \underset{\sim}{n} \end{aligned}$ |  | d 0 0 $\sim$ 7 7 |
| $\left\lvert\, \begin{aligned} & \dot{C}_{1} \\ & a_{1} \\ & \stackrel{F}{\Sigma} \\ & \stackrel{y}{2} \end{aligned}\right.$ | $\begin{aligned} & \text { n } \\ & \text { न̈ } \end{aligned}$ | $\begin{aligned} & \stackrel{n}{n} \\ & \underset{\sim}{\mathrm{~N}} \\ & \hline \end{aligned}$ | $\left\lvert\, \begin{aligned} & \text { n } \\ & \underset{\sim}{0} \\ & \underset{0}{c} \end{aligned}\right.$ | $\left\lvert\, \begin{aligned} & 0 \\ & 0 \\ & 0 \\ & \text { on } \\ & \underset{\sim}{\infty} \\ & \end{aligned}\right.$ | $\begin{aligned} & \text { d } \\ & \underset{\sim}{7} \\ & \underset{\sim}{0} \\ & \sim \end{aligned}$ | $\begin{aligned} & \underset{y}{y} \\ & \text { O} \\ & \underset{\sim}{y} \end{aligned}$ | $\begin{aligned} & \infty \\ & \underset{\sim}{\infty} \\ & \underset{\sim}{n} \\ & \underset{\sim}{2} \end{aligned}$ |  | $\left\lvert\, \begin{aligned} & \infty \\ & \infty \\ & \infty \\ & 0 \\ & 0 \\ & \infty \\ & \infty \end{aligned}\right.$ | $\begin{aligned} & 0 \\ & \underset{N}{1} \\ & \underset{\sim}{N} \\ & \underset{\sim}{0} \end{aligned}$ | $\begin{aligned} & \underset{\sim}{\infty} \\ & \underset{\sim}{\infty} \\ & \infty \\ & \underset{\sim}{7} \end{aligned}$ |  | $\begin{aligned} & \text { o} \\ & 0 \\ & 0 \\ & \\ & \end{aligned}$ | $\left\lvert\, \begin{aligned} & \Gamma \\ & \infty \\ & \infty \\ & \infty \\ & \hline \end{aligned}\right.$ | $\begin{aligned} & \underset{\sim}{m} \\ & \underset{\sim}{0} \\ & \underset{\sim}{\vdots} \\ & \underset{\sim}{2} \end{aligned}$ | $\begin{aligned} & \text { n } \\ & 0 \\ & 0 \\ & \hline \\ & \hline \\ & \end{aligned}$ | $\begin{aligned} & \underset{\sim}{\lambda} \\ & \tilde{N} \\ & \underset{\sim}{N} \end{aligned}$ | $\begin{aligned} & 0 \\ & \underset{\sim}{7} \\ & \underset{\sim}{7} \\ & \underset{\sim}{2} \end{aligned}$ | $\left\lvert\, \begin{gathered} \text { m } \\ \infty \\ \infty \\ \cdots \\ \infty \\ \infty \end{gathered}\right.$ | $\begin{gathered} \underset{\sim}{n} \\ \end{gathered}$ | $\left\lvert\, \begin{gathered} \infty \\ 0 \\ 0 \\ 0 \\ -1 \end{gathered}\right.$ | $\begin{aligned} & -1 \\ & 0 \\ & 0 \\ & \underset{m}{2} \end{aligned}$ | $\begin{aligned} & \infty \\ & o \\ & o \\ & \underset{\sigma}{2} \end{aligned}$ | $\begin{aligned} & \text { d } \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & \sim \end{aligned}$ | $\begin{aligned} & \infty \\ & 0 \\ & 0 \\ & \infty \\ & \infty \\ & 0 \\ & \hline \end{aligned}$ | $\begin{aligned} & 0 \\ & \infty \\ & \infty \\ & \\ & \\ & \end{aligned}$ | $\begin{aligned} & \tilde{N} \\ & \underset{\sim}{\mathrm{O}} \end{aligned}$ | $\underset{\substack{\text { N } \\ \text { N } \\ \text { N } \\ \sim}}{\substack{2}}$ | $\begin{aligned} & \text { m} \\ & \stackrel{y}{n} \\ & \underset{\sim}{y} \\ & \hline \end{aligned}$ | $\begin{aligned} & \underset{\sim}{2} \\ & \infty \\ & \tilde{m} \\ & \underset{\sim}{2} \end{aligned}$ | $\begin{aligned} & \stackrel{n}{\infty} \\ & \infty \\ & \underset{\sim}{\infty} \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { r} \\ & \text { on } \\ & 0 \\ & \text { N} \\ & \hline \end{aligned}$ | $N$ $\infty$ 0 0 0 0 |
| $\left\lvert\, \begin{gathered} \underset{\sim}{\underset{~}{x}} \\ {\underset{\sim}{1}}^{\prime} \end{gathered}\right.$ | $\begin{gathered} \infty \\ \underset{\sim}{\infty} \\ \text { N} \\ \dot{0} \\ \hline \end{gathered}$ | $\begin{aligned} & \vec{\infty} \\ & \text { on } \\ & 0 \\ & \dot{0} \end{aligned}$ | $\begin{aligned} & \text { n } \\ & \text { オ } \\ & \vdots \\ & \dot{0} \end{aligned}$ | $\left\lvert\, \begin{aligned} & 0 \\ & \text { y } \\ & \text { y. } \\ & \dot{0} \end{aligned}\right.$ | $\begin{aligned} & \circ \\ & \stackrel{\circ}{0} \\ & 0 \\ & \dot{0} \end{aligned}$ | $\begin{aligned} & \text { n } \\ & \text { n } \\ & \vdots \\ & \vdots \\ & \dot{0} \end{aligned}$ | $$ | $\begin{aligned} & \text { Y } \\ & \text { J, } \\ & \vdots \\ & \dot{0} \end{aligned}$ | $\begin{aligned} & \text { N} \\ & 0 \\ & 0 \\ & 0 \\ & \dot{0} \end{aligned}$ | $\begin{aligned} & \text { O} \\ & \underset{\text { IN}}{2} \\ & \vdots \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & \vdots \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & \vdots \\ & \dot{\circ} \end{aligned}$ | $\begin{aligned} & \underset{Z}{I} \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \infty \\ & \stackrel{\infty}{m} \\ & 0 \\ & \dot{0} \end{aligned}$ | $\begin{aligned} & \underset{\sim}{n} \\ & \underset{\sim}{0} \\ & \dot{0} \end{aligned}$ | $\begin{gathered} N \\ \\ \vdots \\ \dot{0} \end{gathered}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{0} \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & 0 \\ & \infty \\ & 0 \\ & 0 \\ & 0 \\ & \hline \end{aligned}$ | $\begin{aligned} & 0 \\ & \tilde{m} \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \overrightarrow{0} \\ & \vdots \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \infty \\ & 0 \\ & 0 \\ & \vdots \\ & \dot{0} \end{aligned}$ | $\begin{gathered} \underset{\sim}{N} \\ \underset{\sim}{0} \\ \dot{0} \end{gathered}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & \dot{\circ} \end{aligned}$ | $\left\lvert\, \begin{gathered} m \\ 0 \\ \vdots \\ \vdots \\ 0 \end{gathered}\right.$ | $\begin{gathered} n \\ \infty \\ 0 \\ \vdots \\ \dot{0} \end{gathered}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{0} \\ & \stackrel{1}{0} \\ & 0 \\ & 0 \end{aligned}$ | $\left\lvert\, \begin{aligned} & \mathrm{O} \\ & \mathrm{O} \\ & 0 \\ & 0 \\ & \dot{0} \end{aligned}\right.$ |  | $\begin{gathered} \infty \\ \stackrel{\infty}{0} \\ \vdots \\ \vdots \end{gathered}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{2} \\ & \tilde{0} \\ & \dot{o} \end{aligned}$ | $\begin{aligned} & \infty \\ & \infty \\ & 0 \\ & 0 \\ & \vdots \\ & \dot{0} \end{aligned}$ | $\begin{aligned} & \text { n } \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | con 0 0 0 0 0 |
| $\begin{aligned} & \underset{\sim}{\underset{a}{2}} \end{aligned}$ |  | $\begin{aligned} & 0 \\ & \dot{\circ} \\ & \infty \\ & \dot{m} \\ & \dot{m} \end{aligned}$ |  | $\begin{gathered} \infty \\ \infty \\ \underset{\sim}{n} \\ \dot{\sim} \end{gathered}$ | $\begin{aligned} & \underset{N}{N} \\ & \stackrel{1}{0} \\ & \dot{O} \\ & \vdots \end{aligned}$ | $\begin{aligned} & \underset{Z}{Z} \\ & \underset{\sim}{\mu} \\ & \dot{m} \end{aligned}$ | $\begin{aligned} & \stackrel{n}{N} \\ & \tilde{N} \\ & \dot{\sim} \\ & \dot{N} \end{aligned}$ | $\begin{aligned} & \stackrel{\circ}{\lambda} \\ & \stackrel{n}{r} \\ & \stackrel{i}{2} \end{aligned}$ | $\begin{aligned} & \stackrel{n}{n} \\ & \vec{U} \\ & \dot{\sim} \end{aligned}$ | $\begin{aligned} & \underset{N}{N} \\ & \underset{\sim}{\tilde{m}} \\ & \dot{m} \end{aligned}$ | $\begin{aligned} & \underset{N}{\underset{0}{0}} \\ & \underset{0}{n} \\ & \underset{i}{2} \end{aligned}$ | $\begin{aligned} & n \\ & \stackrel{n}{0} \\ & \stackrel{0}{2} \\ & \dot{\sim} \end{aligned}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{N} \\ & \infty \\ & \infty \\ & \dot{\sim} \end{aligned}$ | $\begin{aligned} & \infty \\ & \\ & 0 \\ & 0 \\ & - \\ & \hline \end{aligned}$ | $\begin{aligned} & \stackrel{0}{2} \\ & \underset{子}{\square} \\ & \dot{\sim} \end{aligned}$ | $\begin{aligned} & \underset{\sim}{n} \\ & \underset{\sim}{\sim} \\ & \dot{m} \end{aligned}$ | $\begin{gathered} \infty \\ 0 \\ \stackrel{0}{\circ} \\ \dot{m} \\ \hline \end{gathered}$ | $\begin{aligned} & \underset{N}{N} \\ & \text { O} \\ & \underset{\sim}{2} \end{aligned}$ | $\begin{aligned} & \stackrel{0}{N} \\ & \stackrel{N}{\sim} \\ & \dot{\sim} \\ & \dot{N} \end{aligned}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{\underset{1}{2}} \\ & \underset{\sim}{2} \\ & \dot{\sim} \end{aligned}$ | $\begin{gathered} \underset{\sim}{\tilde{1}} \\ \underset{\sim}{n} \\ \dot{\sim} \end{gathered}$ | $\begin{aligned} & \text { n } \\ & \underset{\sim}{\sim} \\ & \underset{\sim}{0} \end{aligned}$ | $\begin{aligned} & 0 \\ & \stackrel{0}{-} \\ & \underset{\sim}{n} \end{aligned}$ | $\left\lvert\, \begin{aligned} & 0 \\ & 0 \\ & 0 \\ & \sim \\ & n \\ & n \end{aligned}\right.$ | $\begin{aligned} & 0 \\ & 0 \\ & \infty \\ & \infty \\ & \dot{m} \\ & \hline \end{aligned}$ | $\begin{aligned} & -1 \\ & \infty \\ & \\ & \dot{m} \end{aligned}$ | $\begin{aligned} & \vec{y} \\ & 0 \\ & \infty \\ & \dot{\sim} \end{aligned}$ | $\frac{\underset{\lambda}{\lambda}}{\underset{\sim}{\lambda}}$ | $\stackrel{i}{0}$ | $\begin{aligned} & \tilde{\infty} \\ & 0 \\ & \sim \\ & \\ & \dot{m} \end{aligned}$ | $\begin{aligned} & 0 \\ & \infty \\ & \infty \\ & \infty \\ & \dot{m} \\ & \hline \end{aligned}$ | $\begin{aligned} & \stackrel{\infty}{\infty} \\ & 0 \\ & \\ & i \end{aligned}$ | N N $\sim$ $\sim$ |
| $\left.\begin{gathered} \vec{x} \\ \alpha_{1} \\ 0_{1} \end{gathered} \right\rvert\,$ |  | $\begin{aligned} & \text { y } \\ & \text { H } \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & \vdots \\ & \vdots \\ & 0 \end{aligned}$ | $\left\lvert\, \begin{gathered} - \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \end{gathered}\right.$ | $\begin{aligned} & \text { y } \\ & \text { y } \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \underset{\sim}{7} \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \text { N} \\ & \stackrel{n}{0} \\ & \dot{0} \end{aligned}$ | $\begin{aligned} & 2 \\ & \tilde{n} \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\left\lvert\, \begin{gathered} 0 \\ \infty \\ 0 \\ 0 \\ 0 \\ \hline \end{gathered}\right.$ | $\begin{gathered} o \\ \infty \\ 0 \\ \vdots \\ \vdots \\ \hline \end{gathered}$ | $\begin{gathered} 0 \\ \stackrel{0}{0} \\ 0 \\ \vdots \\ \dot{0} \end{gathered}$ | $\begin{aligned} & \stackrel{\circ}{\circ} \\ & \stackrel{\circ}{\circ} \\ & \dot{0} \end{aligned}$ | $\begin{aligned} & \underset{\sim}{2} \\ & 0 \\ & 0 \\ & \vdots \\ & 0 \end{aligned}$ | $\begin{aligned} & n \\ & \\ & \vdots \\ & 0 \\ & \vdots \\ & 0 \end{aligned}$ | $\begin{aligned} & \infty \\ & \underset{y}{\infty} \\ & \underset{\sim}{0} \\ & \dot{0} \end{aligned}$ | $\begin{aligned} & \tilde{N} \\ & \tilde{N} \\ & 0 \\ & 0 \end{aligned}$ | $\begin{gathered} \text { y } \\ \text { y } \\ \vdots \\ \vdots \end{gathered}$ | $\begin{gathered} \underset{N}{N} \\ \\ \vdots \\ 0 \end{gathered}$ | $\left\lvert\, \begin{aligned} & \text { n } \\ & \text { オ } \\ & \vdots \\ & \dot{0} \end{aligned}\right.$ | $\begin{aligned} & 0 \\ & \vdots \\ & \vdots \\ & \vdots \\ & \vdots \\ & 0 \end{aligned}$ | $\begin{aligned} & m \\ & 7 \\ & 7 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{array}{\|c} \underset{y}{r} \\ \underset{\sim}{2} \\ \vdots \\ \dot{0} \end{array}$ | $\begin{aligned} & n \\ & \stackrel{n}{2} \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & m \\ & \underset{y}{7} \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \text { オ } \\ & \text { y } \\ & 0 \\ & \vdots \\ & 0 \end{aligned}$ | $\begin{aligned} & \infty \\ & \stackrel{\infty}{\square} \\ & \stackrel{1}{0} \\ & \dot{0} \end{aligned}$ | $\begin{array}{\|l\|l} n \\ 0 \\ 0 \\ 0 \\ 0 \end{array}$ | $\left\lvert\, \begin{aligned} & \text { पु } \\ & \text { U' } \\ & 0 \\ & 0 \\ & 0 \end{aligned}\right.$ | $\begin{array}{\|c} \underset{\sim}{r} \\ \\ \vdots \\ \dot{0} \end{array}$ | $\left\lvert\, \begin{aligned} & \text { n } \\ & \\ & 0 \\ & \dot{0} \end{aligned}\right.$ |  | $\begin{array}{\|c} \underset{\sim}{N} \\ \underset{\sim}{0} \\ \vdots \\ 0 \end{array}$ | con |
| $\left\lvert\, \begin{aligned} & \vec{x} \\ & \vec{a} \end{aligned}\right.$ | $\begin{aligned} & \underset{\sim}{7} \\ & \stackrel{\rightharpoonup}{n} \\ & \dot{\sim} \end{aligned}$ | $\stackrel{\underset{\sim}{\underset{N}{N}}}{\underset{\sim}{N}}$ | $\begin{aligned} & \underset{\sim}{\underset{2}{2}} \\ & \underset{\sim}{\sim} \\ & \underset{\sim}{2} \end{aligned}$ | $\begin{gathered} \underset{\sim}{\tilde{N}} \\ \underset{\sim}{\sim} \\ \dot{\sim} \end{gathered}$ | $\begin{aligned} & \pi \\ & \tilde{N} \\ & 0 \\ & \dot{0} \\ & \end{aligned}$ | $\begin{aligned} & \circ \\ & \infty \\ & 0 \\ & m \\ & m \end{aligned}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{n} \\ & \stackrel{\rightharpoonup}{+} \\ & \dot{N} \end{aligned}$ |  | $\underset{\sim}{\underset{\sim}{N}} \underset{\underset{\sim}{\sim}}{ }$ | $\begin{aligned} & \underset{\sim}{m} \\ & \underset{\sim}{\infty} \\ & \dot{m} \end{aligned}$ | $\begin{aligned} & 0 \\ & \infty \\ & \sim \\ & \sim \\ & - \\ & -i \end{aligned}$ | $\begin{aligned} & \underset{o}{0} \\ & \vdots \\ & \vdots \\ & \dot{\sim} \end{aligned}$ | $\begin{gathered} \underset{\sim}{0} \\ \stackrel{n}{n} \\ \dot{\sim} \end{gathered}$ | $\begin{aligned} & \underset{\sim}{x} \\ & \infty \\ & 0 \\ & -i \end{aligned}$ | $\begin{gathered} \underset{\sim}{N} \\ \underset{\sim}{m} \\ \underset{\sim}{2} \end{gathered}$ | $\begin{gathered} \stackrel{\rightharpoonup}{N} \\ \underset{\sim}{\mathrm{~N}} \\ \hline \end{gathered}$ | $\begin{aligned} & \stackrel{\imath}{m} \\ & \underset{\sim}{n} \\ & \underset{m}{2} \end{aligned}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{V} \\ & \tilde{0} \\ & \dot{m} \\ & \dot{N} \end{aligned}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{y} \\ & \underset{y}{0} \\ & \dot{\sim} \end{aligned}$ | $\begin{aligned} & \infty \\ & \infty \\ & \tilde{m} \\ & \underset{\sim}{n} \end{aligned}$ | $\begin{aligned} & \underset{\sim}{\text { J. }} \\ & \underset{\sim}{\sim} \\ & \dot{\sim} \end{aligned}$ | $\begin{aligned} & n \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{gathered} \stackrel{\circ}{n} \\ \stackrel{\rightharpoonup}{\mathrm{~N}} \\ \dot{m} \end{gathered}$ |  | $\begin{aligned} & n \\ & \tilde{m} \\ & \stackrel{\rightharpoonup}{n} \\ & \dot{m} \end{aligned}$ | $\begin{aligned} & \infty \\ & \infty \\ & \stackrel{\infty}{\circ} \\ & \stackrel{m}{2} \end{aligned}$ | $\begin{aligned} & \underset{\sim}{y} \\ & \underset{\sim}{n} \\ & \underset{\sim}{2} \end{aligned}$ | $\left\lvert\, \begin{aligned} & \underset{\sim}{I} \\ & \underset{\sim}{\infty} \\ & \dot{\sim} \end{aligned}\right.$ | $\begin{aligned} & \stackrel{n}{2} \\ & \stackrel{0}{0} \\ & \vdots \\ & \infty \end{aligned}$ | $\begin{aligned} & \stackrel{m}{n} \\ & \stackrel{\sim}{0} \\ & 0 \\ & \dot{m} \\ & \hline \end{aligned}$ | $\begin{aligned} & -\vec{J} \\ & 0 \\ & \vdots \\ & \cdots \end{aligned}$ | $\begin{aligned} & n \\ & 0 \\ & \infty \\ & \vdots \\ & i \end{aligned}$ |  |
| $\begin{array}{\|c} \hline 0 \\ 0 \\ 0 \\ \hline 0 \end{array}$ | $\underset{6}{ }$ | $\checkmark$ | N | $\checkmark$ | $\bigcirc$ | － | ̇ | ת̃ | $\checkmark$ | $\bigcirc$ | ス | H | $\bigcirc$ | ำ | － | $\square$ | － | $\bigcirc$ | － | $\stackrel{\text { N }}{ }$ | N | $\stackrel{\infty}{\sim}$ | \＆ | $\stackrel{\infty}{\square}$ | － | $\curvearrowleft$ | － | $\stackrel{\sim}{\square}$ | － | － | － | $\backsim$ | 앙 |
| $\stackrel{\circ}{2}$ | $\sim$ | $\sim$ | ～ | $\sim$ | N | ～ | $\sim$ | $\sim$ | $\sim$ | $\sim$ | m | N | ～ | $\sim$ | N | $\sim$ | m | ～ | $\sim$ | N | $\sim$ | ～ | $\sim$ | N | m | m | $\sim$ | $\sim$ | $\sim$ | ～ | $\sim$ | ～ | $\sim$ |
| $\begin{aligned} & 0 \\ & . \\ & \stackrel{n}{0} \end{aligned}$ | $\begin{aligned} & \stackrel{\infty}{m} \\ & \text { な } \\ & \stackrel{y}{5} \end{aligned}$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | on <br> 0 <br> 0 <br>  <br>  <br> 0 <br> 0 |  |  |  |  |  |  | n <br> 0 <br> 0 <br>  <br>  <br> 0 |  |  |  |  |  | N m 岂 岕 |  |  |
| \％ | $\stackrel{\sim}{\sim}$ | $\infty$ | $\stackrel{-}{\infty}$ | N | $\stackrel{\infty}{\infty}$ | $\underset{\infty}{+}$ | $\stackrel{\sim}{\infty}$ | $\stackrel{\circ}{\infty}$ | ¢ | $\stackrel{\infty}{\infty}$ | ¢ | 앙 | G | N | n | \％ | ภ | ¢ | ลิ | $\stackrel{\infty}{\circ}$ | の | $\stackrel{\circ}{\square}$ | $\stackrel{\rightharpoonup}{\square}$ | N |  | $\stackrel{\text { ® }}{\square}$ | $\stackrel{\sim}{\square}$ | $\stackrel{\bullet}{\circ}$ | $\stackrel{\text { ® }}{\sim}$ | $\stackrel{\infty}{0}$ | or | $\stackrel{+}{\square}$ | $\xrightarrow{-}$ |

Table 1 （conclusion）：List of Kamdar et al． 2019 objects with spatial movement values derived from GAIA DR2 data

## Counter-Check of Reported Common Origin Pairs

| Obj | Source_ID1 | mass50_1 | teff50_1 | dTeff_1 | x-out_1 | Source_ID2 | mass50_2 | teff50_2 | dTeff_2 | x-out_2 | dmass |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1122704451665024 | 0.82354641 | 5498.7480 | 146.1880 | 0 | 3265589567885580928 | 0.83603233 | 5604.88818 | 336.1582 | 1 | 0.012 |
| 2 | 7692256363002240 | 0.84840083 | 5408.1909 | -34.0991 | 0 | 20971947349799552 | 0.85241753 | 5559.36133 | -11.8789 | 0 | 0.004 |
| 3 | 16154437152168832 | 0.79902482 | 5284.1997 | 128.2896 | 0 | 9880902977577984 | 0.84556007 | 5420.12451 | 84.9746 | 0 | 0.047 |
| 4 | 42551825846558592 | 0.88936770 | 5797.2202 | 147.4404 | 0 | 42084395964513792 | 0.91640741 | 5980.22461 | 162.9248 | 0 | 0.027 |
| 5 | 47857587925189120 | 0.64921528 | 4059.8782 | 158.0081 | 1 | 53789109561985664 | 0.65013093 | 4077.20435 | 153.0745 | 1 | 0.001 |
| 6 | 111189457686432768 | 0.82186818 | 5395.5186 | -109.5015 | 0 | 64763987951737728 | 0.85051018 | 5640.72656 | 44.3364 | 0 | 0.029 |
| 7 | 119917449706594048 | 0.93315661 | 5864.3799 | 85.1597 | 0 | 167938413891896320 | 0.96537900 | 6005.25098 | 267.4912 | 0 | 0.032 |
| 8 | 123121907625515264 | 1.26741958 | 6491.2031 | 99.0132 | 0 | 123122109487531392 | 1.24347806 | 6853.45947 | 390.8696 | 0 | 0.024 |
| 9 | 125696203647091968 | 0.98452550 | 6015.6221 | 76.4819 | 0 | 130090337447459456 | 0.96829951 | 5948.62891 | 91.6890 | 0 | 0.016 |
| 10 | 169924131895224576 | 1.28370500 | 6755.4702 | -26.2397 | 0 | 169924338053653888 | 1.16347039 | 6437.42139 | -196.8584 | 0 | 0.120 |
| 11 | 170420595752139136 | 1.04931045 | 6444.7534 | 274.9937 | 0 | 218583323918729344 | 1.01199782 | 6374.47900 | 313.9390 | 1 | 0.037 |
| 12 | 218585080564110720 | 1.04612553 | 6005.7607 | 309.0010 | 0 | 219558079636107904 | 0.96354043 | 5711.76318 | 93.7334 | 0 | 0.083 |
| 13 | 219306463271848960 | 1.05499613 | 6258.9482 | 12.6782 | 0 | 219306669430278528 | 0.99368417 | 6164.08740 | 105.5674 | 0 | 0.061 |
| 14 | 231950159800091136 | 1.33756506 | 7054.1221 | 453.0620 | 0 | 227143507276539776 | 1.71516764 | 6390.08984 | -224.0200 | 0 | 0.378 |
| 15 | 232399379018288000 | 0.97196430 | 6011.7480 | 495.8979 | 1 | 243874294680919040 | 0.98462313 | 5817.71680 | 259.0269 | 1 | 0.013 |
| 16 | 238163534366737792 | 1.16875386 | 6421.9092 | 440.0693 | 1 | 238164255921243776 | 1.06772888 | 6172.66309 | 225.8433 | 0 | 0.101 |
| 17 | 246940626453544448 | 0.95937753 | 6067.3677 | 41.1675 | 0 | 247256285068674688 | 1.04637659 | 6385.91016 | 248.0503 | 0 | 0.087 |
| 18 | 441835346009033984 | 0.88860452 | 5651.0308 | 122.6509 | 0 | 249672122574436096 | 0.91421098 | 5921.97021 | 213.1802 | 0 | 0.026 |
| 19 | 250841487549136384 | 1.09061027 | 6330.7832 | 25.5132 | 0 | 251596813379978752 | 1.06547701 | 6300.90723 | 151.0771 | 0 | 0.025 |
| 20 | 251334240556370688 | 2.30334878 | 10220.6240 | 2644.8643 | 0 | 443759220778884224 | 1.77058589 | 7417.18799 | -161.9619 | 0 | 0.533 |
| 21 | 261380718817244160 | 1.03973842 | 6310.4351 | -90.9351 | 0 | 261380718817241088 | 1.03881931 | 6374.73389 | 131.6240 | 0 | 0.001 |
| 22 | 324809272581952000 | 1.41529906 | 6508.9897 | 238.0298 | 0 | 324514710840745856 | 1.31336868 | 6583.26660 | 119.8267 | 0 | 0.102 |
| 23 | 341881492706662144 | 1.03255880 | 6162.2339 | -371.4561 | 0 | 341622798236732544 | 1.19063318 | 6680.63037 | 91.4902 | 0 | 0.158 |
| 24 | 454947949883907584 | 1.03670335 | 6178.4028 | -220.7974 | 0 | 359930701929480576 | 1.15079343 | 6465.92969 | -40.6401 | 0 | 0.114 |
| 25 | 375862311879284608 | 1.08074927 | 6232.2070 | -85.0332 | 0 | 375470099761825152 | 1.07294464 | 6282.23291 | -36.3770 | 0 | 0.008 |
| 26 | 378396823620082176 | 0.94099438 | 6020.3726 | 436.8325 | 1 | 2863674156188051968 | 0.91534865 | 5677.97119 | 182.8911 | 0 | 0.026 |
| 27 | 391987680693489152 | 1.15636611 | 6586.3306 | 106.4604 | 0 | 392030252409329152 | 1.22869611 | 6651.58447 | 28.3442 | 0 | 0.072 |
| 28 | 393820124202543744 | 1.23344064 | 6665.6909 | -388.7593 | 0 | 393318025340738304 | 1.21399820 | 6684.61377 | -568.1265 | 1 | 0.019 |
| 29 | 404970954514021248 | 1.48430407 | 6280.6831 | 302.7930 | 0 | 410369144287662848 | 1.21617508 | 6302.15381 | 308.1440 | 0 | 0.268 |
| 30 | 435642209326124672 | 0.94299531 | 6133.8262 | 257.5859 | 1 | 443414833121045376 | 0.97542882 | 6002.25684 | 138.7769 | 0 | 0.032 |
| 31 | 447513503231078400 | 0.94598198 | 5827.5762 | 109.3662 | 0 | 449590205816652416 | 0.92190880 | 5734.71973 | 89.9600 | 0 | 0.024 |
| 32 | 448952656574582912 | 1.38989246 | 6607.0918 | 116.6519 | 0 | 448597243738258176 | 1.28586125 | 6382.28662 | 63.4365 | 0 | 0.104 |
| 33 | 454335594920988288 | 1.25482953 | 6569.2393 | 193.8193 | 0 | 454131631217853312 | 1.09048426 | 6290.67480 | -221.4053 | 0 | 0.164 |
| 34 | 584181183652059520 | 0.93888760 | 5719.1206 | 38.3804 | 0 | 584164828416571264 | 0.86158121 | 5671.30811 | 40.8682 | 0 | 0.077 |
| 35 | 636738934675606784 | 1.59599984 | 6871.2813 | 419.4414 | 0 | 636739179489212032 | 1.51452315 | 6928.14258 | 381.5127 | 0 | 0.081 |
| 36 | 646124645103549312 | 1.33366895 | 6781.7344 | 713.8545 | 1 | 646125297938578944 | 1.40570605 | 6345.96826 | 374.5581 | 0 | 0.072 |
| 37 | 649308177943887744 | 1.28194988 | 6899.8096 | 189.4697 | 0 | 649308177943888256 | 1.41994059 | 7520.42529 | 811.9053 | 1 | 0.138 |
| 38 | 649872875947754368 | 0.96449602 | 5945.3491 | 145.4990 | 0 | 649658509835641984 | 1.07643330 | 6166.14893 | 355.1890 | 1 | 0.112 |
| 39 | 681387387460651392 | 0.98258853 | 6012.6191 | 211.1489 | 0 | 681308634941086592 | 1.01606786 | 6052.39746 | 381.5474 | 1 | 0.033 |
| 40 | 694859604653048320 | 0.84657586 | 5395.4961 | 202.0361 | 1 | 698203249576119424 | 0.85109192 | 5306.54736 | -60.4028 | 0 | 0.005 |

Table 2. List of Kamdar et al. 2019 objects with masses and comparison of effective temperature values

## Counter-Check of Reported Common Origin Pairs

| Obj | Source_ID1 | mass50_1 | teff50_1 | dTeff_1 | x-out_1 | Source_ID2 | mass50_2 | teff50_2 | dTeff_2 | x-out_2 | dmass |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 41 | 700628394269760384 | 0.64959866 | 4521.0425 | 58.2227 | 0 | 712157155941158528 | 0.71526599 | 4770.91699 | 162.7471 | 1 | 0.066 |
| 42 | 743956681482125696 | 1.35093105 | 7145.5308 | 492.9507 | 0 | 743944930451603584 | 1.28289664 | 6508.82422 | -37.8960 | 0 | 0.068 |
| 43 | 759993226077119744 | 0.92548752 | 5661.4824 | 117.1826 | 0 | 757566711288894336 | 0.93561995 | 5696.64600 | -17.2842 | 0 | 0.010 |
| 44 | 772021200385350656 | 0.94391233 | 5885.9219 | -32.8979 | 0 | 771335483086516608 | 0.96646762 | 5955.19873 | 81.3989 | 0 | 0.023 |
| 45 | 809876320578914560 | 1.00969779 | 6152.6660 | -66.9941 | 0 | 809892916332505344 | 1.08645821 | 6430.24365 | 86.2036 | 0 | 0.077 |
| 46 | 1013151556420133760 | 0.83115226 | 5402.2441 | 25.8843 | 0 | 824940179634765184 | 0.88349777 | 5802.80078 | 263.2910 | 1 | 0.052 |
| 47 | 893835131555196928 | 0.85144681 | 5394.0493 | 134.2896 | 0 | 881823826013408384 | 0.84755492 | 5226.06641 | 43.5264 | 0 | 0.004 |
| 48 | 887350001520664320 | 1.20806801 | 6746.3696 | 584.7896 | 1 | 887785476843657216 | 1.15230465 | 6536.98242 | 208.5625 | 0 | 0.056 |
| 49 | 888398321434606336 | 0.82226795 | 5353.3774 | 312.4175 | 1 | 3384590875297070336 | 0.83390433 | 5264.92578 | 33.2856 | 0 | 0.012 |
| 50 | 895244705461615360 | 0.95762074 | 6122.1138 | 49.6538 | 0 | 892704932384262016 | 1.01139796 | 6248.44238 | 210.2822 | 0 | 0.054 |
| 51 | 905618632028815488 | 0.69929254 | 4690.4795 | 260.2393 | 1 | 903078893312116992 | 0.69810742 | 4732.06543 | 181.3057 | 1 | 0.001 |
| 52 | 946471222781159040 | 0.86123198 | 5442.1948 | 473.0249 | 1 | 946435007617437056 | 0.72477883 | 4874.40039 | -26.6694 | 0 | 0.136 |
| 53 | 951384665369338496 | 1.12926197 | 6488.9829 | 22.1230 | 0 | 953385505948551424 | 1.35763168 | 6965.49268 | 310.9727 | 0 | 0.228 |
| 54 | 964632547829087104 | 1.58103168 | 7173.2129 | 500.7930 | 0 | 951885909527428608 | 1.19650888 | 6701.50928 | 132.5293 | 0 | 0.385 |
| 55 | 990900426075527552 | 0.96854061 | 6052.1865 | 12.0967 | 0 | 966619154885685632 | 0.99235028 | 6185.04346 | 49.8232 | 0 | 0.024 |
| 56 | 984419320425738368 | 1.32092071 | 6754.4351 | 253.7949 | 0 | 985206124075128832 | 1.29392290 | 6448.38721 | -41.2529 | 0 | 0.027 |
| 57 | 1001366818996615808 | 0.93009263 | 5673.2266 | 649.2266 | 1 | 1000361659209075584 | 0.86338896 | 5387.76367 | 251.0737 | 1 | 0.067 |
| 58 | 1168180153315910016 | 0.90105122 | 5431.5835 | 1068.2734 | 1 | 1154953573894521856 | 0.70068246 | 4587.10938 | 411.7495 | 1 | 0.200 |
| 59 | 1224871213362969088 | 0.99256539 | 6029.8086 | 321.9087 | 1 | 1206701371397064320 | 0.92105979 | 5934.45117 | 341.0713 | 1 | 0.072 |
| 60 | 1257207742959979776 | 0.89396465 | 5688.6553 | 116.6055 | 0 | 1259706146911669376 | 1.07587135 | 6237.99756 | 562.0874 | 1 | 0.182 |
| 61 | 1281094087612638976 | 0.85173374 | 5623.3350 | 234.4048 | 1 | 1286716440326271616 | 0.87724942 | 5394.81543 | -71.8848 | 0 | 0.026 |
| 62 | 1283252566380133888 | 1.03986490 | 6129.4985 | 348.1587 | 1 | 1283252566380134016 | 0.94234687 | 5829.38037 | 170.2305 | 0 | 0.098 |
| 63 | 1291119606434912384 | 1.07280505 | 6159.2847 | 170.4448 | 0 | 1291120362349158016 | 1.16432071 | 6317.44287 | 359.1631 | 0 | 0.092 |
| 64 | 1304112397900299648 | 0.90432316 | 5936.0337 | 268.3335 | 0 | 1301234048257809664 | 0.88407356 | 5664.48730 | 6.5674 | 0 | 0.020 |
| 65 | 1312512490643339264 | 0.97247839 | 6060.1050 | 153.4448 | 0 | 1312257786198974208 | 0.94912940 | 6049.83057 | 275.2808 | 1 | 0.023 |
| 66 | 1536912922562394880 | 0.70140457 | 4537.9131 | 299.4429 | 1 | 1537081560158538240 | 0.69993162 | 4644.82275 | 337.4028 | 1 | 0.001 |
| 67 | 1541225138449580928 | 1.22434998 | 6302.2651 | 1.3452 | 0 | 1541225310245728640 | 1.41579914 | 6200.96045 | -116.6597 | 0 | 0.191 |
| 68 | 1574123282265128576 | 0.95434684 | 5973.5825 | 23.6123 | 0 | 1574123454063820928 | 0.93853801 | 6068.65332 | 169.0034 | 0 | 0.016 |
| 69 | 1586864388649398656 | 0.80125594 | 5315.1611 | 63.4312 | 0 | 1589711432272952448 | 0.78449023 | 5184.50781 | 113.3677 | 0 | 0.017 |
| 70 | 1588873265111172992 | 1.36576605 | 6336.8926 | 202.9224 | 0 | 1589069356138116608 | 0.97696918 | 6270.41016 | 40.2002 | 0 | 0.389 |
| 71 | 1594637248661284864 | 1.10911000 | 6627.0273 | 353.5674 | 1 | 1594612952030474496 | 1.00242877 | 6135.75049 | -141.3794 | 0 | 0.107 |
| 72 | 1897440689367972096 | 1.20438313 | 6194.0449 | 18.4751 | 0 | 1897440689367972736 | 1.18771827 | 6351.32617 | 140.3560 | 0 | 0.017 |
| 73 | 1935112912675231360 | 1.24469006 | 6474.5093 | 275.2495 | 0 | 1924718984440067584 | 1.03835642 | 6198.70264 | 128.4824 | 0 | 0.206 |
| 74 | 1931948690008232960 | 0.97435808 | 6004.8057 | -19.1143 | 0 | 1935115386576401152 | 1.03344274 | 6228.21582 | 123.1060 | 0 | 0.059 |
| 75 | 1998564659266179840 | 1.05144942 | 6141.1816 | -17.1182 | 0 | 1997199585521495808 | 1.14105678 | 6298.01709 | 134.4170 | 0 | 0.090 |
| 76 | 2052491723180057856 | 1.26357603 | 6604.1338 | 728.7339 | 1 | 2076462137513753600 | 0.97298032 | 6050.66162 | 241.1118 | 1 | 0.291 |
| 77 | 2080536931908027392 | 1.20488608 | 6633.3350 | 98.9048 | 0 | 2128080536248155776 | 1.26988995 | 7061.06836 | 485.2886 | 0 | 0.065 |
| 78 | 2130506398193844352 | 0.79537481 | 5315.3970 | 63.9067 | 0 | 2128941861868381952 | 0.78793889 | 5170.17480 | 15.2646 | 0 | 0.007 |
| 79 | 2130775885920987520 | 0.92900294 | 5973.7861 | 51.5264 | 0 | 2130775782841768192 | 0.98768097 | 6097.32520 | 66.9951 | 0 | 0.059 |
| 80 | 2663390691484861440 | 1.02303529 | 6195.1787 | -52.2515 | 0 | 2663390485326432000 | 1.40682697 | 6382.39063 | 133.0806 | 0 | 0.384 |

## Counter-Check of Reported Common Origin Pairs



## Counter-Check of Reported Common Origin Pairs



Figure 1: Relationship difference Mass50/Teff for GAIA DR2 StarHorse


Figure 2: Relationship difference GAIA DR2 StarHorse Mass50/LAMOST Teff
(Continued from page 49)
ing GAIA DR2 StarHorse Mass50 differences to LAMOST Teff differences (see Figure 2) - no relationship between MASS50 and Teff to be explained by the dTeff<200 cut applied by Kamdar et al. 2019.

GAIA DR2 StarHorse provides also metallicity data - this time the relationship with LAMOST data is somewhat different as $48 \%$ of the LAMOST values are outside the StarHorse 16 and 84 percentile values which can no longer to be explained by statistical means. Table 3 gives the GAIA DR2 StarHorse metallicity values with the LAMOST metallicity values for comparison:

Content description:

- $\quad \mathrm{Obj}=\mathrm{KMD}$ object number
- met16_1 = GAIA DR2 StarHorse percentile 16 metallicity value primary
- met50_1 = GAIA DR2 StarHorse percentile 50 metallicity value primary
- met84_1 = GAIA DR2 StarHorse percentile 84
metallicity value primary
- $\quad[\mathrm{Fe} / \mathrm{H}]_{-} 1=$ LAMOST metallicity value primary
- $\quad$ e $[\mathrm{Fe} / \mathrm{H}] \_1$ = LAMOST metallicity value error primary
- met16_2 = GAIA DR2 StarHorse percentile 16 metallicity value secondary
- met50_2 = GAIA DR2 StarHorse percentile 50 metallicity value secondary
- met84_2 =
- GAIA DR2 StarHorse percentile 84 metallicity value secondary
- $\quad[\mathrm{Fe} / \mathrm{H}] \_2=$ LAMOST metallicity value secondary
- $\quad \mathrm{e}[\mathrm{Fe} / \mathrm{H}] \_2$ = LAMOST metallicity value error secondary

With few exceptions the LAMOST $[\mathrm{Fe} / \mathrm{H}]$ are rather high near the StarHorse met84 values indicating a regular pattern towards higher metallicity values.
(Text continues on page 60)

## Counter-Check of Reported Common Origin Pairs

| Obj | met16_1 | met50_1 | met84_1 | [ $\mathrm{Fe} / \mathrm{H}]{ }^{1}$ | e_[ $\mathrm{Fe} / \mathrm{H}]{ }^{1}$ | met16_2 | met50_2 | met84_2 | [ $\mathrm{Fe} / \mathrm{H}]{ }^{2}$ | e_[ $\mathrm{Fe} / \mathrm{H}] \mathrm{c}^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | -0.504730 | -0.214799 | 0.018472 | 0.174 | 0.024 | -0.533411 | -0.287828 | -0.026535 | 0.135 | 0.027 |
| 2 | -0.329780 | -0.072325 | 0.127610 | 0.068 | 0.027 | -0.419454 | -0.170029 | 0.090333 | -0.058 | 0.026 |
| 3 | -0.336304 | -0.158173 | 0.037918 | 0.212 | 0.036 | -0.339459 | -0.116237 | 0.117755 | 0.141 | 0.029 |
| 4 | -0.486111 | -0.218653 | 0.024346 | 0.112 | 0.032 | -0.617031 | -0.254580 | 0.031695 | 0.012 | 0.033 |
| 5 | -0.034633 | 0.111280 | 0.238216 | -0.255 | 0.064 | -0.015918 | 0.109144 | 0.220651 | -0.099 | 0.055 |
| 6 | -0.411372 | -0.189868 | 0.032739 | -0.159 | 0.026 | -0.491621 | -0.236863 | -0.005390 | -0.100 | 0.017 |
| 7 | -0.465934 | -0.170349 | 0.050404 | 0.172 | 0.016 | -0.543610 | -0.257203 | 0.000011 | 0.110 | 0.026 |
| 8 | -0.478800 | -0.245652 | 0.029777 | -0.264 | 0.013 | -0.591997 | -0.303373 | -0.021739 | -0.350 | 0.011 |
| 9 | -0.538603 | -0.213167 | 0.067907 | 0.115 | 0.019 | -0.497312 | -0.145970 | 0.123018 | 0.028 | 0.019 |
| 10 | -0.571332 | -0.263635 | 0.009952 | 0.016 | 0.013 | -0.461071 | -0.172921 | 0.108874 | 0.045 | 0.063 |
| 11 | -0.684599 | -0.375784 | -0.149648 | -0.017 | 0.018 | -0.705887 | -0.410444 | -0.146290 | 0.096 | 0.027 |
| 12 | -0.604344 | -0.310685 | -0.009425 | 0.140 | 0.024 | -0.735991 | -0.394119 | -0.045994 | 0.142 | 0.025 |
| 13 | -0.478824 | -0.221713 | 0.021298 | 0.006 | 0.015 | -0.554977 | -0.251318 | -0.014994 | 0.035 | 0.019 |
| 14 | -0.486413 | -0.235910 | 0.000006 | -0.056 | 0.036 | -0.220633 | 0.054005 | 0.296181 | 0.031 | 0.032 |
| 15 | -0.454740 | -0.127553 | 0.103158 | 0.173 | 0.026 | -0.223838 | 0.082330 | 0.282742 | 0.291 | 0.029 |
| 16 | -0.490128 | -0.185930 | 0.080898 | 0.186 | 0.012 | -0.444843 | -0.162316 | 0.136129 | 0.108 | 0.009 |
| 17 | -0.567472 | -0.274933 | -0.010450 | -0.168 | 0.033 | -0.678518 | -0.344229 | -0.070892 | -0.467 | 0.048 |
| 18 | -0.373046 | -0.129227 | 0.080382 | 0.113 | 0.017 | -0.520404 | -0.217929 | 0.036367 | 0.249 | 0.021 |
| 19 | -0.552573 | -0.229601 | 0.032306 | -0.058 | 0.015 | -0.553500 | -0.251073 | 0.011146 | 0.188 | 0.063 |
| 20 | -0.445734 | -0.176862 | 0.100036 | -0.124 | 0.084 | -0.475487 | -0.190004 | 0.140063 | -0.075 | 0.032 |
| 21 | -0.591864 | -0.298452 | -0.023195 | -0.017 | 0.030 | -0.668509 | -0.338875 | -0.095745 | -0.038 | 0.037 |
| 22 | -0.460254 | -0.190557 | 0.058184 | 0.107 | 0.014 | -0.536076 | -0.126156 | 0.117013 | 0.087 | 0.016 |
| 23 | -0.492197 | -0.236133 | 0.012984 | -0.013 | 0.021 | -0.580382 | -0.295423 | -0.035518 | -0.044 | 0.011 |
| 24 | -0.417558 | -0.182820 | 0.092814 | 0.089 | 0.025 | -0.496123 | -0.192172 | 0.060389 | 0.006 | 0.012 |
| 25 | -0.411062 | -0.132018 | 0.170217 | 0.204 | 0.015 | -0.504694 | -0.200349 | 0.065168 | 0.076 | 0.014 |
| 26 | -0.528237 | -0.259098 | -0.001440 | 0.152 | 0.027 | -0.336341 | -0.056597 | 0.168923 | 0.287 | 0.042 |
| 27 | -0.571472 | -0.263488 | -0.000101 | -0.111 | 0.044 | -0.494434 | -0.193868 | 0.023416 | -0.047 | 0.019 |
| 28 | -0.545967 | -0.225304 | 0.003052 | -0.112 | 0.011 | -0.631137 | -0.283507 | -0.027862 | -0.144 | 0.023 |
| 29 | -0.446516 | -0.131022 | 0.131600 | -0.300 | 0.084 | -0.420453 | -0.162598 | 0.092312 | 0.135 | 0.021 |
| 30 | -0.654620 | -0.356302 | -0.108279 | 0.095 | 0.028 | -0.522167 | -0.228267 | 0.030653 | 0.121 | 0.034 |
| 31 | -0.394132 | -0.108565 | 0.116114 | -0.138 | 0.088 | -0.317991 | -0.041902 | 0.167132 | 0.182 | 0.046 |
| 32 | -0.347239 | -0.034190 | 0.273158 | 0.043 | 0.029 | -0.269190 | 0.046649 | 0.363325 | 0.090 | 0.079 |
| 33 | -0.464321 | -0.135290 | 0.105043 | -0.007 | 0.020 | -0.462254 | -0.177135 | 0.073527 | -0.048 | 0.046 |
| 34 | -0.316906 | -0.030815 | 0.206894 | 0.069 | 0.038 | -0.504110 | -0.252154 | -0.020714 | -0.073 | 0.017 |
| 35 | -0.516252 | -0.210490 | 0.014526 | -0.148 | 0.011 | -0.520692 | -0.238573 | 0.079828 | -0.280 | 0.009 |
| 36 | -0.417207 | -0.126166 | 0.144058 | 0.331 | 0.034 | -0.323733 | -0.074603 | 0.218316 | 0.316 | 0.035 |
| 37 | -0.603742 | -0.284685 | 0.027087 | -0.169 | 0.012 | -0.686952 | -0.333572 | -0.026532 | -0.139 | 0.010 |
| 38 | -0.392878 | -0.162378 | 0.102092 | 0.142 | 0.072 | -0.420150 | -0.128506 | 0.191816 | 0.101 | 0.014 |
| 39 | -0.467818 | -0.167664 | 0.074249 | 0.164 | 0.023 | -0.416215 | -0.113584 | 0.130606 | 0.251 | 0.025 |
| 40 | -0.291330 | -0.044508 | 0.152809 | 0.007 | 0.025 | -0.188218 | 0.008024 | 0.234578 | 0.090 | 0.035 |

Table 3: Comparison metallicity values GAIA DR2 StarHorse and LAMOST index
Table 3 continues on the next page.

## Counter-Check of Reported Common Origin Pairs

| Obj | met16_1 | met50_1 | met84_1 | [ $\mathrm{Fe} / \mathrm{H}$ ]_1 | e_[ ${ }^{\text {Fe/H] }}$ - ${ }^{1}$ | met16_2 | met50_2 | met84_2 | [ $\mathrm{Fe} / \mathrm{H}]{ }^{2}$ | e_[ $\mathrm{Fe} / \mathrm{H}]{ }^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 41 | -0.467962 | -0.310881 | -0.149665 | -0.490 | 0.024 | -0.326669 | -0.110265 | 0.023105 | -0.530 | 0.037 |
| 42 | -0.576009 | -0.223598 | -0.004569 | -0.173 | 0.009 | -0.530385 | -0.234885 | 0.048425 | -0.157 | 0.014 |
| 43 | -0.156943 | 0.019955 | 0.228716 | 0.241 | 0.057 | -0.317150 | -0.034048 | 0.188641 | 0.207 | 0.032 |
| 44 | -0.426928 | -0.115645 | 0.118799 | -0.009 | 0.013 | -0.442575 | -0.156124 | 0.118844 | 0.368 | 0.026 |
| 45 | -0.482139 | -0.168969 | 0.099184 | -0.116 | 0.016 | -0.590733 | -0.303078 | -0.012615 | 0.034 | 0.012 |
| 46 | -0.403086 | -0.174252 | 0.037731 | 0.080 | 0.025 | -0.610894 | -0.323304 | -0.072447 | 0.013 | 0.020 |
| 47 | -0.311652 | -0.074635 | 0.173896 | 0.344 | 0.039 | -0.161045 | 0.027214 | 0.251074 | 0.403 | 0.090 |
| 48 | -0.576217 | -0.269161 | -0.023067 | 0.009 | 0.054 | -0.574138 | -0.254205 | 0.033766 | 0.257 | 0.016 |
| 49 | -0.337515 | -0.154410 | 0.075029 | 0.092 | 0.088 | -0.283678 | -0.025871 | 0.190184 | 0.222 | 0.030 |
| 50 | -0.539819 | -0.297980 | -0.016171 | -0.645 | 0.015 | -0.627590 | -0.314466 | -0.064572 | -0.493 | 0.045 |
| 51 | -0.351187 | -0.162341 | 0.083178 | -0.158 | 0.036 | -0.411089 | -0.250350 | -0.051253 | -0.477 | 0.042 |
| 52 | -0.285749 | -0.019436 | 0.192397 | 0.397 | 0.056 | -0.327668 | -0.174744 | -0.012329 | -0.233 | 0.265 |
| 53 | -0.558527 | -0.265057 | -0.011548 | -0.045 | 0.019 | -0.418650 | -0.162158 | 0.072589 | 0.006 | 0.016 |
| 54 | -0.405367 | -0.131515 | 0.128203 | -0.251 | 0.009 | -0.580979 | -0.254307 | -0.022309 | -0.056 | 0.030 |
| 55 | -0.597362 | -0.273575 | -0.004613 | 0.066 | 0.046 | -0.613184 | -0.317751 | -0.029745 | -0.131 | 0.032 |
| 56 | -0.488602 | -0.144363 | 0.132350 | 0.061 | 0.013 | -0.474647 | -0.177635 | 0.108953 | 0.073 | 0.012 |
| 57 | -0.203543 | 0.033767 | 0.279636 | 0.079 | 0.137 | -0.252783 | -0.023820 | 0.210767 | 0.257 | 0.074 |
| 58 | -0.033633 | 0.203981 | 0.348046 | -0.205 | 0.032 | -0.244737 | -0.025005 | 0.242549 | -0.149 | 0.050 |
| 59 | -0.467855 | -0.161181 | 0.103636 | 0.198 | 0.021 | -0.574943 | -0.269228 | -0.011852 | 0.189 | 0.028 |
| 60 | -0.397423 | -0.134122 | 0.060205 | 0.158 | 0.031 | -0.427342 | -0.128383 | 0.134354 | 0.408 | 0.021 |
| 61 | -0.494288 | -0.248751 | -0.007749 | 0.109 | 0.020 | -0.227273 | 0.010158 | 0.272978 | 0.172 | 0.018 |
| 62 | -0.464254 | -0.137565 | 0.143238 | -0.069 | 0.024 | -0.382538 | -0.070400 | 0.158991 | -0.071 | 0.031 |
| 63 | -0.426165 | -0.124629 | 0.170746 | 0.183 | 0.024 | -0.461149 | -0.136512 | 0.149124 | 0.306 | 0.020 |
| 64 | -0.653075 | -0.323214 | -0.060079 | 0.466 | 0.022 | -0.431629 | -0.210161 | 0.040055 | -0.002 | 0.126 |
| 65 | -0.517179 | -0.217378 | 0.069149 | 0.062 | 0.014 | -0.620444 | -0.314410 | -0.029756 | 0.236 | 0.019 |
| 66 | -0.105293 | 0.071622 | 0.275868 | -0.072 | 0.044 | -0.332103 | -0.106805 | 0.169103 | -0.093 | 0.052 |
| 67 | -0.500502 | -0.211273 | 0.055008 | -0.181 | 0.015 | -0.495982 | -0.161934 | 0.070655 | -0.223 | 0.009 |
| 68 | -0.544916 | -0.209462 | 0.032198 | -0.089 | 0.019 | -0.598782 | -0.318575 | -0.051733 | -0.030 | 0.017 |
| 69 | -0.411865 | -0.195246 | -0.003980 | -0.030 | 0.024 | -0.407800 | -0.211744 | -0.017349 | -0.062 | 0.102 |
| 70 | -0.556978 | -0.254612 | 0.043373 | -0.038 | 0.014 | -0.744829 | -0.368586 | -0.125936 | -0.630 | 0.144 |
| 71 | -0.847239 | -0.435922 | -0.103198 | -0.053 | 0.011 | -0.514046 | -0.178073 | 0.086689 | -0.123 | 0.169 |
| 72 | -0.393255 | -0.106633 | 0.192288 | 0.158 | 0.011 | -0.403667 | -0.096065 | 0.126546 | 0.098 | 0.011 |
| 73 | -0.427884 | -0.105269 | 0.157445 | -0.006 | 0.021 | -0.598389 | -0.247903 | 0.026764 | 0.028 | 0.075 |
| 74 | -0.446517 | -0.114970 | 0.126011 | -0.100 | 0.266 | -0.519610 | -0.251212 | 0.029192 | -0.344 | 0.191 |
| 75 | -0.385577 | -0.089389 | 0.164477 | 0.072 | 0.016 | -0.406135 | -0.139310 | 0.111336 | 0.034 | 0.025 |
| 76 | -0.417588 | -0.114350 | 0.145133 | -0.041 | 0.013 | -0.476157 | -0.191820 | 0.009347 | -0.013 | 0.016 |
| 77 | -0.517962 | -0.206479 | 0.027314 | 0.156 | 0.012 | -0.619084 | -0.324270 | -0.124792 | -0.043 | 0.007 |
| 78 | -0.455724 | -0.207887 | -0.022782 | 0.147 | 0.028 | -0.347699 | -0.161017 | 0.028828 | 0.104 | 0.039 |
| 79 | -0.597275 | -0.287297 | -0.023274 | -0.401 | 0.015 | -0.582232 | -0.287810 | -0.013766 | -0.362 | 0.010 |
| 80 | -0.516644 | -0.232680 | 0.013543 | -0.115 | 0.020 | -0.497026 | -0.163418 | 0.105706 | -0.101 | 0.012 |

Table 3: Comparison metallicity values GAIA DR2 StarHorse and LAMOST index
Table 3 concludes on the next page.

## Counter-Check of Reported Common Origin Pairs

| Obj | met16_1 | met50_1 | met84_1 | [ $\mathrm{Fe} / \mathrm{H}$ ] ${ }^{1}$ | e_[ $\mathrm{Fe} / \mathrm{H}]$ _ ${ }^{1}$ | met16_2 | met50_2 | met84_2 | [ $\mathrm{Fe} / \mathrm{H}$ ] ${ }^{2}$ | e [ $[\mathrm{Fe} / \mathrm{H}]]^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 81 | -0.245099 | -0.025589 | 0.182754 | 0.200 | 0.038 | -0.274318 | -0.052505 | 0.204684 | 0.289 | 0.030 |
| 82 | -0.255055 | -0.047360 | 0.178653 | 0.045 | 0.098 | -0.250551 | -0.028803 | 0.210143 | 0.069 | 0.026 |
| 83 | -0.415291 | -0.221541 | -0.049659 | -0.097 | 0.031 | -0.585354 | -0.360555 | -0.161202 | -0.008 | 0.040 |
| 84 | -0.539896 | -0.302899 | -0.074394 | -0.139 | 0.052 | -0.560185 | -0.302825 | -0.038305 | -0.104 | 0.036 |
| 85 | -0.661037 | -0.357688 | -0.063986 | -0.150 | 0.050 | -0.293320 | 0.013868 | 0.266422 | 0.139 | 0.065 |
| 86 | -0.472739 | -0.113192 | 0.196950 | -0.114 | 0.014 | -0.495787 | -0.235061 | 0.079860 | -0.051 | 0.017 |
| 87 | -0.491012 | -0.194485 | 0.076004 | -0.048 | 0.012 | -0.710104 | -0.386832 | -0.151339 | 0.072 | 0.014 |
| 88 | -0.745131 | -0.381134 | -0.115921 | 0.414 | 0.034 | -0.428539 | -0.170137 | 0.144898 | 0.026 | 0.014 |
| 89 | -0.620113 | -0.321608 | -0.035361 | -0.157 | 0.020 | -0.574907 | -0.236641 | 0.011743 | -0.175 | 0.088 |
| 90 | -0.508143 | -0.181210 | 0.033789 | 0.119 | 0.046 | -0.513228 | -0.234314 | -0.013196 | 0.109 | 0.017 |
| 91 | -0.675210 | -0.348837 | -0.053654 | 0.005 | 0.029 | -0.695992 | -0.451026 | -0.187099 | 0.209 | 0.039 |
| 92 | -0.513469 | -0.229146 | 0.026655 | -0.049 | 0.012 | -0.546004 | -0.231167 | 0.047707 | -0.052 | 0.014 |
| 93 | -0.722804 | -0.384410 | -0.099262 | 0.103 | 0.010 | -0.624944 | -0.336379 | -0.071526 | -0.183 | 0.163 |
| 94 | -0.438684 | -0.191086 | 0.085585 | 0.120 | 0.035 | -0.544101 | -0.278767 | -0.005878 | 0.157 | 0.051 |
| 95 | -0.324121 | -0.087987 | 0.146839 | 0.256 | 0.096 | -0.400773 | -0.133940 | 0.161750 | 0.272 | 0.036 |
| 96 | -0.579761 | -0.314241 | -0.033525 | 0.107 | 0.019 | -0.490528 | -0.190337 | 0.091962 | -0.120 | 0.012 |
| 97 | -0.508968 | -0.246256 | 0.019071 | 0.131 | 0.073 | -0.612042 | -0.301296 | -0.022261 | -0.042 | 0.052 |
| 98 | -0.421883 | -0.178091 | 0.124905 | -0.112 | 0.019 | -0.498384 | -0.193039 | 0.097248 | -0.118 | 0.016 |
| 99 | -0.576596 | -0.299978 | -0.010733 | -0.219 | 0.024 | -0.613788 | -0.328775 | -0.082957 | -0.211 | 0.028 |
| 100 | -0.415308 | -0.186204 | -0.030741 | -0.485 | 0.148 | -0.657282 | -0.416836 | -0.235046 | -0.656 | 0.176 |
| 101 | -0.565219 | -0.262246 | -0.003038 | -0.348 | 0.011 | -0.539020 | -0.216841 | 0.029790 | -0.328 | 0.010 |
| 102 | -0.461958 | -0.213569 | 0.016547 | -0.063 | 0.035 | -0.603614 | -0.312158 | -0.029837 | 0.179 | 0.015 |
| 103 | -0.417991 | -0.149785 | 0.102371 | -0.137 | 0.016 | -0.471003 | -0.210515 | 0.032888 | 0.086 | 0.030 |
| 104 | -0.551161 | -0.244398 | 0.020161 | -0.285 | 0.009 | -0.487894 | -0.149511 | 0.123009 | 0.594 | 0.024 |
| 105 | -0.610076 | -0.316874 | -0.027733 | -0.585 | 0.017 | -0.601856 | -0.312392 | -0.053280 | -0.595 | 0.018 |
| 106 | -0.207767 | 0.002264 | 0.233144 | 0.180 | 0.051 | -0.467373 | -0.268151 | -0.033045 | 0.118 | 0.072 |
| 107 | -0.075377 | 0.299468 | 0.457746 | -0.099 | 0.152 | -0.401706 | -0.243814 | -0.028263 | -0.017 | 0.043 |
| 108 | -0.615981 | -0.264281 | 0.014048 | -0.053 | 0.031 | -0.411049 | -0.121928 | 0.115080 | -0.046 | 0.215 |
| 109 | -0.628613 | -0.284778 | 0.011912 | -0.061 | 0.090 | -0.504638 | -0.186563 | 0.100059 | 0.017 | 0.072 |
| 110 | -0.388020 | -0.093996 | 0.170338 | 0.188 | 0.054 | -0.430975 | -0.143925 | 0.115032 | -0.001 | 0.015 |
| 111 | -0.417117 | -0.176035 | 0.106590 | -0.146 | 0.023 | -0.278125 | -0.030774 | 0.211766 | -0.363 | 0.015 |

Table 3 (conclusion) Comparison metallicity values GAIA DR2 StarHorse and LAMOST index

## Counter-Check of Reported Common Origin Pairs



Graph 3: Comparison GAIA DR2 StarHorse metallicity (met16 and met84) with LAMOST [Fe/H] for the primary

## (Continued from page 56)

These noticeable differences between GAIA DR2 StarHorse and LAMOST metallicity values remain in this context without explanation but comparing the StarHorse met50 values for primary and secondary shows in $\sim 67 \%$ a difference of less than 0.1 indicating similar metallicity also with StarHorse catalog values.

## 4. Discussion

The question how stars are born and how they move over time is certainly fascinating. A star with a moderate fast spatial velocity of $20 \mathrm{~km} / \mathrm{s}$ needs only 150 million years to change is position relative to our Solar system by $\sim 10$ lightyears. Taking the age of the Sun with $\sim 4.6$ billion years this makes then about 300 lightyears or nearly 100 parsecs. As the speed and direction of the movement of stars is quite diverse this means that the neighborhood of a star might change significantly over its lifetime.

This report basically confirms with some caveats the data given in the Kamdar et al. 2019 paper but does not necessarily confirm the conclusion that the reported pairs have to be indeed of common origin due to the following reasons:

- Several seemingly co-moving pairs are despite very similar data values for parallax, proper motion and radial as well as spatial velolicty moving in directions different enough to question the "co -moving" property (see comments below Table 1). With a bit more restrictive thresholds for the
direction of the spatial velocity even most of the listed KMD objects would not be assessed as comoving
- Several KMD objects might if with a small likelihood be very well bound by gravitation according to a Monte Carlo simulation with a sample size of 120,000 using GAIA DR2 data for RA/ Dec and Plx with the given error range used as standard deviation
- The KMD objects are listed as pairs and with few exceptions most are confirmed as such as no other objects with similar parameters are to be found in GAIA DR2. In a few cases such additional objects were found but could not be identified as co-moving members. This confirms the proposition that the listed pairs are most likely no longer part of a cluster - on the other side it seems a bit surprising that the "common origin" property should be restricted to pairs
- More or less all KMD objects come with parallax error values larger than $0.5 \%$ which means that at least for this parameter the data quality for the selected objects might be questionable. As the derived values like the spatial velocity are directly depending on the parallax data this casts a shadow at the final assessment of these objects
- With distances between the components of in average 40 light years the tidal forces of the Galaxy are no longer identical and such differences would over time counter-act against comovement questioning the overall setup of the Kamdar et al. 2019 paper


## Counter-Check of Reported Common Origin Pairs

- The total number of GAIA DR2 objects with parallax $>1$ and existing radial velocity data is 3,129,408 suggesting some likelidhood for pairs to have by chance similar values for parallax, proper motion and radial velocity - so it seems possible that the presented pairs are just random even if the additional criteria "metallicity" is considered
- Several KMD objects are already known double stars listed in the WDS catalog rendering these WDS pairs as likely opticals.


## 5. Common Origin/Common Movement Pairs in the WDS

The WDS catalog contains per June 2019 about 25,000 objects with code "V" for common proper motion. These objects offer a good chance to detect stars born together without being close enough for gravitational relationship by differentiating three szenarios:

- Doubles with a high likelihood for a spatial distance between components smaller than 1 parsec allowing for potential gravitational relationship binaries
- Doubles with proper motion values by chance similar but with parallaxes and radial velocities and as a result spatial velocity far too different to be born together - opticals
- Doubles with all parameters similar enough to be considered to be born together but with parallaxes different enough make potential gravitational relationship rather unlikely either from the very beginning or by splitting up wide binaries later on - common origin.

After eliminating all pairs with separation less than 0.4 " or more than 9999.9 " plus the objects with insufficient RA/Dec data 24,635 objects remained for cross matching with GAIA DR2. The first X-match run with 5 " radius around the primary position yielded 33,232 matches. The second X-match run with calculated J2000 positions for these objects and again 5 " search radius yielded 55,882 objects with the unavoidable self matches and double matches for doubles with a separation smaller than $5^{\prime \prime}$. After eliminating all self-matches and likely wrong matches with delta in separation larger than $20 \%$, delta in position angle larger than $15^{\circ}$, delta in M1 and M2 larger than 4 and eliminating the remaining multiple matches due to dense star fields 23,476 objects remained considered as likely correct cross-matches.

To be able to calculate spatial velocity all objects with missing parallax data or values below 1 mas or missing radial velocity values were eliminated as well
reducing the object count drastically to 2,654. From these 2,203 objects have similar spatial movement (spatial velocity delta below $10 \%$ of the mean spatial velocity of both components and less than $10^{\circ}$ delta in spatial movement direction - this allows for a few pairs a larger delta in spatial velocity than the $1.5 \mathrm{~km} / \mathrm{s}$ cut used by Kamdar et al. 2019 but the additional cut with the direction of the spatial velocity should overcompensate this generosity) and from these 1,030 have a spatial distance between the components (calculated with the given parallax values and the angular separation) of less than 1 parsec considered as threshold for potential gravitational relationship. From the remaining 1,173 objects a few with a spatial distance larger than 100 lightyears as threshold for the diameter of star forming molecular clouds had to be eliminated leaving 1,137 objects.

This selection of WDS objects is based only on the available astrometric data. An attempt to check the aspect of similar metallicity with LAMOST was not very successful due to the limited LAMOST DR4 coverage - only 32 pairs have $[\mathrm{Fe} / \mathrm{H}]$ values for both components with 25 of them up to a delta of 0.1 meeting the cut applied by Kamdar et al. 2019.

The GAIA DR2 StarHorse catalog offers $\sim 98 \%$ coverage but the spread of the given metallicity values indicated by the percentile 16 and 84 data is significantly larger than the spread of the $[\mathrm{Fe} / \mathrm{H}]$ values in LAMOST. With a doubled cut value of 0.2 applied on the GAIA DR2 StarHorse median metallicity value 904 of the listed WDS code "V" objects qualify for similar metallicity and 233 pairs are despite common movement most likely not of common origin due to different metallicities including some outliers with large differences up to 1.96.

This means that out from the sample of V-coded WDS objects with data available for assessment $44 \%$ show common spatial movement and $34 \%$ have additionally similar metallicity suggesting common origin. It might make sense to add an additional WDS notes code for such objects - for example "G" for "proper motion, parallax and radial velocity suggest common origin and common movement".

Table 4 lists 20 such objects as stub with the full table available as flat text file "WDS V common origin and movement" for download.

- WDS = WDS ID
- Disc = Discoverer ID
- $\quad$ C $\quad=$ Components ( AB if blank, in Table 2 all AB)
- PA $=$ Position angle from GAIA DR2 positions
- *e_PA = Error position angle
(Text continues on page 63)


## Counter-Check of Reported Common Origin Pairs

| WDS | Disc | c | PA | Sep | P1x1 | pmra1 | pmdec1 | rv1 | P1x2 | pmra2 | pmdec2 | rv2 | v1 | v1D | v2 | v2D | D_1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 00013+0504 | UC 304 |  | 53.647 | 5.33981 | 8.2162 | -56.863 | -23.221 | 12.43 | 8.3387 | -57.641 | -28.541 | 14.57 | 37.55 | 247.79 | 39.36 | 243.66 | 5.83 |
| $00061+2649$ | GRV |  | 19.102 | 7.27753 | 2.6503 | 18.946 | -8.057 | -51.42 | 2.6719 | 19.038 | -8.275 | -44.36 | 63.24 | 113.04 | 57.65 | 113.49 | . 949 |
| 00093+2517 | GIC | AB | 237.130 | 29.61193 | 24.8900 | 175.348 | -158.970 | 0.67 | 22.4794 | 171.422 | $-145.634$ | 4.71 | 45.08 | 132.20 | 47.66 | 130.35 | 14.052 |
| 00094-3321 | TDS1322 |  | 277.574 | 9.46621 | 7.0201 | 167.998 | -20.353 | 2.82 | 7.1058 | 167.660 | -21.768 | 3.42 | 114.30 | 96.91 | 112.83 | 97.40 | 5.604 |
| 00102+0417 | GRV 12 |  | 174.406 | 45.84872 | 4.7830 | 42.249 | -25.818 | -0.02 | 4.5758 | 42.832 | -26.555 | -1.77 | 49.07 | 121.43 | 52.23 | 121.80 | 30.879 |
| 00110-6309 | UC 331 |  | 311.131 | 35.11078 | 3.7097 | 51.032 | 20.737 | -25.51 | 3.7404 | 51.617 | 21.094 | -25.32 | 74.86 | 67.89 | 75.06 | 67.77 | . 218 |
| 00141-0602 | KPP 52 |  | 196.820 | 3.50176 | 2.4400 | 2.805 | -14.824 | -57.53 | 2.4279 | 3.351 | -14.913 | -59.73 | 64.57 | 169.29 | 66.77 | 167.34 | 6.662 |
| $00159+1706$ | GRV 18 |  | 359.979 | 16.09166 | 4.2445 | 25.805 | -30.356 | -38.99 | 4.2743 | 25.842 | -30.438 | -37.03 | 59.16 | 139.63 | 57.72 | 139.67 | 5.358 |
| $00174+0221$ | STF 21 |  | 51.638 | 7.65779 | 5.6454 | 12.511 | 51.421 | -61.17 | 5.6817 | 13.230 | 50.902 | -60.20 | 75.61 | 13.67 | 74.49 | 14.57 | 3.691 |
| 00175-6142 | KPP1737 |  | 130.295 | 13.40715 | 3.0868 | 40.117 | 7.579 | -1.60 | 3.0467 | 40.374 | 7.870 | -1.78 | 62.71 | 79.30 | 64.02 | 78.97 | 13.907 |
| 00185-3005 | KPP1566 |  | 335.377 | 11.81386 | 2.6561 | -9.674 | -31.869 | -2.41 | 2.6430 | -8.606 | -31.498 | -3.69 | 59.48 | 196.89 | 58.68 | 195.28 | 6.087 |
| 00185-5325 | UC 351 |  | 223.453 | 28.96995 | 7.4621 | 47.531 | -24.364 | -10.30 | . 5238 | 46.284 | -22.534 | -11.19 | 35.46 | 117.14 | 34.31 | 115.96 | 3.585 |
| $00196+6457$ | CBL 568 |  | 12.476 | 24.68793 | 4.6139 | 73.984 | 19.281 | -14.47 | 4.6375 | 73.907 | 18.312 | -12.28 | 79.87 | 75.39 | 78.79 | 76.08 | 3.598 |
| $00211+5447$ | CBL |  | 332.118 | 18.03992 | 5.7191 | 75.425 | 8.714 | 8.82 | 5.6807 | 75.713 | 8.555 | 10.43 | 63.54 | 83.41 | 64.43 | 83.55 | 3.855 |
| 00215-6744 | HJ 3361 | ${ }^{\text {AB }}$ | 293.406 | 5.06627 | 4.4129 | -53.342 | -20.102 | 17.27 | 4.3495 | -52.848 | -21.031 | 16.94 | 63.62 | 249.35 | 64.26 | 248.30 | 10.7 |
| 00248+5030 | KPP1866 |  | 211.506 | 14.46339 | 3.4277 | 40.495 | -16.087 | 1.67 | 3.4742 | 40.499 | -16.434 | 1.16 | 60.28 | 111.67 | 59.64 | 112.09 | 12.736 |
| 00250-5904 | SPM |  | 3.943 | 24.51298 | 5.8662 | 82.091 | 66.293 | 17.47 | 5.9561 | 82.592 | 67.876 | 20.27 | 87.03 | 51.08 | 87.46 | 50.59 | 8.392 |
| $00276+1616$ | GWP 52 |  | 73.467 | 19.52921 | 2.5648 | 50.140 | -4.752 | -0.67 | 2.6465 | 50.352 | -4.498 | 0.32 | 93.08 | 95.41 | 90.54 | 95.10 | 39.258 |
| 00280-3051 | UC 375 |  | 335.925 | 39.77174 | 4.4577 | -30.718 | -44.846 | -3.77 | 4.5852 | -28.046 | -44.646 | -3.86 | 57.92 | 214.41 | 54.64 | 212.14 | 20.346 |
| $00298+0727$ | LOC 1 |  | 77.057 | 9.27217 | 3.4029 | -15.519 | -30.537 | -7.81 | 3.4884 | -16.535 | -31.076 | -7.98 | 48.35 | 206.94 | 48.49 | 208.02 | 23.492 |

Table 4. WDS pairs assumed to be of common origin

## Counter-Check of Reported Common Origin Pairs

(Continued from page 61)

- Sep $=$ Separation from GAIA DR2 positions in arcseconds
- $\quad$ e_Sep $=$ Error separation
- *Vest1 = Vmag1 estiamted from GAIA DR2 G/B/

R-mags

- *Vest2 = Vmag2 estiamted from GAIA DR2 G/B/

R-mags

- Plx1 = Parallax 1 in mas
- pmral $=$ Proper motion RA 1 in mas/yr
- pmdec $1=$ Proper motion Dec 1 in mas/yr
- rV1 $=$ Radial velocity 1 in km/s
- Plx2 $=$ Parallax 2 in mas
- pmra2 $=$ Proper motion RA 2 in mas/yr
- pmdec $2=$ Proper motion Dec 2 in mas/yr
- rV2 $=$ Radial velocity 2 in $\mathrm{km} / \mathrm{s}$
- *Vt1 $=$ Transverse velocity $1 \mathrm{in} \mathrm{km} / \mathrm{s}$
- V1 $\quad=$ Spatial velocity $1 \mathrm{in} \mathrm{km} / \mathrm{s}$
- V1D $=$ Velocity 1 direction
- *Vt2 $=$ Transverse velocity 2 in $\mathrm{km} / \mathrm{s}$
- V2 $\quad=$ Spatial velocity $2 \mathrm{in} \mathrm{km} / \mathrm{s}$
- V2D $=$ Velocity 2 direction
- D_1-2 = Spatial Distance between the components in lightyears calculated by inverting the given parallaxes
- *met50_1 = GAIA DR2 StarHorse median metallicity $1 \overline{\text { in }}$ dex
- ${ }^{\text {met50_2 }}$ = GAIA DR2 StarHorse median metallicity 2 in dex
- *dmet50 $=$ Metallicity difference between the components
* $\quad=$ Data given only in download file

A side result of this matching process is that WDS objects BPM 489/490/491/492/493/494 have an identical primary.

## 6. Acknowledgements

The following tools and resources have been used for this research:

- DSS2 images
- Aladin Sky Atlas v10.0
- GAIA DR2 catalog
- LAMOST DR4 catalog
- GAIA DR2 StarHorse catalog
- Washington Double Star Catalog
- CDS VizieR
- GAIA Archive (ADQL Search)
- Gaia@AIP Services hosted by the LeibnizInstitute for Astrophysics Potsdam (AIP)


## 7. References

Anders, F., et al., 2019, "Photo-astrometric distances, extinctions, and astrophysical parameters for Gaia DR2 stars brighter than $\mathrm{G}=18^{\prime \prime}$, Astronomy \& Astrophysics, 10.1051/0004-6361/201935765.
Harshil Kamdar, et al., 2019, "Stars that Move Together Were Born Together", arXiv:1904.02159 [astroph.GA]. Submitted to ApJL
Knapp, Wilfried R. A., 2018, "A New Concept for Counter-Checking of Assumed Binaries", Journal of Double Star Observations, 14 (3), 487.

## Counter-Check of Reported Common Origin Pairs

## Appendix

## Description of the PGR assessment procedure (according to Knapp 2018)

GAIA DR2 data for RA/Dec and Plx are used for a Monte Carlo simulation assuming a normal distribution for these parameters with the given error range as standard deviation. The distance between the components is calculated from the inverted simulated parallax data and the simulated angular separation using the law of cosines

$$
\sqrt{a^{2}-2 a b \cos \gamma+b^{2}}
$$

with $a$ and $b=$ distance vectors for the stars A and B in lightyears calculated as $(1000 / \mathrm{Plx}) * 3.261631$ and $\gamma=$ angular separation in degrees calculated as

$$
\gamma=\arccos [\sin (D E 1) \sin (D E 2)+\cos (D E 1) \cos (D E 2) \cos (|R A 1-R A 2|)]
$$

The likelihood for potential gravitational relationship (LPGR) is the percentage of simulation results $<200,000$ $\mathrm{AU}(\sim 1 \mathrm{parsec})$ out of the simulation sample with a size of 120,000 corresponding with the likelihood that the real distance is smaller than $200,000 \mathrm{AU}$ with an margin of error of $0.37 \%$ at $99 \%$ confidence.

The smallest, median and largest distance is the smallest, median and largest result of the simulation sample.

# TYC 2036-1173-1: An Optical Triple Star System in Corona Borealis? 

Trygve Prestgard<br>trygvep@hotmail.fr


#### Abstract

TYC 2036-1173-1 is a V $=12.4$ mag star in Corona Borealis, neighboured by two fainter stars of $V=15.7 \mathrm{mag}$ and $\mathrm{V}=17.1 \mathrm{mag}$. While their colours and comparable proper motion might be suggestive of a true visual triple star system, astrometric data from Gaia DR2 and UCAC5 indicate that they may potentially be unrelated.


## Introduction

While hunting for uncataloged multiple stars in SDSS images, I came across an interesting group of three stars that give the appearance of a visual triple star system in survey imagery (see figure 1). The system is composed of TYC 2036-1173-1 ( $\mathrm{V}=12.36$ ), UCAC4 579-051312 (V=15.65) and UCAC4 579051311 ( $\mathrm{V}=17.13$ ), which I have decided to label A, B and C respectively throughout the rest of this paper. The group appears to be absent from the SIMBAD, Stelle Doppie and Vizier database.

In the case of A, the Visual Magnitude and the B-V colour index were extracted from the APASS (Henden, 2016) catalogue. However, due to the fainter nature of B and C, their B-V indices and Visual magnitude could only be determined from GSC2.3 (STScI, 2006) data.

The $\mathrm{B}-\mathrm{V}$ and the Gaia $\mathrm{Bp}-\mathrm{Br}$ colour indices (Gaia Collaboration, 2018) of these stars indicate that they have the following spectral types: G 9.5 V or K 1 V (A), K4V or K 6.5 V (B), and K6.5Vor M1.5V (C). These colours reflect the appearance of the stars in survey imagery (e.g. figure 1), as well as those of a potentially true trinary system. Based on the absolute magnitude of the Gaia G filter (MG) and the $\mathrm{Bp}-\mathrm{Br}$ colours (Gaia Collaboration, 2018) it appears that A and B are of the main sequence (Gaia Collaboration, 2018b). The lack of Gaia DR2 parallax data for the faintest component (see Table 3) makes it impossible to calculate its MG. However, the object's faint Mid-IR response in WISE suggests that it is most likely a main-sequence star rather than a K or M-type giant. The photometric properties of these stars are summarized in Table 1.


Figure 1: Discovery SDSS image extract showing the group of three stars. Their visual appearance in this image is much like that of a triple star system. From left to right: TYC 2036-1173-1 (A), UCAC4 579-051312 (B) and UCAC4 579-051311 (C). The image was taken on B2003.4818.

| Star | Mag | MG | G | Bp | B-V | Bp-Br |
| :---: | :---: | :---: | :---: | :---: | :---: | ---: |
| A | 12.4 | 3.5 | 12.3 | 12.7 | 0.83 | 0.96 |
| B | 14.0 | 7.3 | 14.0 | 14.6 | 1.62 | 1.32 |
| C | 16.0 | -- | 15.4 | 16.2 | 2.17 | 1.69 |

Table 1. Photometry (APASS, Gaia DR2 and GSC2.3)

TYC 2036-1173-1: An Optical Triple Star System in Corona Borealis?

| Star | UCAC5 PM <br> (RA) | UCAC5 PM <br> (DEC) | URAT1 PM <br> (RA) | URAT1 PM <br> (DEC) |
| :---: | :---: | :---: | :---: | :---: |
| A | -9.3 | 12.3 | -6.4 | 17.7 |
| B | -7.4 | 13.8 | -5.5 | 21.6 |
| C | -13.8 | 7.0 | -5.1 | 17.2 |

Table 2: Proper Motion (UCAC5 and URAT1)

## Astrometry

In regards to the astrometric properties of this system, the URAT1 (Zacharias, 2015) catalogue indicates that all three stars have a relatively similar proper motion, especially A and C (see Table 2), very much like a true common proper motion triplet. UCAC5 (Zacharias, 2017) shows comparable proper motion values to URAT1, but indicates that A and B are significantly more similar in terms of their proper motion than C (see Table 2).

Interestingly, astrometric data from Gaia DR2 shows that A and B are likely unrelated based on their parallax. Indeed, B appears to be significantly closer than A ( $\sim 950$ ly and ~1860 ly respectively), even when including the measurement uncertainties . Gaia DR2 measurements were also used to calculate the apparent separation (Sep) and the Position angle (PA). The Gaia DR2 astrometry measurements for these stars are summarized in Table 3. Note that the UCAC5 measurements are significantly more similar to Gaia DR2 in comparison to URAT1.

Unfortunately, Gaia DR2 does not list any astrometric data for C , with the exception of positional data. Hence, it is not possible to rule out the possibility of C being at a similar distance to A or B . Indeed, the apparent separation is such that C could theoretically be gravitationally bound to either A or B, based on the parallax measurements of the latter two. More specifically, assuming C is at the same distance as A , the physical distance would be $\sim 2.5 \mathrm{ly}$. In comparison, the separation would be $\sim 0.5$ ly if at the same distance (bound) to B. However, due to the differences in the proper motion measured by UCAC5, it is possible that C may be unrelated to A and B .

## Conclusion

While the colors and the relatively similar proper motion of TYC 2036-1173-1, UCAC4 579-051312 and UCAC4 579-051311 are indicative of a trinary star system (especially according to URAT1 astrometry), Gaia DR2 shows that the two brightest members are likely unrelated based on their parallax. Furthermore, UCAC5 astrometry indicates that the faintest member differs significantly in its proper motion from the two others (despite being relatively similar), hence indicating that this star may perhaps be unrelated to the two others. Further study may be needed to assess the true nature of this system, especially in the case of UCAC4 579051311 in relation to the two others.

## Acknowledgments

This work has also made use of data from the European Space Agency (ESA) mission Gaia (https:// www.cosmos.esa.int/gaia), processed by the Gaia Data Processing and Analysis Consortium (DPAC, https:// www.cosmos.esa.int/web/gaia/dpac/consortium). Funding for the DPAC has been provided by national institutions, in particular the institutions participating in the Gaia Multilateral Agreement.

## References

Evans et al., 2018, "Gaia Data Release 2: Photometric content and validation", ArXiv e-prints
Gaia Collaboration, 2016, "The Gaia mission", $A \& A$ 595, A1.
Gaia Collaboration, 2018, "Gaia Data Release 2: Summary of the contents and survey properties". ArXiv e-prints.
Gaia Collaboration, 2018b, "Observational Hertzsprung -Russell diagrams", $A \& A, 616$, A10.
Henden, A., et al., 2016, "VizieR Online Data Catalog: AAVSO Photometric All Sky Survey (APASS) DR9"

Lindegren, L., et al., 2018, "Gaia Data Release 2: The astrometric solution", ArXiv e-prints.
Luri, X., et al., 2018, "Gaia Data Release 2: using Gaia parallaxes", ArXiv e-prints.

| Star | Coordinates (RA+DEC) | PM (RA) | PM (DEC) | Plx (mas ) | Sep | PA |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | $154525.06+253740.2$ | -9.25 | 11.81 | 1.75 | -- | -- |
| B | $154524.29+253740.3$ | -7.50 | 14.29 | 3.44 | $10.6^{\prime \prime}$ | $271^{\circ}$ |
| C | $154523.74+253740.3$ | -- | -- | -- | $17.8^{\prime \prime}$ | $270^{\circ}$ |

TYC 2036-1173-1: An Optical Triple Star System in Corona Borealis?

STScI., 2006, "The Guide Star Catalogue, Version 2.3.2".

Riello, M. et al., 2018, "Gaia Data Release 2: processing of the photometric data", ArXiv e-prints.
Zacharias, N., et al., 2015, "VizieR Online Data Catalog: URAT1 Catalogue"

Zacharias, N., et al., 2017, "VizieR Online Data Catalog: UCAC5 Catalogue"

# Measurement of Rasalgethi with a DSLR Camera 

Blake Nancarrow<br>Bradford, Ontario, Canada<br>RASC Carr Astronomical Observatory, Blue Mountains, Ontario, Canada<br>astronomy@computer-ease.com


#### Abstract

In an effort to apply new methods learned, digital images were acquired of Rasalgethi aka STF 2140 AB and nearby calibration stars with a DSLR affixed to a C14 to determine the position angle and separation. On analysing the images in REDUC software, a position angle of 103.3 degrees and separation of 4.90 arc-seconds was calculated.


## Introduction

While I learned to visually measure doubles with the Celestron Micro Guide, I was inclined to use a DSLR camera. Ernõ Berkó helped me understand the equipment I would need and process that I should use.

I chose Rasalgethi or Alpha Herculis, WDS ID $17146+1423$ (STF2140AB), to practice my data gathering workflow. I prepared a list of calibration stars in the vicinity using the SkyTools planning software.

My observations were made at Carr Astronomical Observatory. I used a Canon 40D (unmodified) camera body on an unguided Celestron 14 -inch f/11 SCT with Optec TCF-S temperature-controller focuser. The 'scope was permanently mounted atop a Paramount ME controlled by TheSky6. The OTA primary mirror was not locked. The camera was controlled remotely with the EOS Utility software running on a small netbook computer. The camera was set to use ISO 1000 and capture in RAW format ( $3888 \times 2592$ pixels). The focal length of the C14 is approximately 3.9 metres; no further magnification (i.e. Barlow) was used.

In all the included images, north is up and east is left.

## Methods

My observations were made on 15 July 2015 (2015.53) between 12:30 AM and 3:30 AM Eastern Standard Time. Work was conducted at this time to avoid a meridian flip with the equatorial mount. Approximately 100 digital images in total were captured. The altitude of the target stars ranged from $+62^{\circ}$


Figure 1. Image captured during drifting process showing alignment to the declination axis.
through +33 during the session.
Using the camera Live View, I focused on the star Unukalhai aka Alpha Serpentis initially moving the C14 primary, then using the Optec hand paddle.

I rotated the camera in the focuser to align the long edge of the frame with the declination axis and captured a couple of long exposure images, drifting Unukalhai with the mount tracking off, to refine the alignment. The camera was not touched after this time. I then slewed Unukalhai to the bottom edge of the frame from centre so to ease verification (Figure 1). I programmed a dozen shots to record the camera orientation, repositioning Unukalhai at the east edge of the field before each exposure.

## Measurement of Rasalgethi with a DSLR Camera



Figure 2. Calibration star HD 159481 aka 52185 exposed at 30 seconds.

The mount was then slewed to previously selected calibration stars. Several images were captured for each star. Various exposures were taken to ensure the dimmest element remained visible while avoiding overexposing the bright stars. For each calibration star 3 to 8 images were captured. Exposure times ranged from 30 seconds to $1 / 125$ th of a second. The digital image of HD 159481 (STF 2185) in Ophiuchus revealed a multistar system with over 8 elements (Figure 2).

The altitude of Rasalgethi was approximately $+36^{\circ}$ when imaged. Over 15 digital images were captured for Rasalgethi, again at various exposures. The exposure of Alpha Her at $1 / 15$ th of a second reveals the delicate companion B aside the bright primary (Figure 3).

After the session, the drift alignment, calibration star, and target star shots were processed and converted to JPG format using the Canon Digital Photo Professional software. The evening's proceedings with a selection of images were documented on my astronomy blog.

## Analysis

All images were converted at full size from Canon CR2 RAW format to Windows BMP with IrfanView 4.51 .

REDUC 5 software by Florent Losse was used with Automatic Centering enabled.

A total of 16 drift images were captured for declination analysis (Table 1). The $\Delta$ or image inclination in relation to the celestial equator was measured for each image. The average was calculated at -0.20 with a standard deviation of 0.04 .

The calibration images were visually assessed. The best ones selected showed good exposure and little or no vibration. Similarly, the best exposure (file IMG_5171) of Rasalgethi was selected for final analysis.


Figure 3. Image of Rasalgethi aka 22140 captured at 1/15th of a second.

In September 2015, the first pass at calculating the image scale or E value of the calibration star images was performed using separation values from the SkyTools 3 Professional software. From the 12 images, an average value of 0.56 was obtained. When the inclination and scale average values were applied a single image of Rasalgethi, the position angle of 103 and separation of 4.8 were derived. These compared well to the values shown in SkyTools for alpha Her: 103 and 4.6.

A later pass was conducted using the separation values as found at the time in the WDS. The scale average value 0.56 was obtained. When calculations were performed for the target star, REDUC yielded the values 102 and 5.3.

| image file | $\boldsymbol{\Delta}$ | notes |
| :---: | :---: | :---: |
| IMG_5105 | -1.16 | different position |
| IMG_5106 | -0.15 |  |
| IMG_5107 | -0.17 |  |
| IMG_5108 | -0.18 |  |
| IMG_5109 | -0.25 |  |
| IMG_5110 | -0.23 |  |
| IMG_5111 | -0.28 |  |
| IMG_5112 | -0.19 |  |
| IMG_5113 | -0.21 |  |
| IMG_5114 | -0.21 |  |
| IMG_5115 | -0.18 |  |
| IMG_5116 | -0.22 |  |
| IMG_5117 | -0.20 |  |
| IMG_5118 | -0.19 |  |
| IMG_5119 | -0.14 |  |
| IMG_5120 | -0.21 |  |

Table 1. Image files for drift analysis with inclination values from REDUC.

## Measurement of Rasalgethi with a DSLR Camera

Another attempt was made in July 2018 again drawing the reference separation values for the calibration stars from the WDS (Table 2). It was noted a couple of the entries had reports post-dating the period when the image data was captured. Nevertheless, E was computed to be 0.28 with a standard deviation of 0.00 . The inclination and scale averages were applied to the Rasalgethi image yielding theta 103.3 and rho 4.90. The published values for $17146+1423$ [STF2140AB] from 2017 were 104 and 4.8.

## Conclusions

I set out to practice digital imaging a double star to be measured and calibration stars for determining the image scale.

The double star STF 2140AB was measured with a DSLR. The separation was found to be 4.9 arc seconds and the position angle was determined to be 103.3 degrees (Table 3).

## Acknowledgements

I am grateful for Geoff Gaherty's support while at the early stages of my fascination with double stars. I thank Ernõ Berkó for helping me understand how to capture doubles with a DSLR. I found Jolyon Johnson's video and the text in Bob Argyle's book very helpful in preparing a paper. I thank Dr Roberto Abraham for his guidance in writing a scientific paper. I appreciate the access to the research-grade telescope and mount equipment at the CAO offered through my RASC Toronto Centre membership. And thanks for Florent Losse for providing the REDUC software.

## References

The Baader Micro Guide is equivalent to the Celestron Micro Guide. https://www.baader-planetarium.com/en/ micro-guide-eyepiece-with-log-pot-illuminator.
SkyTools 3 Professional planning software. https:// www.skyhound.com/skytools.html
REDUC software. http://www.astrosurf.com/hfosaf/uk/ tdownload.htm
IrfanView software, https://www.irfanview.com/
Author blog post noting local weather conditions with Clear Sky Chart 3 hours prior to session. http:// blog.lumpydarkness.com/2015/07/the-sun-through-clear-band.html.

| image file | WDS ID | last | $\boldsymbol{\rho}$ | E |
| :---: | :---: | :---: | :---: | :---: |
| IMG_5151 | $16315+0818$ <br> SHJ 233 | 2015 | 59.2 | 0.28 |
| IMG_5158 | $16435+2043$ <br> STTA149 | 2011 | 97.7 | 0.28 |
| IMG_5135 | $16442+2331$ <br> STF2094AC | 2013 | 24.7 | 0.28 |
| IMG_5191 | $17249+1320$ <br> STF2159 | 2015 | 26.4 | 0.28 |
| IMG_5197 | $17348+0601$ <br> PWS 16AH | 2001 | 187.3 | 0.28 |
| IMG_5200 | $17433+1741$ <br> STH 4 | 2010 | 15.5 | 0.28 |
| IMG_5156 | $16354+1703$ <br> WEB 6 | 2015 | 155.5 | 0.28 |
| IMG_5168 | $17037+1336$ <br> STFA 33AB | 2014 | 304.9 | 0.28 |
| IMG_5203 | $18057+1200$ <br> STF2276AC | 2016 | 63.8 | 0.28 |
| IMG_5129 | $16081+1703$ <br> STF2010AB | 2017 | 27.0 | 0.28 |
| IMG_5142 | $17150+2450$ <br> STF3127AD | 2009 | 191.6 | 0.28 |
| IMG_5192 | $17016+1457$ <br> H 4 122 | 2014 | 18.9 | 0.28 |

Table 2. WDS separation values for the calibration stars with scale E values from REDUC.

Author blog post with a fast exposure colour image of Rasalgethi showing the B companion. http:// blog.lumpydarkness.com/2015/07/shot-rasalgethi.html
Author blog post featuring detailed report of imaging run with drift and all calibration star images. http:// blog.lumpydarkness.com/2015/07/performed-full-double-star-imaging-run.html
Washington Double Star catalog. http:// www.astro.gsu.edu/wds
How to Write a Double Star Research Paper. Jolyon Johnson YouTube video, 2015, https:// www.youtube.com/watch? $\mathrm{v}=\mathrm{akF} 3 \mathrm{~T} 4 \mathrm{on} 0 \mathrm{~mA}$
Observing and Measuring Visual Double Stars, 2/e, Bob Argyle, Ed, 2012. Springer, New York.
"Observing Double stars for Fun and Science", Ronald Tanguay, Sky and Telescope, Feb 1999, https:// www.skyandtelescope.com/observing/celestial-objects-to-watch/observing-double-stars-for-fun-and-science/

| Name | RA+Dec | PA | Sep | Date | N | Note |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| STF2140AB | $17146+1423$ | 103.3 | 4.90 | 2015.53 | 1 | 1 |

1. Canon 40D (unmodified) camera and Celestron 14 inch f/11 SCT.

Table 3. Final calculated separation and position values for Rasalgethi.

## Measurement of Rasalgethi with a DSLR Camera

"Double-Star Measurement Made Easy", Thomas Teague, Sky and Telescope, pg 112, July 2000.
"Double Star Measures Using a DSLR Camera", Ernõ Berkó, JDSO, 4(4), 144, 2008.
"Double Star Measurements in the Pleiades Cluster Using a DSLR Camera", Michel Michaud, JDSO, 8 (4), 249, 2012.

> Blake Nancarrow has observed double stars as a young amateur astronomer. His first log entry of a double was made in June 1991. He learned and practiced the Celestron steps, Ronald Tanguay's techniques, and Tom Teague's method, developing his own refined workflow. On reading Ernõ Berkó's 2008 and Michel Michaud 2012 JDSO papers on using a DSLR to capture double star data, he decided to go the digital imaging route.
> The Carr Astronomical Observatory on Blue Mountains in south-western Ontario is owned and operated by the Royal Astronomical Society of Canada - Toronto Centre. Nancarrow is one of the site supervisors.

# CCD and Gaia Measurements Indicate that WSD 12095 + 3356 is a Physical System 

Alexa Brammer ${ }^{1}$, Jessica Padron-Loredo ${ }^{1}$, Charmain Brammer ${ }^{1}$, and Cameron Pace ${ }^{2}$<br>1. SUCCESS Academy<br>2. Southern Utah University


#### Abstract

The Double Star System WSD $12095+3356$ was observed using the Great Basin Observatory telescope. The images were plate solved and calibrated, and then position angle and separation were measured using AstroImageJ. Our measurements $\left(\theta=324.131^{\circ}\right.$ and $\rho=63.490^{\prime \prime}$ ) were compared to historical observations. Parallax and proper motion data from the Gaia database indicate that the stars in this system are physically associated.


## Introduction

The goal of this project was to provide additional measurements of the separation and position angle to determine if WSD $12095+3356$ is a binary system. WSD $12095+3356$ is recorded as a double star with components A and B. It was first observed in 1998 and most recently in 2015, with a total of six observations. This system was selected because it could be viewed with the Great Basin Observatory (GBO) Telescope. This research was done by high school students from SUCCESS Academy (an early college high school) in collaboration with Southern Utah University.

## Methods

This research was conducted using the telescope at the Great Basin Observatory in Great Basin National Park (Figure 1). The GBO is the first research grade telescope in a national park. It is managed by the Great Basin National Park and the Great Basin National Park Foundation, in collaboration with Concordia University, Southern Utah University, University of NevadaReno, and Western Nevada College. The telescope has an aperture of 27 inches (Anselmo et al. 2018).

The images of WSD $12095+3356$ (Figure 2) were acquired remotely on February 22, 2018. A total of 26 images were taken with an exposure time of 180 seconds with the V filter. The exposure time was chosen so the target stars would not be over-exposed. The images were binned 1 x 1 . Images were plate solved using astrometry.net and then the images were calibrated by applying dark, bias, and flats using AstroImageJ version 3.2.27 (Collins et. al 2017). Position angle ( $\theta$ ) and separation ( $\rho$ ) were measured through AstroImageJ.


Figure 1. The Great Basin Observatory and the control room (Anselmo et al. 2018).


Figure 2. Image of $A$ and $B$ components of $W S D$ $12095+3356$. Plate scaled at 0.4 arcsec/pixel.

# CCD and Gaia Measurements Indicate that WSD 12095 + 3356 is a Physical System 

| WDS No. | ID | Nights | Date | Observations |  | $\boldsymbol{\theta}^{\circ}$ | $\boldsymbol{\rho}^{\prime \prime}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $12095+3356$ | GRV 853 | 1 |  |  |  | Mean | 324.131 |

Table 1. Observations from the data collected. $\theta, \rho$, mean, standard deviation, and standard error.

The centroid feature was used to improve accuracy by measuring from the center of each star. All data was exported into Excel to calculate the mean, standard deviation, and standard error for $\theta$ and $\rho$.

## Results

The mean, standard deviation, and standard error were measured for the star system, and the results are shown in Table 1.

## Discussion

Table 2 shows the historical data for WSD $12095+$ 3356. The measurements cover 1998 to 2015 with six observations in total. In Figure 3, the measurements are shown, with the primary star at $(0,0)$. All of the data is in the same range, but there are two possible outliers. As seen in Figure 4, either the original observation in 1998 or the 2004 data could be an outlier. Stars do not move several arcseconds away then return to their previous positions. This could mean that the data from 2004 is be spurious, as Figure 4 shows that with the exception of the 2004 data point, the other observations suggest a more linear form for the motion of the B component.

To further determine if the stars are physically associated, we retrieved data from the Gaia database (Gaia Collaboration, 2018) for the parallax and proper


Figure 3. This graph shows our measurements (orange triangle) along with historical measurements (blue circles) around the primary star. The measurements are in arcseconds. In this graph, the primary star is located at the origin.

| Epoch | $\theta^{\circ}$ | $\rho^{\prime \prime}$ |
| :--- | :--- | :---: |
| 1998.19 | 324.0 | 63.770 |
| 2000 | 324.0 | 63.600 |
| 2002.04 | 324.0 | 63.568 |
| 2004.205 | 324.500 | 63.395 |
| 2010.5 | 324.100 | 63.560 |
| 2015 | 324.064 | 63.581 |
| 2018.17 | 324.131 | 63.490 |

Table 2. Historical Data for WSD $12095+3356$.


Figure 4. A closer look at the historical data (blue circles) and collected data (orange triangle). The data points suggest the secondary star is orbiting the primary star, but there is one discrepant data point, which is the 2004 measurement.

CCD and Gaia Measurements Indicate that WSD 12095 + 3356 is a Physical System

| Component | Right <br> Ascension | Declination | Parallax <br> [mas] | Parallax <br> Error <br> [mas] | Proper Motion Proper Motion <br> RA |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | 182.391 | 33.944 | 5.847 | 0.077 | -59.935 | 12.358 |
| B | 182.379 | 33.958 | 5.840 | 0.045 | -59.319 | 12.661 |

Table 3. Data obtained from the ESA Gaia telescope (Gaia Collaboration, 2018). Both the parallax and proper motion data indicate that the stars are indeed a binary.
motion of each star. The coordinates of the stars are given in the second and third column of Table 3. The parallax is the measurement of how far away the stars are from Earth. When the parallax is converted to parsecs we find that both stars are at a similar distance (171 parsecs). However, since Gaia's accuracy begins to drop off around 5 mas, we can only conclude that the stars in this system are physically associated. Table 3 shows that both stars have similar proper motion in declination and right ascension, which further suggests they are physical stars. The distance data agrees with the proper motion data; both stars are at a similar distance and moving the same way, which indicates that the system is physical. To determine if this pair forms a binary system, an orbit would need to be computed.

## Acknowledgments

This project was made possible by the collaboration with Southern Utah University and the Great Basin Observatory (GBO). We would like to thank those who operate the GBO. This research made use of AstroImageJ, Astrometry.net, and the Washington Double Star Catalog maintained by the U.S. Naval Observatory. This work has made use of data from the European Space Agency (ESA) mission Gaia (https:// www.cosmos.esa.int/gaia), processed by the Gaia Data Processing and Analysis Consortium (DPAC, https:// www.cosmos.esa.int/web/gaia/dpac/consortium). Funding for the DPAC has been provided by national institutions, in particular the institutions participating in the Gaia Multilateral Agreement.

## References

Anselmo, Dallas, et al., 2018, "CCD Measurements of AB and AC Components of WDS 20420+2452", Journal of Double Star Observations, 14 (3). http:// www.jdso.org/volume14/number3/Anselmo_492_495.pdf
Collins, K. A., Kielkopf, J. F., Stassun, K. G., \& Hessman, F. V., 2017, The Astronomical Journal, 153 (2), 77.

Gaia Collaboration Brown, A. G. A. Vallenari, A. Prusti, T. de
Bruijne, J. H. J. et al., 2018, "Gaia Data Release 2. Summary of the contents and survey properties." Astronomy \& Astrophysics, 616, A1.
Great Basin Observatory, 2016, retrieved Feb. 22,2018, from http://planewave.com/great-basinobservatory/
Mason, Brian D, 2019, The Washington Double Star Catalog, Astronomy Department, U.S. Naval Observatory.

# Astrometric Measurement of WDS 12459-7511 HJ 4545 

Isabel Zheng ${ }^{1}$, Yael Brynjegard-Bialik ${ }^{1}$, Jackie Roche ${ }^{1}$, Pat Boyce ${ }^{2}$, and Grady Boyce ${ }^{2}$<br>1 West Ranch High School, Stevenson Ranch, California, USA<br>2 Boyce Research Initiatives and Education Foundation, San Diego, California, USA


#### Abstract

This paper reports the astrometric measurements of the double star system WDS 12459-7511 HJ4545 using the Las Cumbres Observatory. We found the relative position of the AB pair to have a separation of $9.15^{\prime \prime}$ and position angle of $192.6^{\circ}$ for epoch 2019.294. Additionally, the relative position of the AC pair was also measured to have a separation of $36.9^{\prime \prime}$ and position angle of $238.0^{\circ}$ at for epoch 2019.294. When combined with GAIA parallax and proper motion data, the results strongly suggest that the AB pair is gravitationally bound and the AC pair is optical.


## Introduction

The binary star HJ 4545 was selected because it met the following requirements:

- Observable from the Southern Hemisphere in the spring
- Angular separation of at least 5 arcseconds
- Difference in magnitude of no more than 3

HJ 4545, Figure 1, was first observed by John Herschel in 1835. Since the initial measurement, HJ 4545 AB has been observed 22 times with the latest measurement in 2000 (Sordiglioni, G.).

The primary star has a spectral type of A5V (Mason and Hartkopf, 2015), and is a main sequence star. According to data from Tycho in the visible band, the A star has a magnitude of 9.1, the B star has a magnitude of 9.25 , and the C star has a magnitude of 11.17. The initial and most recent measurements for each pair are outlined in Table 1. These measurements do not include those reported in this paper.

## Materials and Methods

The images of HJ 4545 were taken in Sutherland, South Africa by an SBIG 6303 camera on a 0.4 -meter Meade telescope, part of the Los Cumbres Observatory (LCO) system. A total of 7 images were measured and


Figure 1. Image of WDS 12459-7511 cropped to show target stars.

| Pair | Epoch | Theta | Rho |
| :---: | :---: | :---: | :---: |
| AB | 1835 (Initial) | $189.0^{\circ}$ | $12.000^{\prime \prime}$ |
|  | 2000 (Most recent) | $192.0^{\circ}$ | $9.100^{\prime \prime}$ |
| AC | 1895 (Initial) | $235.9^{\circ}$ | $36.262^{\prime \prime}$ |
|  | 2000 (Most recent) | $237.9^{\circ}$ | $36.930^{\prime \prime}$ |

Table 1. Initial and most recent measurements of each star pair, excluding the measurements in this paper.

Astrometric Measurement of WDS 12459-7511 HJ 4545

| Epoch 2019.294 | Postion Angle <br> $(\theta)$ | Separation <br> $(\boldsymbol{\rho})$ |
| :---: | :---: | :---: |
| Mean | 192.6 | 9.15 |
| Standard Deviation | 0.264 | 0.023 |
| Standard Deviation <br> of the Mean | 0.1 | 0.009 |

Table 2. Results of Mira Pro astrometric measurements of WDS 12459-7511 for $A B$.
taken on 2019.294 with an exposure time of 15 seconds using a Bessell B filter.

The images were calibrated by Our Solar Siblings (OSS) data pipeline (Fitzgerald 2018). The software, MiraPro x64, was used to measure the position angle $(\theta)$ and separation ( $\rho$ ). These separation and position angle measurements were then entered into Google Sheets for calculations of the mean, standard deviation, and standard deviation of the mean for $\theta$ and $\rho$.

## Results

A total of seven images were measured for the separation and position angle as shown in Table 2 and Table 3 for AB and AC , respectively.

## Discussion

Astrometry, derived from these current results, are plotted together with historical data from the WDS, Figure 2. There is no apparent trend in the positions of the B or C star in relation to A over the period from first observation to our 2019 observation. The only points outside the tight patterns for both are the first two for B and could well be measurement error.

We found that Stelle Doppie (Stelle Doppie Web) has classified the AC pair as physical and the AB pair as uncertain. In Richard Harshaw's analysis of the WDS data merged with the Gaia DR2 data, he classified the AC pair as "unknown" largely because the Gaia DR2 data was missing from his merged database. He classified the AB pair as very likely to be physical. These opposing classifications arise from the differences in proper motions for the stars between the WDS and Gaia DR2, Table 3. The proper motions in RA for all three components are comparable between WDS and the more accurate Gaia DR2 data. The proper motions in DEC for the A and B stars are substantially different in the WDS but almost identical in Gaia DR2. Thus, Stelle Doppie's reliance on the WDS proper motions could have led to the conclusion that AC is physical and AB is not. Conversely, when the more accurate Gaia DR2 is applied, as in Harshaw, the nearly identical proper motions for the A and B stars yield a compelling case for their being physical and possibly weaken the case for AC to be physical. Figure 3 depicts the proper

| Epoch 2019.294 | Postion Angle <br> $(\theta)$ | Separation <br> $(\rho)$ |
| :---: | :---: | :---: |
| Mean | 236.5 | 36.3 |
| Standard Deviation | 1.081 | 1.111 |
| Standard Deviation <br> of the Mean | 0.120 | 0.123 |

Table 3. Results of Mira Pro astrometric measurements of WDS 12459-7511 for AC.


Figure 2. Historical positions of HJ4545 B and C compared to $A$ at the origin.

| Star | Proper Motion <br> Data | RA | DEC |
| :---: | :---: | :---: | :---: |
| A | WDS | -028 | +007 |
|  | Gaia DR2 | -28.239 | 2.368 |
| B | WDS | -030 | -006 |
|  | Gaia DR2 | -28.231 | 2.26 |
| C | WDS | -038 | +007 |
|  | Gaia DR2 | -38.924 | 7.32 |

Table 3. Comparison of WDS and Gaia DR2 data for HJ 4545

## Astrometric Measurement of WDS 12459-7511 HJ 4545



Figure 3. Image from Aladin 10 showing proper motion vectors for HJ4545
motion vectors of the system using ALADIN with GAIA data displayed.

Gaia DR2 parallax of the A star yields a distance of 789.29 light years and that of the B star yields a distance of 785.07 light years. Thus, with nearly identical proper motions and distances, this suggests the possibility that these two stars may be gravitationally bound.

The C star of the system has a parallax measure from Gaia DR2 that yields a much greater distance than the A and B at 1106.59 light years. The C star is likely to be optical as its proper motion vectors are different and its distance is distinct from the A star and B star. The parallax for the A star is $1.2669 \times 10^{-3}$ arcseconds and the parallax for the B star is $1.2738 \times 10^{-3}$ arcseconds while the parallax for C is drastically different at $9.0368 \times 10^{-4}$ arcseconds.

## Conclusion

After using the 0.4 m telescope at LCO's South African observatory, we were able to provide additional astrometric data of the double star system WDS 124597511 HJ 4545. By applying data from Gaia DR2, we able to strongly suggest that s the AB pair is likely to be gravitationally bound and noted as a T in the WDS. The AC pair appears to be an optical double only and noted as an S in the WDS classification system.

## Acknowledgments

The authors thank the United States Naval Observatory for providing historical measurement data and LCO for the use of their service, and we appreciate the ability to have Simbad and Gaia access from the CDS Strasbourg Database. We also thank Boyce Research Initiatives and Education Foundation (B.R.I.E.F.). Additionally, we thank Jerry Hilburn for his guidance and effort to the research team. Appreciation is extended to Christine Hirst for providing us the opportunity to participate in this project and supporting our team. Measurements were made using Mira Prox64, which provided accurate astrometric measurements for our double star system.

## References

Fitzgerald, M.T. (2018, accepted), "The Our Solar Siblings Pipeline: Tackling the data issues of the scaling problem for robotic telescope based astronomy education projects.", Robotic Telescopes, Student Research and Education Proceedings
Mason, B. and Hartkopf, W. The Washington Double Star Catalog, October 2015. Astrometry Department, U.S. Naval Observatory. https:// ad.usno.navy.mil/wds/Webtextfiles/ wdsnewframe3.html
O'Connor, J J, and E F Robertson. "John Frederick William Herschel." John Herschel (1792-1871), July 1999, www-history.mes.st-andrews.ac.uk/ Biographies/Herschel.html.

Stelle Doppie Web: Sordiglioni, Gianluca Stella Doppie, Double Star Catalog. https:// www.stelledoppie.it/index2.php?iddoppia=54877 (Accessed Sept 15, 2019)

# Astrometric Measurements of OSO 51 AB 

Shreya Goel ${ }^{1,2}$, Shabdika Gubba ${ }^{1,2}$, Pat Boyce ${ }^{3}$, Grady Boyce ${ }^{3}$<br>1. BE WiSE<br>2. Thurgood Marshall Middle School<br>3. Boyce Research Initiatives and Education Foundation (BRIEF)


#### Abstract

The double star system WDS 13111+1220 OSO 51 AB was studied by a team of students using the Las Cumbres Observatory (LCO) to analyze the nature of the double star. Images were measured for position angle (Theta) and separation distance (Rho) through image analysis software AstroImageJ. The position angle was measured to be $299.33^{\circ}$ and the angular separation was measured to be 11.467 " arcseconds. It was determined through measurements and Gaia parallax data that the double is most likely an optical double and not a physical binary.


## Introduction

The objective of this research was to observe and measure double star system WDS 13111+1220 OSO 51 AB. OSO 51 AB was discovered by Maria Rosa Zapatero Osorio in 1994 (Stelle Doppie Web) and has been observed 7 times from 1994 to 2001.

A primary factor considered in the selection of OSO 51 included choosing a double star system with the necessary Right Ascension (RA) for observation, between 08-14 hours, as observations were taken in spring. The difference in magnitude was also taken into consideration with the magnitudes for the A and B stars of 14.22 and 15.50 respectively, resulting in a delta magnitude of 1.28 , low enough for neither star to obscure the other from view in a CCD image. The third star, C, of OSO 51 AC , with a magnitude of 19.1, was too dim to be seen in an image even with a high exposure time and could not be studied with the resources available.

The nature of OSO 51 as an optical double star system or as a binary system was to be reviewed by these observations. Proper motion and parallax are often used to determine the nature of double stars. The team used Aladin10 to find proper motion vectors for the two stars, Figure 1. Parallax is the angle subtended by the
star on opposite sides of the Earth's orbit. This can be used to find the distance of the star from Earth. Stars which are very different distances from Earth, and therefore far from each other, are usually not gravitationally bound.

After requesting historical WDS data for the system, it was noted there are 7 historical measurements with the first in 1994 and the latest, before this paper, in 2001. These measurements are plotted in Figure 2. Then, a linear trendline was compared to a polynomial trendline, Figure 3 and Figure 4, for the data, and there seemed to be some sort of non-linear trend that might indicate an orbit.

## Methods and Materials

The Las Cumbres Observatory (LCO) telescope network was used to acquire images. Images were captured by a $0.4-\mathrm{m}$ Meade 6303 telescope, Figure 5, with an SBIG CCD camera resulting in a pixel size of $0.571^{\prime \prime}$ binned 1x1. The images had a field of view of 29'x19' and were taken at Haleakala Observatory, Hawaii, at 10,000 feet above sea level. Due to the dim nature of both stars in the system, no filter was used while ob-
(Text continues on page 80)

## Astrometric Measurements of OSO 51 AB



Figure 1. WDS 13111+1220 as shown on Aladin 10 with proper motion vectors.


Figure 2. Graph of historical data.

## Astrometric Measurements of OSO 51 AB



Figure 3. Graph of historical data with linear trendline

| \# | Theta ( ${ }^{\circ}$ ) | Rho (") |
| :---: | :---: | :---: |
| 1 | 299.51 | 11.393 |
| 2 | 296.84 | 11.923 |
| 3 | 301.96 | 11.337 |
| 4 | 299.05 | 11.551 |
| 5 | 299.29 | 11.510 |
| 6 | 300.14 | 11.000 |
| 7 | 298.25 | 11.523 |
| 8 | 298.86 | 11.530 |
| 9 | 299.02 | 11.529 |
| 10 | 299.22 | 11.572 |
| 11 | 299.83 | 11.344 |
| 12 | 299.11 | 11.373 |
| 13 | 299.18 | 11.430 |
| 14 | 299.96 | 11.515 |
| 15 | 299.78 | 11.473 |
| Avg | 299.33 | 11.467 |

Table 1. Measurements from new observations with final average. All measurements were made on Besselian date 2019.284, JD 2458588.


Figure 4. Graph of historical data with polynomial trendline
taining images with an exposure time of 300 seconds. 15 images were taken on 2458588.291667 (BJD).

Images were processed through the Our Solar Siblings (OSS) pipeline (Fitzgerald 2018) to ensure image quality and ease for measurement, by embedding World Coordinate System (WCS) coordinates, calibration, and photometry. AstroImageJ was used to measure separation distance and position angle.

## Data and Results

Two team members independently took measurements which were then averaged to ensure greatest accuracy. Measurements of Theta and Rho are provided, Table 1. The mean position angle was calculated to be $299.33^{\circ}$, and the mean separation was $11.467^{\prime \prime}$, Table 2. The standard deviation of position angle was 1.079 and the standard deviation of separation was 0.191 , indicating the data was fairly consistent, Table 2 . A combination of the historical measurements with Theta and Rho from this paper are provided, Table 3. The average of all measurements is plotted in Figure 6. The data was graphed, showing how it made sense and seemed rea-
(Text continues on page 82)

| Epoch | \# of <br> obs. | Mean <br> PA | Std. <br> Dev. | Mean <br> Sep. | Std. <br> Dev. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2019.284 <br> JD2458588 | 15 | 299.3 | 1.1 | 11.47 | 0.19 |

Table 2. Mean Position Angle (Theta) and Angular Separation (Rho) with related statistics.

## Astrometric Measurements of OSO 51 AB



Figure 5. 0.4-m Meade telescope provided by $L C O$.


Figure 6. Historical data graphed alongside the average of the measurements presented in this paper. The gap between current and historic measurements is expected for the time passage.

| Epoch | PA | Sep |
| :---: | :---: | :---: |
| 1994.109 | 273.1 | 11.43 |
| 1994.09 | 273.2 | 11.41 |
| 1995.429 | 375.4 | 11.4 |
| 1995.47 | 374.4 | 11.44 |
| 1995.47 | 274.7 | 11.36 |
| 2000.33 | 279.7 | 11.24 |
| 2001.103 | 280.5 | 11.13 |
| 2019.284 | 299.3 | 11.5 |

Table 3. Historical measurements for OSO 51 AB with Theta and Rho from this paper.


Figure 6. Historical data graphed alongside the average of the measurements presented in this paper. The gap between current and historic measurements is expected for the time passage.

## Astrometric Measurements of OSO 51 AB

(Continued from page 80)
sonable as the average of the group's data lined up well with historical data, Figure 6.

## Discussion

With the historical and current data sets combined, a seeming linear trend was observed, Figure 7. Figure 8 was used as a comparison between a polynomial trend and a linear trend. After comparing a linear trend and a polynomial trend, the polynomial trend had a slightly lower error by 0.0018 , but the difference between the linear and polynomial trends was so minimal, that it was decided that, it is likely a linear trend, or possibly a nonlinear trend that we do not have enough data to see. With current measurements, it was observed that the system most likely was not a binary system. In the 2019 measurement, the position angle, Theta, increased, relative to historical data, Table 3. Graphing all measurements of Rho, Figure 9, the overall trend is consistent over time (shown with the most recent data point). Given that there is only 25 years of history, conclusions could be premature, but only further measurements in the future could prove or disprove any further theories. Relative to the primary star, the secondary star appears to be following a linear path. Without a sign of an arc, the trend line suggests the possibility that these stars are not binary.

In Aladin10, Figure 1, the proper motion vectors, displayed from GAIA data, are visible for each star. It is further suggested that the stars may not be gravitationally bound as the paths are divergent, given that the error estimate for RA was 0.02196 and the error estimate for DEC was 0.02264 . Adding GAIA data for parallax, the primary star's parallax was 2.813 . However, because of the low precision of GAIA's instruments, this figure may not be significant. The secondary star's parallax was 5.294, as shown in Table 4. Converting to lightyears (and parsecs) from Earth, the primary star is approximately 1158.89 lightyears, or 355.49 parsecs from Earth, while the secondary star is approximately 615.83 lightyears, or 188.9 parsecs from Earth, Table 5. Noting the stars' separation from each other, and lack of parallactic overlap, this adds to the belief that OSO 51 AB is an optical double, and not a gravitationallybound physical binary.

## Conclusion

The star system, OSO 51 AB , was studied to study whether a double star or gravitationally bound system. After independently collecting data, the average position angle was $299.33^{\circ}$ and the average separation was $11.5^{\prime \prime}$. These measurements were compared alongside historical data, and it was seen that the position angle changed while the separation remained relatively con-


Figure 7. Historical data with average of new data along with linear trend line.


Figure 8. Historical data with average of new data along with polynomial trend line.


Figure 9. Graph showing Rho decreasing and then increasing over time.

Astrometric Measurements of OSO 51 AB

| Inputs | Parallax | Parallax Error |
| :---: | :---: | :---: |
| Star A | 2.812 | 0.025 |
| Star B | 5.294 | 0.216 |

Table 4. Parallax and parallax error for OSO 51 AB .
stant. The addition of the European Space Agency's Gaia archives parallax data seems to support the position that these are not gravitationally bound stars based on a parallax assessment. After considering multiple factors, it is proposed that OSO 51 AB is most likely not a physical double.

## Acknowledgments

The team would like to thank the Boyce Research Initiatives and Education Foundation (BRIEF) for their teaching, support, and guidance through this whole project. The team would also like to thank their mentors, Hilde Van den Bergh and Ana Parra for their encouragement and advice. Finally, the team would like to thank Better Education for Women in Science and Engineering (BEWiSE) for providing the gateway for the team to get involved in the program. Dates were converted from Gregorian date to Julian date using the USNO Julian Date Converter.

This work has made use of data from the European Space Agency (ESA) mission Gaia (https:// www.cosmos.esa.int/gaia), processed by the Gaia Data Processing and Analysis Consortium (DPAC, https:// www.cosmos.esa.int/web/gaia/dpac/consortium). Funding for the DPAC has been provided by national institutions, in particular the institutions participating in the Gaia Multilateral Agreement.

| Star | Min Distance | Midpoint | Max Distance |
| :---: | :---: | :---: | :---: |
| Primary | 352.31 | 355.49 | 358.72 |
| Secondary | 181.49 | 188.9 | 196.95 |

Table 5. Distance from Earth for the primary and secondary stars expressed in parsecs.

## References

Fernique, F; Boch, T; Oberto, A and Bonnarel, A; Strasbourg astronomical Data Center, Aladin Sky Atlas 10

Gaia Collaboration, 2018, "Gaia Data Release 2: Summary of the contents and survey properties". ArXiv e-prints.
Gaia Collaboration, 2016, "The Gaia mission", $A$ \& $A$ 595, A1.
Las Cumbres Observatory; Haleakala Observatory, Hawaii

Luri et al. (2018): On the use of Gaia parallaxes.
Fitzgerald, M.T. (2018, accepted), "The Our Solar Siblings Pipeline: Tackling the data issues of the scaling problem for robotic telescope based astronomy education projects."; Robotic Telescopes, Student Research and Education Proceedings

Washington Double Star Catalog. Stelle Doppie Web Double Star Database https://www.stelledoppie.it/ index2.php
Astronomical Applications Department. USNO Julian Date Converter https://aa.usno.navy.mil/data/docs/ JulianDate.php

# Discovery of a Wide Binary in the Solar Neighborhood 

Wilfried R.A. Knapp<br>Vienna, Austria<br>wilfried.knapp@gmail.com


#### Abstract

During the work on a report with the topic of star systems in the solar neighbourhood up to 10 parsecs a so far unknown wide binary was discovered at a distance of $\sim 8.1$ parsecs from the Sun. This comes rather as a surprise as stars in the solar neighborhood are most likely the best investigated stellar objects


Part of the work on a report on star systems in the solar neighborhood up to 10 parsecs (currently in progress) was the selection of GAIA DR2 objects with parallax $>100$ mas and parallax error $<0.5 \%$ and Gmag $<$ 18. The resulting 34 objects at a distance up to 10 parsecs included a pair so far not listed in the WDS catalog or other catalogs with binaries/multiples in the solar neighborhood.

The primary is TYC 3980-1081-1 (Gaia DR2 2202703050388170880 ) at J2000 position RA 2151 38.297 Dec +59 1738.456 with Gmag 9.3832 and parallax 123.0568 mas with error 0.5944 which means a distance of $\sim 8.1$ parsecs from the Sun. The secondary is UCAC4 747-070768 (Gaia DR2 2202703050401536000 ) at J2000 position RA 215140.108 Dec +591734.854 with Gmag 14.3852 and parallax 118.1243 mas with error 0.0208. Using the DR2 data for a Monte Carlo simulation calculating spatial distance between the components (see Appendix) results in a minimum distance of $\sim 33,000 \mathrm{AU}$, a median distance of $\sim 70,000 \mathrm{AU}$ and a maximum distance of $\sim 105,000$ AU suggesting strongly a potential gravitational relationship (see Figure 1).

It seems a bit surprising to detect a new likely physical pair this close to the Sun, but this might be explained by the rather large delta parallax of $\sim 5 \mathrm{mas}$ between the two components making a potential gravitational relationship not very obvious at first glance.

Even more surprising is the fact that not even the primary is listed as close to the neighboring object in, for example, the RECONS project - this asks for additional research. Finch et al. 2016 give a parallax of


Figure 1. Distance distribution for newly detected binary in 1000 AU .
154.8 mas with a rather large error of 12.1 mas and for this reason this object was not included in the Henry et al. 2018 paper on new discoveries within 10 pc . So there might be some caveats regarding this object but nonetheless the currently given data suggests a so far not identified physical system at a distance within 10 parsecs from the Sun even if DR2 proper motion data are somewhat different.

Gaia DR2 lists neither for the primary nor for the secondary a duplicated_source indication but the RUWE value for the primary is $>16$ suggests that the Gaia DR2 single-star model does not provide very good fit to the astrometric observations - in this case just indicating for good reasons that the source is a non-single object.

## Discovery of a Wide Binary in the Solar Neighborhood

The Gaia DR2 StarHorse catalog (Anders et al. 2019) provides a median mass for the primary of $\sim 0.5$ Sun mass but no such value for the secondary - an estimation based on magnitude difference gives $\sim 0.15$ Sun mass for the secondary. Based on these values a potential orbit (see Appendix) would have a minimum period of several million years, which means that most likely no human time frame will deliver enough observations for a reliable calculation of such a long period orbit.

Data for KPP4430 (WDS 21516+5918) based on Gaia DR2 2015.5 values are as follows:

- 327.90966427 RA J2000 in degrees
- 59.29440774 Dec J2000 in degrees
- $111.954 \quad$ Position angle J2015.5
- $0.003 \quad$ Error position angle
- 14.64214 Separation in arcseconds
- 0.00078 Error separation
- 10.93868 Estimated Vmag primary
- 0.00286 Error estimated Vmag Primary
- 14.58762 Estimated Vmag secondary
- 0.00227 Error estimated Vmag secondary
- 123.0568 Parallax primary in mas
- 0.5944 Error parallax primary
- 8.12633 Distance primary from the Sun in parsecs
- $\quad 118.1243$ Parallax secondary in mas
- 0.0208 Error parallax secondary
- 8.46566 Distance secondary from the Sun in parsecs
- -79.190 Proper motion RA primary in mas/ yr
- $10.517 \quad$ Proper motion Dec primary in mas/ yr
- -86.799 Proper motion RA secondary in
- -19.190 Proper motion Dec secondary in mas/yr
- 33,031 Minimum spatial distance between the components in AU
- $\quad 0.50268 \quad$ StarHorse median mass for primary in Solar masses
- $\quad 0.15000 \quad$ Estimated mass for secondary in Solar masses
- 7,472,048 Minimum period of a potential orbit in years


## Acknowledgements

The following tools and resources have been used for this research:

- DSS2 images
- 2MASS images
- Aladin Sky Atlas v10.0
- GAIA DR2 catalog
- GAIA DR2 StarHorse catalog
- Washington Double Star Catalog
- CDS VizieR
- CDS TAPVizieR
- Gaia@AIP Services hosted by the LeibnizInstitute for Astrophysics Potsdam (AIP)


## References

F. Anders, et al., 2019, "Photo-astrometric distances, extinctions, and astrophysical parameters for Gaia DR2 stars brighter than $\mathrm{G}=18^{\prime \prime}$, Astronomy \& Astrophysics, 628, A94.
Finch, Charlie T., Zacharias, Norbert, 2016, "Parallax Results from URAT Epoch Data", The Astronomical Journal, 151, Issue 6, article id. 160, 12 pp.
Todd J. Henry, et al., 2018, "The Solar Neighborhood XLIV: RECONS Discoveries within 10 parsecs", The Astronomical Journal, 155:265 (23pp).
Knapp, Wilfried R. A., 2018, "A New Concept for Counter-Checking of Assumed Binaries", Journal of Double Star Observations, 14 (3), 443.

## Discovery of a Wide Binary in the Solar Neighborhood

## Appendix

## Description of the Potential Gravitational Relationship assessment procedure (according to Knapp 2018):

GAIA DR2 data for RA/Dec and Plx are used for a Monte Carlo simulation assuming a normal distribution for these parameters with the given error range as standard deviation. The distance between the components is calculated from the inverted simulated parallax data and the simulated angular separation using the law of cosines

$$
\sqrt{a^{2}-2 a b \cos \gamma+b^{2}}
$$

with $a$ and $b=$ distance vectors for the stars A and B in lightyears calculated as $(1000 / \mathrm{Plx}) * 3.261631$ and $\gamma=$ angular separation in degrees calculated as

$$
\gamma=\arccos [\sin (D E 1) \sin (D E 2)+\cos (D E 1) \cos (D E 2) \cos (|R A 1-R A 2|)]
$$

The likelihood for potential gravitational relationship (LPGR) is the percentage of simulation results < 200,000 AU ( $\sim 1$ parsec) out of the simulation sample with a size of 120,000 corresponding with the likelihood that the real distance is smaller than $200,000 \mathrm{AU}$ with an margin of error of $0.37 \%$ at $99 \%$ confidence.

The minimum, median, and maximum distance is the smallest, median, and largest result of the simulation sample.

Ignoring the likely effects of eccentricity the smallest/median/largest distance is used as estimation for the value for the semi-major axis of a potential orbit allowing for the calculation of a minimum/median/maximum orbit period assuming zero inclination using either median mass data from Anders et al. 2019 or if not available mass estimation from other sources.

# UCAC4 337-189531, Discovery of Stellar Duplicity During Asteroidal Occultation by (3130) Hillary 

Carles Perello<br>Agrupacio Astronomica de Sabadell, MPC-619, Catalonia, Spain International Occultation Timing Association (IOTA-ES)<br>rigilk436@gmail.com<br>Eric Frappa<br>Euraster, Faycelles, France<br>frappa@euraster.net<br>Tomas Janik<br>Usti nad Labem, Czech Rep.<br>Czech Astronomical Society<br>Teplice Observatory and Planetarium<br>jazzer@centrum.cz<br>Bjoern Kattentidt<br>MPC K71, Neutraubling, Germany<br>International Occultation Timing Association (IOTA-ES)<br>bjoern@kattentidt-astro.de<br>Jiri Polak<br>Observatory Rokycany and Pilsen, Plzen-Lhota, Czech Rep.<br>jiri.polak@centrum.cz<br>Michal Rottenborn<br>Observatory Rokycany and Pilsen, Plzen, Czech Rep.<br>rotmi@seznam.cz<br>Antoni Selva<br>Agrupacio Astronomica de Sabadell, MPC-619, Catalonia, Spain International Occultation Timing Association (IOTA-ES)<br>antoni.selva@gmail.com


#### Abstract

The occultation of the star UCAC4 337-189531 by the asteroid 3130 Hillary on July 23rd, 2019 has shown the duplicity of the star. Three observations carried out from Czech Rep. and Spain enable the determination of the parameters of this double star. A separation of $193.5 \pm 18.7$ milliarcseconds (mas) and a position angle (PA) of $54.5 \pm 18.7$ degrees has been calculated. From the magnitude drop in the light curves the estimated magnitudes without filter are $12.6 \pm 0.04$ and $13.4 \pm 0.2$. We suggest that this pair be included in the WDS catalog.


## UCAC4 337-189531, Discovery of Stellar Duplicity During Asteroidal Occultation by (3130) Hillary

## Circumstances

On July 23, 2019 the asteroid (3130) Hillary occulted the star UCAC4 337-189531. This prediction was published in the occultation's feed IBEROC *[6] (Figure 1) and it was observable from the south-east of Spain to the north-east of Europe. UCAC4 337-189531 is a 12.2 V magnitude star with the equatorial coordinates RA $19 \mathrm{~h} \quad 30 \mathrm{~m}$ 57.37s, Dec. $-22^{\circ} \quad 36^{\prime}$ 13.26" (J2000.0).

The magnitude of the asteroid (3130) Hillary in the moment of the occultation was 15.5 . This value has been obtained in the ephemeris web page of the Minor Planet Center *[3]. Also, star and asteroid magnitudes were in the prediction

## Observations

Five separate sites observed this occultation: three with positive results and two negatives. Table 1 gives the geographical coordinates and instrumentation used.

All the stations recording a positive occultation had results that did not match with the predicted ones, specifically the predicted time and the predicted magnitude drop.

## Data Analysis

Using the magnitudes of the UCAC4 catalogue and the MPC, the predicted combined magnitude of the target was 12.26 V (star UCAC4 [12.312V] plus asteroid MPC [ 15.5 V$]$ ) and the predicted drop magnitude was 3.24 V .


Figure 1. Prediction map of the occultation computed with Occult *[1].

The \#1 station recorded the occultation 32.9 s after the predicted time, and the event had a measured magnitude based in its light flux value of $12.50 \mathrm{~V} \pm 0.01$ and a magnitude drop of only 0.33 V . That's allow us to compute the magnitude of the star not occulted: 12.57 V $\pm 0.04$.

The \#2 station recorded a delayed disappearance of 53.0s and the event had a measured magnitude based in its light flux value of $13.29 \mathrm{~V} \pm 0.01$ and a magnitude

| \# | Station Team | Longitude, Latitude, \& Altitude | Telescope | Equipment | Integration used |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | C. Perello \& A. Selva | $\begin{gathered} 2^{\circ} 05^{\prime} 24.6^{\prime \prime} \mathrm{E} \\ 41^{\circ} 33^{\prime} 00.2^{\prime \prime} \mathrm{N} \\ 224 \mathrm{~m} \end{gathered}$ | $\begin{aligned} & \text { Newton } \\ & 50 \mathrm{~cm} \mathrm{f/4} \end{aligned}$ | TV Camera <br> Watec 910HX \& KIWI inserter time | 0.08 s |
| 2 | J. Polak | $\begin{gathered} 13^{\circ} 16^{\prime} 30.6^{\prime \prime} \mathrm{E} \\ 49^{\circ} 56^{\prime} 02.1^{\prime \prime} \mathrm{N} \\ 533 \mathrm{~m} \end{gathered}$ | Newton $30.3 \mathrm{~cm} \mathrm{f} / 4$ | TV Camera <br> Watec 120N \& TIM-10 Inserter time | 0.16 s |
| 3 | T. Janik | $\begin{gathered} 14^{\circ} 02^{\prime} 25,77^{\prime \prime} \mathrm{E} \\ 0^{\circ} 40^{\prime} 59,5 "^{\prime \prime} \mathrm{N} \\ 378 \mathrm{~m} \end{gathered}$ | $\begin{gathered} \text { Schmidt- } \\ \text { Cassegrain } 15.3 \\ \mathrm{~cm} / 10 \end{gathered}$ | TV Camera <br> Watec 120N+ \& TIM-10 Inserter time | 0.64 s |
| 4 | M. Rottenborn | $\begin{array}{ccc} 13^{\circ} & 15^{\prime} & 37.8^{\prime \prime} \\ 49^{\circ} & 50^{\prime} 09.5^{\prime \prime} & \mathrm{N} \\ 427 \mathrm{~m} \end{array}$ | Newton $30.3 \mathrm{~cm} \mathrm{f} / 4$ | TV Camera <br> Watec 120N \& TIM-10 <br> Inserter time | 0.16 s |
| 5 | B. Kattentidt | $\begin{array}{ccc} 12^{\circ} & 12^{\prime} 57.3^{\prime \prime} & \mathrm{E} \\ 48^{\circ} 59^{\prime} 23.1^{\prime \prime} & \mathrm{N} \\ 335 \mathrm{~m} \end{array}$ | $\begin{gathered} \text { Schmidt- } \\ \text { Cassegrain }+ \text { FR } \\ 27.9 \mathrm{~cm} \mathrm{f} / 6,3 \end{gathered}$ | TV Camera Watec 910HX \& DL7USM-2006 V2.0 inserter time | 0.16 s |

Table 1. Geographical coordinates and equipment of each station

## UCAC4 337-189531, Discovery of Stellar Duplicity During Asteroidal Occultation by (3130) Hillary



Figure 2. Estimated magnitude of the A component.
drop of only $1,07 \mathrm{~V}$. That's allow us to compute the magnitude of the star not occulted: $13.44 \mathrm{~V} \pm 0.2$.

The \#3 station recorded a delayed disappearance of 53.0s (the observer didn't report the delayed time in his report, he only reported a similar delay to station \#2) and the event had a measured magnitude based in its light flux value of $13.27 \mathrm{~V} \pm 0.01$ and a magnitude drop of only $1,08 \mathrm{~V}$. That's allow us to compute the magnitude of the star not occulted: $13.42 \mathrm{~V} \pm 0.2$.

All the results match with the explanation that one component of the system did not be occulted. Based on the signal measured in the recorded videos we think the \#1 station recorded the occultation of the B component (the fainter) and the \#2 and \#3 stations recorded the occultation of the A component (the brighter). The tool used to estimate the magnitude for each star based in the recording was the Magnitude Calculator Tool from Occult *[1] (Figures 2 to 4 ) assuming the magnitude assigned to the asteroid is 15.5 V . These magnitudes are an approximation since there were no photometric filters placed in front of the detectors, and based in the predicted asteroid magnitude and the magnitude drop recorded.

We have also used the same comparison star to estimate the magnitude of the both components: UCAC4 337-189601 with a catalogue magnitude of 11.93 V .

All timings obtained with TV-cameras have been


Figure 3. Estimated magnitude of the B component.


Figure 4. Estimated magnitude of the B component.
corrected following the values obtained by G. Dangl * [4]. See Table 2.

Using the software Occult $*[1]$ we fit the circular shape limits of the asteroid, obtaining a result of $\sim 24.4$ km being the negative result of the \#4 station a good reference for the shadow limit (see Figure 9) but not enough to be fully reliable about the shape. A separation and position angle of the occulted double star were also obtained and the values are listed in Table 3.

## Conclusions

The casual occultation caused by an asteroid of the star UCAC4 337-189531 revealed its duplicity. The number of observers registered was enough to allow us to determine the parameters of this binary system.

| $\#$ | D2 | D1 | R2 | R1 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | $234038.47 \pm 0.08$ | - | $234041.39 \pm 0.04$ | - |
| $\mathbf{2}$ | - | $23: 39: 24.08 \pm 0.16$ | - | $23: 39: 25.84 \pm 0.16$ |
| $\mathbf{3}$ | - | $23: 39: 19.32 \pm 0.32$ | - | $23: 39: 21.56 \pm 0.32$ |

Table 2. Timings of the occultation. D1 and R1 are the disappearance and the reappearance of the brighter component of the double star, while D2 and R2 are, respectively, the disappearance and the reappearance of the secondary component.

| Distance (mas) | $193.5 \pm 18.7$ |
| :---: | :---: |
| PA (degrees) | $54.5 \pm 18.7$ |
| Mag. (A) | $12.6 \mathrm{~V} \pm 0.04$ |
| Mag. (B) | $13.4 \mathrm{~V} \pm 0.20$ |

Table 3. Parameters of the double star UCAC4 337-189531

## UCAC4 337-189531, Discovery of Stellar Duplicity During Asteroidal Occultation by (3130) Hillary



Figure 5. A. Selva and C. Perello light curve obtained with Tangra *[2].


Figure 6. T.Janik light curve obtained with Tangra *[2].


Figure 7. J. Polak light curve obtained with Tangra *[2].

## UCAC4 337-189531, Discovery of Stellar Duplicity During Asteroidal Occultation by (3130) Hillary



Figure 8. Chart obtained with Occult *[1] before double star settings applied. The line is present if the star is visible.



Figure 9. Double star solution obtained by Eric Frappa with Occult *[1]. See *[5] for more information. The chords in the circle are the time that the star is occulted by the asteroid for the different observer's station. The continuous line below are the two negative chords (star always visible) and the dotted line is the predicted path.

## UCAC4 337-189531, Discovery of Stellar Duplicity During Asteroidal Occultation by (3130) Hillary

(Continued from page 89)
*References
Occult v4.7.2 (last stable version is v4.9.1) Occultation prediction software by David Herald, http:// www.lunar-occultations.com/iota/occult4.htm

Tangra 3.6.17 software for reducing astronomical video observations, http://www.hristopavlov.net/Tangra3/

Minor Planet Center as a feed of orbital elements used to create the predictions, https://
minorplanetcenter.net/
Dangl, Gerhard, Video exposure time analysis from recordings with video time insertion, http:// www.dangl.at/ausruest/vid_tim/vid_tim1.htm

Herald, Dave, et al., "New Double Stars from Asteroidal Occultations, 1971 - 2008", Journal of Double Star Observations, 6(1), 88-96, 2010. http:// www.jdso.org/volume6/number1/herald.pdf
IBEROC Occultation Feed for Occult Watcher, maintained by Carles Perelló and available at http:// ocultacions.astrosabadell.org/IBEROC/

# Astronomical Association of Queensland 2017 Program: Blue Star Observatory Measurement of Six Neglected Southern Multiple Stars 

Peter N. Culshaw, Diane Hughes, John Hughes, Des Janke, Graeme Jenkinson<br>Astronomical Association of Queensland, Australia. bluestars@iprimus.com.au


#### Abstract

This paper presents the final results of a 2017 programme of photographic measurements of six southern multiple stars. All results were obtained using an Atik 460EX mono CCD camera used in conjunction with an equatorially mounted 400 mm F4.5 Newtonian reflector.


## Introduction

These latest results are part of an ongoing programme commenced in 2008 by the Double Star Section of the Astronomical Association of Queensland. The target stars were selected from the Washington Double Star Catalogue (WDSC) and were observed in Queensland, Australia from a latitude of approximately $27^{\circ} \mathrm{S}$.

The results are presented in Table 1. The mean $95 \%$ confidence intervals for the new measures were $\pm$ $0.638^{\circ}$ in PA and $\pm 0.128^{\prime \prime}$ in separation.

## Method

Nightly sets of one hundred images were obtained with the equipment described above, after which the images were stacked using Atik DAWN software and then analysed using the astrometric double star program REDUC (Losse, 2008). Approximately ten stacked images of each target were taken per night for seven nights and the results averaged to obtain measures of separation and position angle with sufficient confidence.

Full details of the method are given in Napier-
(Text continues on page 97)

| System | Last listed measure |  |  | New measure |  |  | Comment |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | PA ${ }^{\circ}$ | Sep. " | Epoch | PA - | Sep. " | Epoch* |  |
| RSS331 | 254 | 8.2 | 1976 | 255.77 | 7.90 | 2017.396 | Minimal change in 43 years |
| I 1250 | 149 | 4.6 | 1932 | 149.06 | 4.88 | 2017.448 | Little probable change |
| B2777 | 106 | 5.7 | 1934 | 85.24 | 3.73 | 2017.456 | Clear movement |
| CPO453 | 337 | 3.2 | 1928 | 334.29 | 3.53 | 2017.525 | Slight changes |
| DON779 | 294 | 6.0 | 2015 | 299.10 | 5.90 | 2017.530 | Change in PA |
| DON779 | n/a | n/a | n/a | 235.05 | 11.64 | 2017.530 | New "C" component? |
| B2385 | 228 | 6.3 | 2015 | 229.94 | 6.17 | 2017.552 | Minimal change |
| B2385 | $\mathrm{n} / \mathrm{a}$ | $\mathrm{n} / \mathrm{a}$ | $\mathrm{n} / \mathrm{a}$ | 26.30 | 8.35 | 2017.552 | New "C" component? |

Table 1. Measurements of Nine Southern Multiple Stars

[^1]
## Astro. Assn. of Queensland 2017 Program: Blue Star Observatory Measurement of Six Multiple

| $\begin{gathered} \text { RSS331 } \\ \text { Centaurus } \end{gathered}$ | RA. 1422.7 | DEC. -30 36 | Last Measure 1976 |
| :---: | :---: | :---: | :---: |
|  | MAG. 9.89 \& $\mathrm{n} / \mathrm{a}$ | PA. $254{ }^{\circ}$ | SEP. 8.2" |
| Date | No. images | PA ${ }^{\circ}$ | Sep" |
| 16 May 2017 | 10 | 256.16 | 8.111 |
| 24 May 2017 | 10 | 255.80 | 7.939 |
| 25 May 2017 | 10 | 255.86 | 7.834 |
| 26 May 2017 | 10 | 255.77 | 7.793 |
| 27 May 2017 | 10 | 255.97 | 7.873 |
| 30 May 2017 | 10 | 255.05 | 7.856 |
| 02 June 2017 | 10 | 255.75 | 7.861 |
| Mean |  | 255.766 | 7.895 |
| Std. dev. |  | 0.346 | 0.105 |
| 95\% CI +/- |  | 0.320 | 0.097 |
| P(t) movement |  | 0.000 | 0.000 |
| COMMENTS <br> Slight increase | PA. Little pro | e change in | eparation. |



| $\begin{gathered} I 1250 \\ \text { Centaurus } \end{gathered}$ | RA. 1443.6 | DEC. -39 39 | Last Measure 1932 |
| :---: | :---: | :---: | :---: |
|  | MAG. 8.11 \& 12.7 | PA. $149^{\circ}$ | SEP. 4.6" |
| Date | No. images | PA ${ }^{\circ}$ | Sep" |
| 02 June 2017 | 10 | 148.97 | 4.895 |
| 04 June 2017 | 10 | 149.8 | 4.794 |
| 09 June 2017 | 10 | 149.98 | 4.783 |
| 20 June 2017 | 10 | 149.33 | 4.76 |
| 21 June 2017 | 10 | 148.86 | 4.796 |
| 23 June 2017 | 10 | 148.89 | 4.93 |
| 25 June 2017 | 10 | 147.57 | 5.222 |
| Mean |  | 149.057 | 4.883 |
| Std. dev. |  | 0.792 | 0.162 |
| 95\% CI +/- |  | 0.733 | 0.150 |
| $\mathrm{P}(\mathrm{t})$ movement |  | 0.000 | 0.000 |
| COMMENTS <br> Very little if 1925. | y apparent moveme | since the | st measure in |



| $B 2777$ Centaurus | RA. 1456.0 | DEC. -33 04 | Last Measure 1934 |
| :---: | :---: | :---: | :---: |
|  | MAG. 8.52 \& 14.1 | PA. $106^{\circ}$ | SEP. 5.7" |
| Date | No. images | PA ${ }^{\circ}$ | Sep" |
| 02 June 2017 | 10 | 84.56 | 3.632 |
| 04 June 2017 | 10 | 85.66 | 3.585 |
| 21 June 2017 | 10 | 85.63 | 3.744 |
| 23 June 2017 | 10 | 84.57 | 3.74 |
| 25 June 2017 | 10 | 85.77 | 3.782 |
| 30 June 2017 | 10 | 85.26 | 3.892 |
| Mean |  | 85.242 | 3.729 |
| Std. dev. |  | 0.552 | 0.109 |
| 95\% CI +/- |  | 0.579 | 0.115 |
| $\mathrm{P}(\mathrm{t})$ movement |  | 0.000 | 0.000 |
| COMMENTS <br> Reductions in PA measures of 1930 | and separation ar \& 1934. | consistent | h the two previous |



## Astro. Assn. of Queensland 2017 Program: Blue Star Observatory Measurement of Six Multiple

| CPO453 <br> Norma | RA. 1604.7 | DEC. -49 15 | Last Measure 1928 |
| :---: | :---: | :---: | :---: |
|  | MAG. 11.21 \& 13.0 | PA. $337^{\circ}$ | SEP. 3.2" |
| Date | No. images | PA ${ }^{\circ}$ | Sep" |
| 30 June 2017 | 10 | 334.21 | 3.548 |
| 02 July 2017 | 10 | 335.19 | 3.606 |
| 17 July 2017 | 10 | 334.43 | 3.485 |
| 18 July 2017 | 10 | 333.4 | 3.509 |
| 21 July 2017 | 10 | 335.17 | 3.588 |
| 22 July 2017 | 10 | 334.37 | 3.515 |
| 23 July 2017 | 10 | 333.28 | 3.48 |
| Mean |  | 334.293 | 3.533 |
| Std. dev. |  | 0.756 | 0.049 |
| 95\% CI +/- |  | 0.699 | 0.046 |
| $\mathrm{P}(\mathrm{t})$ movement |  | 0.000 | 0.000 |
| COMMENTS <br> PA has decrease measures in 190 same period. | in contrast to sl \& 1928. Probable | ht increase ight increas | tween previous in separation over |



| $\begin{gathered} \text { DON779 } \\ \text { TrA } \end{gathered}$ | RA. 1610.9 | DEC. -66 21 | Last Measure 2015 |
| :---: | :---: | :---: | :---: |
|  | MAG. 8.09 \& 14.1 | PA. $294^{\circ}$ | SEP. 6.0" |
| Date | No. images | PA ${ }^{\circ}$ | Sep" |
| 02 July 2017 | 10 | 299.75 | 5.438 |
| 11 July 2017 | 10 | 299.12 | 6.129 |
| 17 July 2017 | 10 | 299.77 | 5.842 |
| 18 July 2017 | 10 | 298.73 | 5.772 |
| 21 July 2017 | 10 | 298.10 | 5.929 |
| 22 July 2017 | 10 | 299.13 | 6.097 |
| 23 July 2017 | 10 | 299.12 | 6.083 |
| Mean |  | 299.103 | 5.899 |
| Std. dev. |  | 0.579 | 0.245 |
| 95\% CI +/- |  | 0.535 | 0.226 |
| P(t) movement |  | 0.000 | 0.000 |
| COMMENTS <br> Clear change in | Little probabl | movement in | paration. |



| DON779 | RA. 1610.9 | DEC. -66 21 | Last Measure n/a |
| :---: | :---: | :---: | :---: |
| TrA | MAG. 8.09 \& n/a | PA. $\mathrm{n} / \mathrm{a}$ | SEP. n/a |
| Date | No. images | PA ${ }^{\circ}$ | Sep" |
| 02 July 2017 | 10 | 234.69 | 11.572 |
| 11 July 2017 | 10 | 234.59 | 11.383 |
| 17 July 2017 | 10 | 234.15 | 11.72 |
| 18 July 2017 | 10 | 234.98 | 11.621 |
| 21 July 2017 | 10 | 235.59 | 11.863 |
| 22 July 2017 | 10 | 235.65 | 11.786 |
| 23 July 2017 | 10 | 235.71 | 11.522 |
| Mean |  | 235.051 | 11.638 |
| Std. dev. |  | 0.611 | 0.165 |
| 95\% CI +/- |  | 0.566 | 0.152 |
| $\mathrm{P}(\mathrm{t})$ movement |  |  |  |
| OMMENTS ossible "C" | nent not previo | y recorded. |  |

Astro. Assn. of Queensland 2017 Program: Blue Star Observatory Measurement of Six Multiple ...

| B2385 Norma | RA. 1626.0 | DEC. -44 43 | Last Measure 2015 |
| :---: | :---: | :---: | :---: |
|  | MAG. 9.06 \& 13.1 | PA. $228{ }^{\circ}$ | SEP. 6.3" |
| Date | No. images | PA ${ }^{\circ}$ | Sep" |
| 11 July 2017 | 10 | 228.44 | 6.203 |
| 21 July 2017 | 10 | 230.16 | 6.215 |
| 22 July 2017 | 10 | 231.61 | 6.154 |
| 23 July 2017 | 10 | 231.11 | 6.198 |
| 29 July 2017 | 10 | 229.98 | 6.106 |
| 30 July 2017 | 10 | 229.11 | 6.24 |
| 01 August 2017 | 10 | 229.14 | 6.09 |
| Mean |  | 229.936 | 6.172 |
| Std. dev. |  | 1.139 | 0.057 |
| 95\% CI +/- |  | 1.054 | 0.053 |
| $P(t)$ movement |  | 0.000 | 0.000 |

## COMMENTS

Minimal changes evident since the original 1901 measure.

| B2385 Norma | RA. 1626.0 | DEC. -44 43 | Last Measure 2015 |
| :---: | :---: | :---: | :---: |
|  | MAG. 9.06 \& 13.1 | PA. $228{ }^{\circ}$ | SEP. 6.3" |
| Date | No. images | PA ${ }^{\circ}$ | Sep" |
| 18 July 2017 | 10 | 26.26 | 8.251 |
| 21 July 2017 | 10 | 27.00 | 8.087 |
| 22 July 2017 | 10 | 26.54 | 8.364 |
| 23 July 2017 | 10 | 25.12 | 8.249 |
| 29 July 2017 | 10 | 26.76 | 8.632 |
| 30 July 2017 | 10 | 25.91 | 8.310 |
| 01 August 2017 | 10 | 26.50 | 8.557 |
| Mean |  | 26.299 | 8.350 |
| Std. dev. |  | 0.625 | 0.189 |
| 95\% CI +/- |  | 0.578 | 0.174 |
| P(t) movement |  |  |  |



## COMMENTS

Possible new "C" component not previously recorded.

# Astro. Assn. of Queensland 2017 Program: Blue Star Observatory Measurement of Six Multiple ... 

(Continued from page 93)
Munn and Jenkinson (2009). Subsequent work on the errors inherent in the method is described in NapierMunn and Jenkinson (2014). As proficiency has grown in the use of this equipment with the 400 mm reflector, close doubles with considerable magnitude difference between the components have been successfully measured.

Fellow AAQ members Culshaw, Hughes and Hughes provided invaluable assistance with image processing using Losse's REDUC software, and Janke with processing the original FITS image files into JPEG photographs.

## Results

For all of the systems shown below the WDSC information is first reproduced, showing the epoch 2000 position, magnitudes, separation, PA, and the last recorded measurement. The new measurements are then given in tabular form, including the mean and standard deviation and $95 \%$ confidence limits. Any uncertainties between the images and the last recorded measurements are discussed. Finally a conclusion is given as to whether any movement of the component stars has occurred in PA or separation, based on the P-value for the t-test comparing the new mean values with the catalogued value ( $\mathrm{P}<0.05$ is considered as evidence of change).

As detailed in the tabulated results above possible previously unlisted "C" components were found and imaged for both DON779 Triangulum Australe and B2385 Norma.

The pair CPO453 Norma shows a slight decrease in PA in comparison to an increase recorded in previous measures of 1902 and 1928.

Please note that all attached images are aligned with North to the bottom and East to the right.

## Acknowledgements

This research has made use of the Washington Double Star Catalog maintained at the U.S. Naval Observatory.

The Edward Corbould Research Fund administered by the Astronomical Association of Queensland for granting of funds to upgrade imaging camera and observatory computer to suit.

## References

Losse, F. Reduc software, V4.5.1. http:// www.astrosurf.com/hfosaf/uk/tdownload.htm
Napier-Munn, T.J. and Jenkinson, G., 2009,
"Measurement of some neglected southern multiple stars in Pavo", Webb Society Double Star Section Circular, 17, 6-12.

Napier-Munn, T.J and Jenkinson G., 2014, "Analysis of Errors in the Measurement of Double Stars Using Imaging and the Reduc Software", Journal of Double Star Observations, 10 (3), 193-198.
Argyle, R.W., 2012, Observing and Measuring Visual Double Stars 2nd edition, Springer.

# Astrometric Measurement and Analysis of Celestial Motion for Double Star WDS 02176+5920 

Marielle Cooper ${ }^{1}$, Theophilus Human ${ }^{1}$, Grady Boyce ${ }^{2}$, Pat Boyce ${ }^{2}$, Jae Calanog ${ }^{1}$<br>1. San Diego Miramar College<br>2. Boyce Research Initiatives and Education Foundation (BRIEF)


#### Abstract

Our team observed and analyzed the double star system WDS 02176+5920 (STI 1828) using the Las Cumbres Telescope network. The data was analyzed in AstroImageJ to measure position angle and separation. A mean position angle of $140.0^{\circ} \pm 0.2^{\circ}$ (theta) and separation of $11.76^{\prime \prime} \pm 0.05^{\prime \prime}$ (rho) was measured on Julian date 2458426.85100 The historical trend shows relatively no change in position angle and separation. However, the Harshaw calculation of the pair is low at 0.0017 (Harshaw 2014) and measurements on the proper motion of the right ascension and declination suggest the stars are moving in the same direction. These combined data points suggest a high probability that STI 1828 is common proper motion pair.


## Introduction

The purpose of our research was to observe and analyze the position angle and separation of a double star system. The data was then analyzed to determine the nature of the selected double star to see if it could be classified as a physical or optical pair. A subcategory of physical double stars are common proper motion pairs which share similar proper motions but have very little relative change in separation over time (Greaves 2004). Classification of a physical binary can allow the total mass of the system to be determined, if an orbital solution is obtained and the distance to the system is known. As the nature of the STI 1828 system (shown in Figure 1) is unknown, the term double star shall be used throughout this paper.

To study double star pairs and determine their nature, the position angle $(\theta)$ and distance $(\rho)$ between the stars is measured and compared to historical values. Graphing these values over time, it can be determined whether the stars may be gravitationally bound. Our research focused on WDS $02176+5920$ (STI 1828). STI 1828 is in Cassiopeia and was selected as it met the requirements of falling between 00-08 RA hours, ( 02 h 17 m RA), having a separation greater than $5^{\prime \prime}$ (arcseconds), and a magnitude difference less than 3. Right Ascension was restricted between $00-08$ RA as


Figure 1. WDS 02176+5920 A is the primary star and B is the secondary
this is the portion of sky visible at night during October -November. The constraints of separations of greater than 5 " and magnitude difference less than 3 were applied to ensure stars could be visibly separated in the telescope images. Additional selection criteria included a desire for magnitude visible through binoculars and a position in the Northern Hemisphere.

The first observation of this double star was recorded in 1908 by Johan Stein at the Vatican observatory with $(\theta)$ of $139.4^{\circ}$ and ( $\rho$ ) of $12.499^{\prime \prime}$. Since 1908 elev-
(Text continues on page 99)

## Astrometric Measurement and Analysis of Celestial Motion for Double Star WDS 02176+5920

| Epoch | Position Angle | Separation |
| :---: | :---: | :---: |
| 1908.05 | 139.4 | 12.499 |
| 1909.5 | 140.0 | 12.0 |
| 1910.97 | 140.2 | 11.408 |
| 1919.94 | 139.6 | 11.5 |
| 1921.85 | 138.9 | 11.66 |
| 1999.71 | 139.5 | 11.70 |
| 2003.68 | 139.8 | 11.742 |
| 2007.175 | 139.17 | 11.76 |
| 2010.5 | 140.0 | 11.76 |
| 2012.585 | 140.07 | 11.776 |
| 2013.066 | 139.863 | 11.745 |
| 2015.000 | 140.0 | 11.76 |
| 2018.843 | 140 | 102 |

Table 1: Historical Values of Position Angle and Separation Distance for STI 1828 02176+5920. Position angle is in degrees and Separation Distance in arc seconds. (Mason 2018)
en more observations have been recorded. The most recent measurement was in 2015 with $(\theta)$ of $139.8^{\circ},(\rho)$ of $11.745^{\prime \prime}$, and a difference in magnitude of 0.60 .

## Observations and Analysis

The Las Cumbres Observatory (LCO) was used for observation as its global network of telescopes allows for continuous visibility across the night sky. A 0.4meter telescope was used mounted on a C-ring equatorial mount. The scientific camera used was an SBIG STX6303 mounted at the Cassegrain focus. Light was detected with the CCD capable of capturing 2048 x 3072 pixels with a resolution of 0.57 arcseconds per pixel. Default binning of $1 \times 1$ was used. The first data set was collected from the Haleakala site with telescope 2 (telescope id kb82) in Hawaii, USA. The second data set was collected from the McDonald Observatory using telescope 1 (telescope id kb92) in Texas, USA.

Two data sets were collected. The first one consist-
ed of 18 images with the second having 15 images. The filters selected for the first data set were SDSS r', SDSS g', and clear. Filters were selected as the primary star had a spectral classification of G5V. For the second data set the same selection plus Bessel-B was used. Bessel-B was added to compare filter effects. Exposure times ranged from under a second for the clear to 12 seconds for the red filter. Due to telescope tracking issues for the images in the first data set, only the second data set was included for analysis. 15 images were analyzed from the second data set.

Data was returned after being processed through the Our Solar Sibling Pipeline (Fitzgerald 2018) which attached WCS coordinates, removed hot pixels, image artifacts, and flat fielded the images. The software AstroImageJ (AIJ) was used to analyze and measure the images.

The position angle and separation distance between the double stars was measured. 15 images were analyzed, four with SDSS red filters, four with SDSS green filters, three with Bessel blue filters, and four with no filter. The red filters at 12 seconds produced the clearest images while no filter and blue were lower quality images. AIJ then provided a measurement of position angle in degrees, separation distance in arcseconds, and delta magnitude between the stars. From the data the average standard deviation, and error were calculated for both position angle and separation. All data points were included as there were no obvious outliers and there was confidence in the measurement capabilities even at lower image qualities. For Epoch 2018.821 the average position angle was $140.0^{\circ} \pm 0.2^{\circ}$ and $11.76 " \pm$ 0.05 ".

## Discussion

The collected data was added and analyzed against historical data. Historical data suggested that no significant movement (movement greater than a standard deviation from the 2018 measurements) between the two stars had occurred in the past century. The data shown in Table 2 supports this conclusion. The movement recorded (Figures 2 and 3) through the century follows no obvious trend and it seems likely given the small differences that most or all movement falls within a standard deviation of each observation's measurement.

| Epoch | Number of <br> Images | Mean Position <br> Angle ( ${ }^{\circ}$ ) | Standard <br> Deviation ( |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2018.843 | 15 | 140.0 | Mean <br> Separation <br> Distance (") | Standard <br> Deviation (") |  |
| 2015.000 | - | 139.863 | 0.2 | 11.76 | 0.05 |

Table 2. Summary of the measurement of Position Angle and Separation Distance for STI 1828 02176+5920 from this paper compared to the last reported measurement in 2015. (Mason 2

## Astrometric Measurement and Analysis of Celestial Motion for Double Star WDS 02176+5920

Of the 12 past observations, eight of the position angles recorded fall within the standard deviation of the 2018 position angle and seven within the standard deviation of the separation distance, Tables 1 and 2.

Graphs comparing position angle over time, separation over time, and the proper motion (Figures 2-4) demonstrate no clear relationship between the data points. Comparing the position angle data (Table 1 and Figure 2) from the early portion of the 20th century to the early 21 st century, the range of the position angle remains within about one degree, with no obvious movement (Figure 2). While it is possible that there was a significant movement in both directions that then averaged out, it seems more probable that the accuracy of measurements improved over time, resulting in smaller standard deviations.

Graphing the separation distance against time, Figure 2, showed very little movement. Data from the late 20th/early 21 st century is consistent, showing little change.

Figure 3 shows that separation distance in approximately the last 20 years has moved less than 0.2 arc seconds. While the early 20th century measurements show more variable separation distances, when combined with the modern measurements no clear pattern emerges.

The parametric graph also shows no clear movement in separation, Figure 4. Dates are marked on the


Figure 2. Graph of position angle ( ${ }^{\circ}$ ) against the time passed since the first observation in 1908.05 including historical data and observations conducted. The red triangle marks the most recent data.


Figure 3. Graph of separation distance (") against the time passed since the first observation in 1908.05 including historical data and observations conducted. The red triangle marks the most recent data.


Figure 4: Graph of STI 1828 movement relative to the primary. The red triangle marks the most recent data. The $R^{2}$ for a linear fit is 0.6774 which does not fit a linear model.

## Astrometric Measurement and Analysis of Celestial Motion for Double Star WDS 02176+5920



Figure 5. Graph showing potential distance overlap ranges between primary and secondary star.
graph to show the progression of time. The data is irregular, with 1998-2018 datapoints occurring clumped together while other epochs are spread apart. Analysis of the graph suggests a poor fit for either a linear or elliptic curve model, however it does fit a common proper motion pair model. As the movement is over a small range of values it is possible most movement is due to standard deviation in recording.

The proper motion of the star was also collected from Gaia release 2 data. It is summarized in Table 3.

Comparison shows that both the proper motion of the right ascension and declination of the primary and secondary are relatively close. This suggests that they are moving in the same direction.

There is evidence of stars having stellar companions (i.e. currently or previously gravitationally bound) with a distance of up to 8 parsecs between them (Shaya \& Olling 2011). Gravitationally unbound stellar companions may be part of a stellar association, a very loose star cluster whose stars share a common origin but have become gravitationally unbound. Stellar companions with large distance between them are considered wide or very wide physical pairs (depending on distance). If the GAIA suggested 1.1 parsec separation between the primary and secondary star of STI 1828 is accurate it could comfortably fall into the wide binary range. Wide binaries often share similar proper motions making them common proper motion pairs or multiples (Shaya \& Olling 2011). Common proper motion pairs are stars that have very similar proper motions but little
to no relative motions between themselves over timescales of a century. The similarity of space motion however suggests they are related by origin. Additionally, it is common for their separation to be large enough to cause uncertainty of whether they orbit one another (Greaves 2004). STI 1828 seems to fit the categorization of common proper motion pair as it has a high degree of similarity between the star's proper motions.

The Harshaw Method (Harshaw 2014) was used to provide further analysis. There is a correlation between values close to zero being physical binaries and values close to 1 tending to be optical doubles. The result of the Harshaw calculation 0.0017 which suggests that STI 1828 is a physical double star though this method cannot predict the nature of the double's gravitational link.

There is a strong possibility that STI 1828 is a common proper motion pair as the proper motion suggests both stars are moving in the same direction. the Harshaw calculation predicts a physical pair, and the position angle and separation distance are relatively constant. The stars could also be gravitationally bound as the parallax suggests that the stars are close enough for a gravitational link to be possible. It is suggested that the position angle and separation distance be taken every five to ten years, to confirm consistency in the distance between the stars.

## Conclusion

The data on STI 1828 suggests that stars are physically bound in a common proper motion pair. The new data acquired continued the trend of little movement in both position angle and separation distance over the last twenty years. The parallax data suggests the stars are close enough for gravitational linkage while the proper motion suggests the stars are heading in the same direction. The Harshaw analysis of this movement also suggests that that the primary and secondary star are physically bound.

## Acknowledgements

This paper would not have been possible without the aide and support of the Boyce Research Initiatives and Education Foundation (BRIEF), Theophilius Human, Pat Boyce, Grady Boyce, and Jae Calanog. I thank them all for their time and feedback. The analysis and

| Star | Proper Motion <br> Right Ascension <br> (mas/yr) | RA Proper Motion <br> Uncertainty | Declination <br> Proper Motion <br> (mas/yr) | Dec Proper Motion <br> Uncertainty |
| :---: | :---: | :---: | :---: | :---: |
| Primary | -43.411 | 0.06 | -43.705 | 0.078 |
| Secondary | -43.481 | 0.125 | -43.903 | 0.127 |

Table 3: Summary of the Proper Motion of STI 1828. Data acquired from GAIA Release 2.

## Astrometric Measurement and Analysis of Celestial Motion for Double Star WDS 02176+5920

work contained within also relied upon the LCOGT network and made use of the SIMBAD and Aladin databases operated by the CDS in Strasbourg, France.

This work has made use of data from the European Space Agency (ESA) mission Gaia (https:// www.cosmos.esa.int/gaia), processed by the Gaia Data Processing and Analysis Consortium (DPAC, https:// www.cosmos.esa.int/web/gaia/dpac/consortium). Funding for the DPAC has been provided by national institutions, in particular the institutions participating in the GAIA Multilateral Agreement.

## References

Boyce, G. and Boyce, P. Boyce, Research Initiatives and Education Foundation (BRIEF), http:// www.boyce-astro.org/home.html.
Fitzgerald, M.T., 2018, accepted, "The Our Solar Siblings Pipeline: Tackling the data issues of the scaling problem for robotic telescope based astronomy education projects.", Robotic Telescopes, Student Research and Education Proceedings.

Gaia Collaboration, 2016, "The Gaia mission", $A$ \& $A$ 595, A1.

Gaia Collaboration, 2018, "Gaia Data Release 2: Summary of the contents and survey properties". ArXiv e-prints.

Greaves, J., 2004, "New Northern hemisphere common proper-motion pairs", Monthly Notices of the Royal Astronomical Society, 355, Issue 2, 585-590, https://doi.org/10.1111/j.1365-2966.2004.08341.x
Las Cumbres Observatory Network https://lco.global/ observatory/sites/
Mason, B, 2018. Washington Double Star Catalog. Astronomy Department, United States Naval Observatory, http://ad.usno.navy.mil/proj/WDS/.
Sordiglioni, G. 2018. Stella Doppie Double Star Catalog, http://www.stelledoppie.it/index2.php? menu $=29$ \&iddoppia $=48579$.
Shaya, E.J \& Olling, R.P., 2011, "Very Wide Binaries and Other Comoving Stellar Companions: A Bayesian Analysis of the Hipparcos Catalogue", The Astrophysical Journal Supplement Series, 192:2 (17 pp).
Harshaw, R., 2014, "Another Statistical Tool for Evaluating Binary Stars", Journal of Double Star Observations, 10, 32.

Wenger et al., 2000. "The SIMBAD astronomical database", $A \& A S, 143: 9$.

## Journal of Double Star Observations

January 1, 2020
Volume 16, Number 1

## Editors

R. Kent Clark

Russ Genet
Richard Harshaw
Jo Johnson
Rod Mollise
Assistant Editors
Vera Wallen
Student Assistant Editor
Eric Weise
Advisory Editors
Brian D. Mason
William I. Hartkopf

Web Master<br>Michael Boleman

The Journal of Double Star Observations (ISSN 2572-4436) is an electronic journal published quarterly. Copies can be freely downloaded from http://www.jdso.org.

No part of this issue may be sold or used in commercial products without written permission of the Journal of Double Star Observations.
© 2020 Journal of Double Star Observations
Questions, comments, or submissions may be directed to rclark@southalabama.edu or to rmollise@bellsouth.net

The Journal of Double Star Observations (JDSO) publishes articles on any and all aspects of astronomy involving double and binary stars. The $J D S O$ is especially interested in observations made by amateur astronomers. Submitted articles announcing measurements, discoveries, or conclusions about double or binary stars may undergo a peer review. This means that a paper submitted by an amateur astronomer will be reviewed by other amateur astronomers doing similar work.

Submitted manuscripts must be original, unpublished material and written in English. They should contain an abstract and a short description or biography ( 2 or 3 sentences) of the author(s). For more information about format of submitted articles, please see our web site at http://www.jdso.org

Submissions should be made electronically via e-mail to rclark@southalabama.edu or to rmolise@bellsouth.net. Articles should be attached to the email in Microsoft Word, Word Perfect, Open Office, or text format. All images should be in jpg or fits format.


[^0]:    Table 1 (conclusion). Results for measured WDS objects
    3) SNR for A or B or both <20: Indicates that the Vmag measurement results might be a bit less precise than desired
    due to a low SNR value but this is already included in the calculation of the magnitude error range estimation
    4) Overlapping star disks: Indicates that the star disks overlap to the degree of an elongation and that the measure$B$ seems to be an optical double, Vmag thus likely too bright
    6) $\quad A$ is double itself
    7) $\quad B$ is double itself

[^1]:    * Epochs of new measures given in Besselian years as the average of the observations making up the measure.

