The Number of Binaries in the Sky Compared to a Random Distribution of Similar Stars

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Abstract: The likelihood of a pair of stars orbiting one another is investigated using statistics generated from the Gaia DR2 data[1]. Stars brighter than 11, 15, and 18 G band magnitude (Gmag) were searched for pairs with separations less than 60”. The Gaia DR2 stars’ positions were then randomized and again searched. The differences between pairs found in the real sky vs the randomized sky indicate that the likelihood that a pair is physical is inversely proportional to the pairs’ separation.

When two stars are close together in the sky, they are often listed as double stars. Whether a particular pair actually orbit one another or is simply in the same line of sight as seen from the earth has been debated for at least a century. Below is an excerpt from R. Aitken's 1918 book The Binary Stars[2].

> *The data consist of all visual double stars as bright as 0.9 B. D. magnitude which fall within the distance limits set by the following 'working' definition of a double star proposed by me in 1911:*
>
> *(1) Two stars shall be considered to constitute a double star when the apparent distance between them falls within the following limits:*
>
> *1" if the combined magnitude of the components is fainter than 11 .0*
>
> *3" if the combined magnitude of the components is fainter than 9.0 B. D.*
>
> *5" if the combined magnitude of the components lies between 6.0 and 9.0 B. D.*
>
> *10" if the combined magnitude of the components lies between 4.0 and 6.0 B. D.*
>
> *20" if the combined magnitude of the components lies between 2.0 and 4.0 B. D.*
>
> *40" if the combined magnitude of the components is brighter than 2.0 B. D.*
>
> *(2) Pairs which exceed these limits shall be entitled to the name double star only when it has been shown*
>
> *(a) that orbital motion exists;*
>
> *(b) that the two components have a well defined common proper motion, or proper motions of the 61 Cygni type;*
>
> *(c) that the parallax is decidedly greater than the average for stars of corresponding magnitude.*

Our knowledge of this has greatly improved since Aitken's time as the numbers and accuracies of proper motion, parallax, and radial velocity studies have increased significantly. There are, however, other ways to try to analyze this question.

In this paper, a statistical treatment of all of the Gaia DR2 stars brighter than 11.0 Gmag (1,240,319 stars), 15.0 Gmag (35,399,780 stars), and 18.0 Gmag (299,758,720 stars) was done using the programs that can be obtained from the SourceForge site(4). Each of these stars was searched in this way:

The area within a radius of 60” of the given star was searched for the star that was closest to that star and would be selected as the other member of the pair.

The pair found was tested for its commonality of distance, spatial velocity, and orbital velocity. If the parameters were within certain limits (see below), the pair was counted as a binary.

Once a star had been identified as a member of a pair, it was flagged as such and removed from the database of stars being searched.

These searches were then repeated using the same stars, with their positions randomized. Randomization of a star’s position consisted of moving the star an arc minute from its J2000 position in a random direction. The results of the 6 runs that searched for the closest star are shown here, with the number of pairs or binaries found on the vertical axis, and their separation in arcseconds on the horizontal axis. Note that the magenta line represents pairs and binaries found in the real

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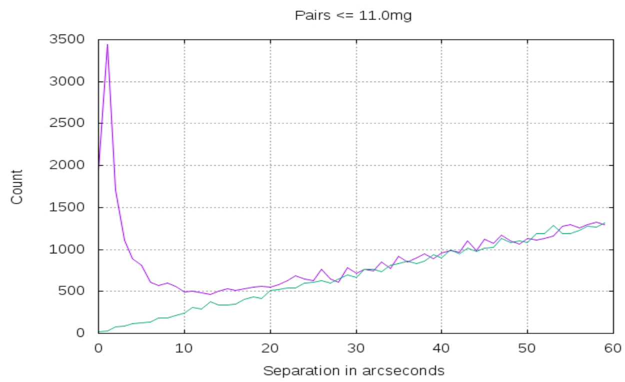

Figure 1

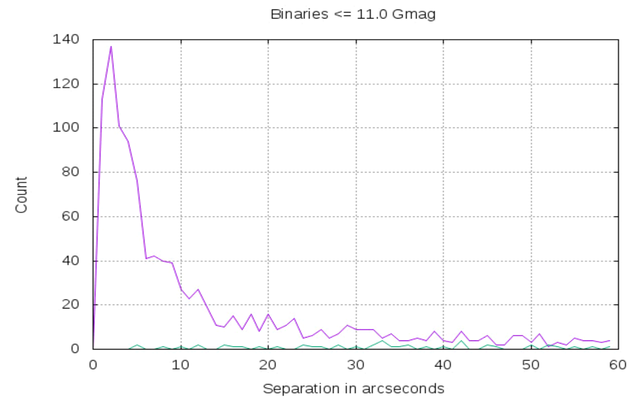

Figure 2

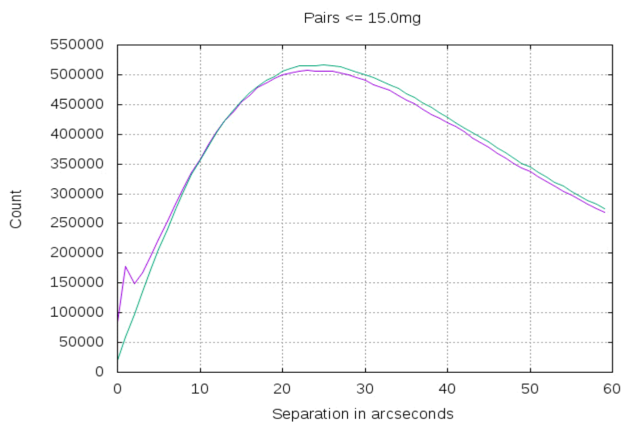

Figure 3

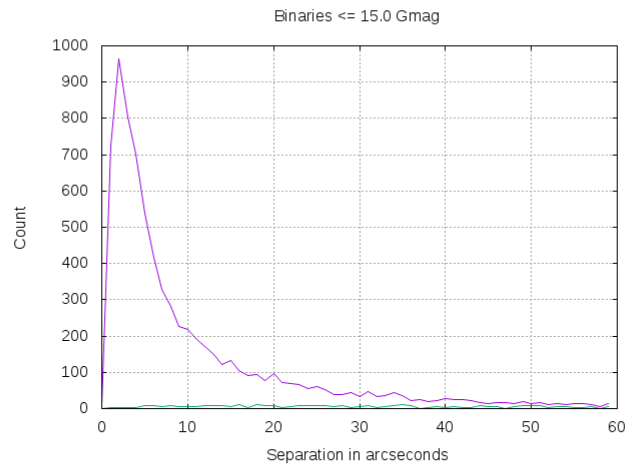

Figure 4

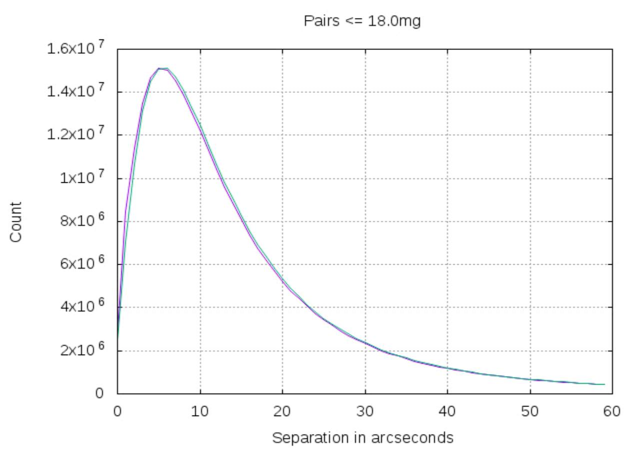

Figure 5

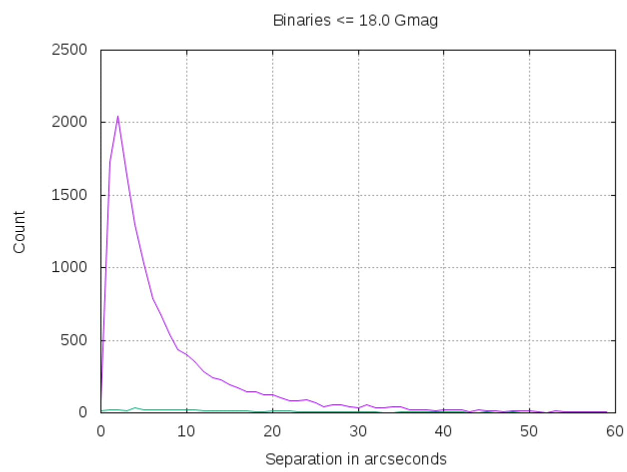

Figure 6

The Number of Binaries in the Sky Compared to a Random Distribution of Similar Stars

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sky, and the green line represents pairs and binaries found in the randomized sky. Comparing the real sky with the randomized sky gives an idea as to the number of actual binaries vs the number of optical pairs in the sky.

Pairs within 2.0 Gmag of One Another.

A majority of the pairs in the WDS were discovered visually. Pairs with a large brightness difference were either ignored or not seen, as the difficulty of resolving a close pair is a function of the difference in brightness of that pair. The Figures 1 - 6 do not consider this fact. Would the results be different if the stars were required to be close in brightness as well as the other criteria? The programs were revised so as to exclude pairs that were more than 2.0 Gmag different in brightness. Figures 7 - 12 are the results for the 11.0 Gmag, 15.0 Gmag, and 18.0 Gmag runs with pairs required to be within 2.0 Gmag of one another. These six plots show the results for the closest pairs found within an arcminute of a given star.

The number of pairs and binaries is much reduced by limiting the brightness differential to 2 Gmag, but the overall shape of the curves is unchanged.

Average Distances of Binaries with Respect to Their Separation.

When the program found that a pair was a possible binary, the distance to the pair, that is the average distance (in light years) of both members from the sun, was computed and saved in its arc second separation bin. When the run ended, the average distance of all binaries found in that separation bin was computed. Please note that Figures 13 - 18 do not show the number of binaries found in each separation bin. Refer to Figures 7 - 12 for that data, although in most cases, the spikes on Figures 13 - 18 only represent one or two stars, so making any statistical trends based on this data inaccurate, with the possible exception of the 18 Gmag data (Figures 17 and 18).

Discussion

The number of pairs less than 1 to 2 arc seconds apart is under counted as it is close to the resolution on the Gaia telescope, especially in a crowded star field.

In order for a pair to be considered for binary membership, both stars needed to have a Gaia DR2 parallax recorded for them. Over 98% of all stars brighter than 18.0 Gmag did.

The chance of a pair of stars being close to one another in the sky is a function of the number of stars in the sky. Note that there are about 30 times more stars brighter than 15 Gmag than 11 Gmag, and about 240 times as many 18 Gmag stars than 11 Gmag. This is reflected in the above plots, which show the ratio of the number of pairs and binaries found in the actual sky compared to those found in the randomized sky decreases as the number of stars searched increases. Note that there are so many pairs found in the 18 Gmag data that most stars are found to be part of a pair, and only when they are checked for binary membership do we see a difference between the real and randomized sky.

The criteria for determining if a pair was mutually orbiting one another were as follows. The ratio of the star's spatial velocities over their estimated orbital velocities needed to be within a factor of 10 of each other and their parallaxes needed to indicate that the stars' distances were within a light year of one another. This distance was chosen as the average distance of stars in the Milky Way is about 5 light years [3]. Two solar mass stars in a circular orbit at this distance take about 6 million years, and after a few orbits, there is a significant chance that the pair will encounter other stars that will disrupt their orbit, so a light year was thought to be a good limit on a stable binaries' orbital diameter.

The stars' orbital velocities were calculated as described by Rica[4]:

$$M = L^{(1/3.5)}$$

$$m = l^{(1/3.5)}$$

$$V = \sqrt{\frac{1.9891 \times 10^{30}\, G\left(M + m\right)}{dr}}$$

Where:

- dr is the average distance to the stars multiplied by their apparent separation in radians.
- L is the first star's luminosity, in solar units.
- l is the second star's luminosity, in solar units.
- M is the mass of the first star.
- m is the mass of the second star.
- V is the orbital velocity, in m/s.
- G is the gravitational constant, 6.67408×10^{-11}.
- The mass of the sun is 1.989×10^{30} kg.

The stars are assumed to be main sequence stars, making the relationship approximately valid.

Note that this simple calculation makes the unlikely assumptions that the orbits are circular, perpendicular to the plane of the sky, with their velocity vectors pointing at the sun. The generous limits of a light year's separation and an order of magnitude between this orbital velocity and the observed spatial velocity are used to

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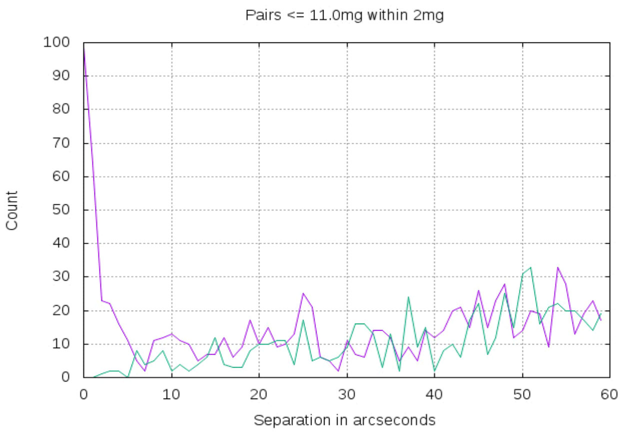
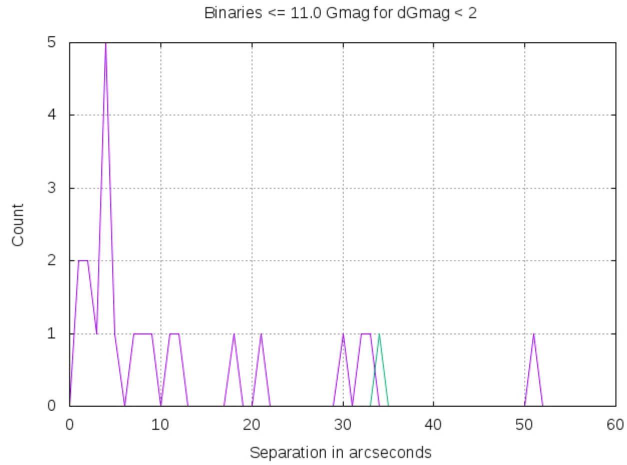


Figure 7.



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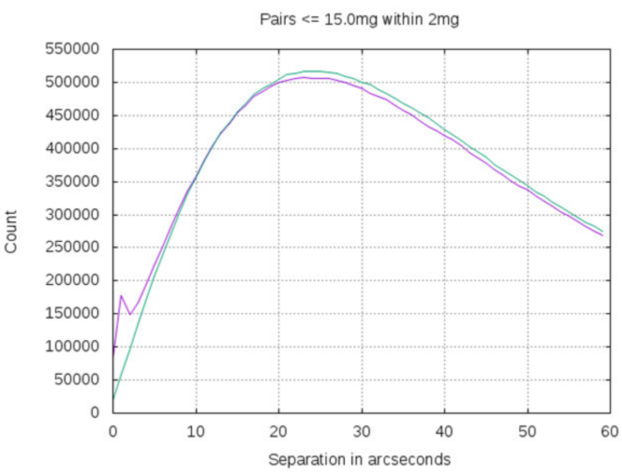

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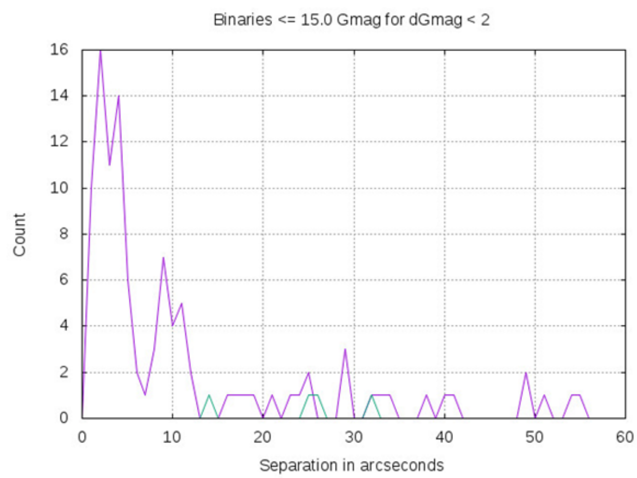

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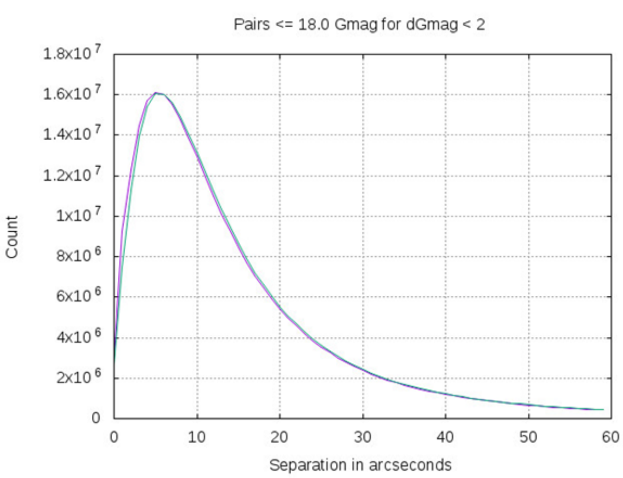

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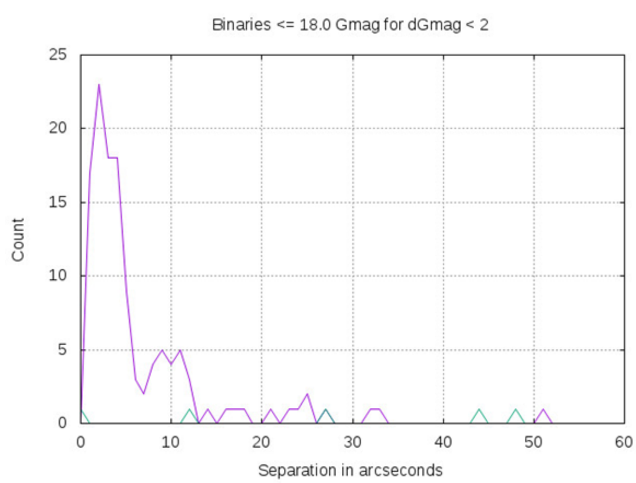

Figure 12.

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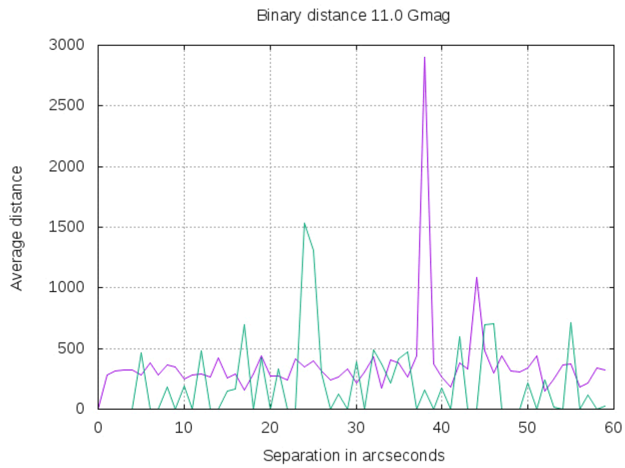
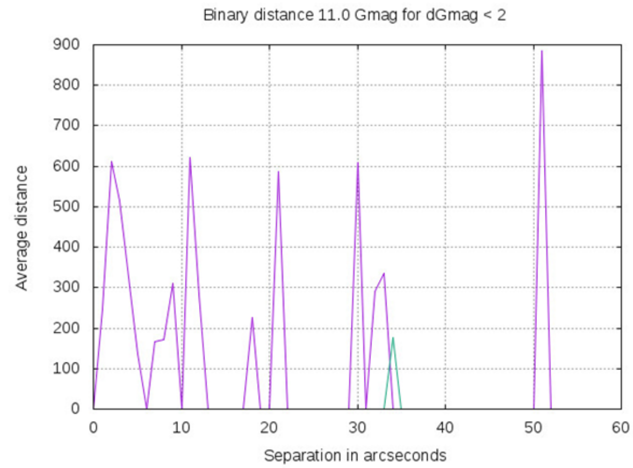


Figure 13.



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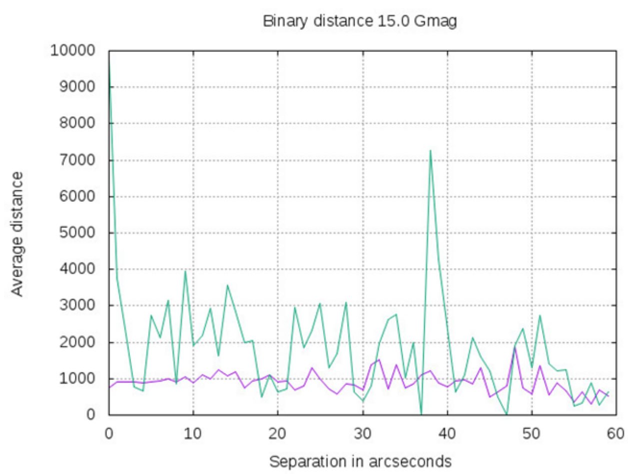

Figure 15.

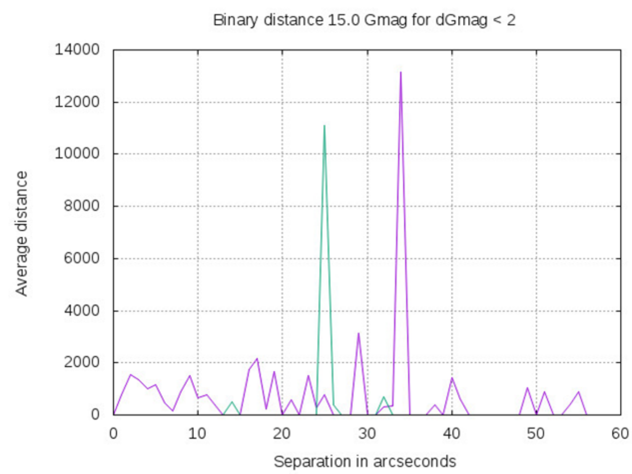

Figure 16.

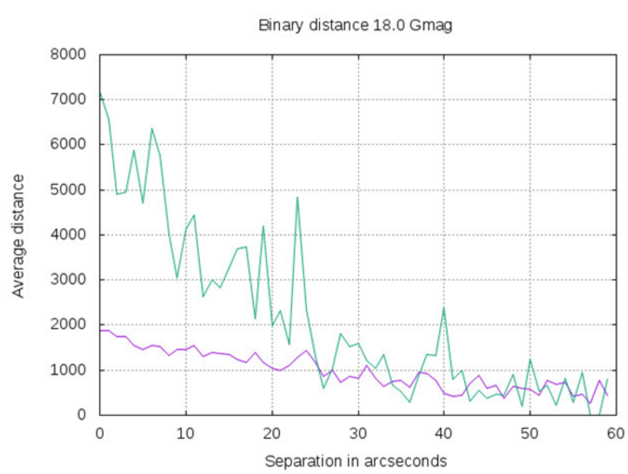

Figure 17.

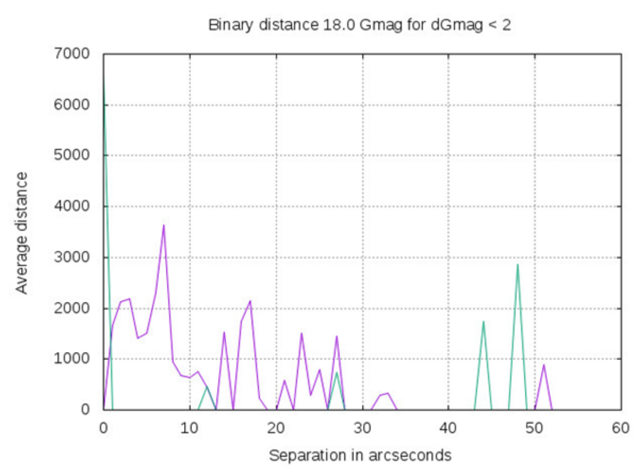

Figure 18.

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offset the assumptions made about the orbits. Even with these wide margins, the number of binaries found drops markedly when compared to the number of pairs found.

Initially, it was thought that if the parallax uncertainty limits of the two stars in a pair overlapped, then the pair would be considered to be at approximately the same distance from the sun. This method was abandoned when it was found that over 99% of pairs met this criterion. It was replaced by the requirement that the parallaxes of the two stars needed to indicate that they were within a light year of one another.

The results for the 18.0 Gmag runs simply indicate that virtually all stars have another star 18 Gmag or brighter within an arc minute of them. This was somewhat true of the 15.0 Gmag runs as well. These often optical pairs were minimized in the 11.0 Gmag run. In all runs, however, the above results indicate that the number of binaries found favors the nearer pairs.

Conclusion

Aitken's 1911 guesses were perhaps a bit too conservative, but the basic idea, the closer a given pair, the more likely it is to be a binary, is sound.

While careful measurements over time of the position angle, separation, radial velocities, and parallaxes of a pair are required to determine if the pair is indeed a binary, an initial guess can be made simply from the pair's separation. While using these graphs to determine if a pair is a binary is only a rough estimate of a given pair's binary nature, they might well be useful in selecting which pairs should be chosen for further study, much like Aitken's 1918 criteria were.

Acknowledgements

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References

[1]The Gaia DR2 data can be downloaded here: http://cdn.gea.esac.esa.int/Gaia/gdr2/gaia_source/csv/

[2] R. G. Aitken, The Binary Stars, 1918, The University of California.

[3] http://boojum.as.arizona.edu/~jill/EPO/Stars/galaxy.html

[3] F. M. Rica, "Determining the Nature of a Double Star: The Law of Conservation of Energy and the Orbital Velocity", *Journal of Double Star Observations*, **7** (4), 254, 2011.

[4] The C programs used to create, search and randomize the Gaia RD2 data can be downloaded here: http://sourceforge.net/p/realvsrandomdoublestarcount. The plots were created using gnuplot: http://www.gnuplot.info/