The Southern Double Stars of Carl Rümker III: Quantified Probability of Boundedness and Preliminary Grade 5 Orbits for Some Very Long Period Doubles

Roderick R. Letchford[1]
Graeme L. White[2]
Allan D. Ernest[3]

1. Vianney College Seminary, NSW Australia, rodvianney@yahoo.com.au
2. Astrophysics Group, Computational Engineering and Science Research Centre, University of Southern Queensland, Toowoomba, Australia QLD 4350, graemewhiteau@gmail.com
3. Charles Sturt University, NSW Australia, aernest@csu.edu.au

Abstract: All of the Southern double star pairs published by Carl Rümker in 1832 may be classified as very wide and measures have been obtained for only a small fraction of a single orbit. We attempt to find orbital elements of some Rümker doubles, and give an objective probability as to their binary status. Of the 28 pairs discovered by Rumker, we show some evidence for orbital (bound) movement for pairs RMK 4 and 8 and propose grade 5 orbits for them.

1. Introduction

In 1990 Charles E. Worley published an article with the rather provocative title "Is this Orbit really Necessary?" (Worley, 1990). It followed an earlier article by Willem H. van den Bos (1962) with the same title and argument. Both authors criticised the publication of orbits that were either unreliable or useless. Unreliable because they were based on too few observations over too short an arc, or useless because the quality of recalculated orbits rarely increases.

Notwithstanding the opinions of these eminent astronomers, we wish to offer a justification and a new simple method for attempting to find orbital parameters of wide binaries that have been observed over a short arc.

The justification is easily stated. If the residuals from an elliptical orbit are smaller than those obtained from a presumed rectilinear motion then it can be said that it is more likely than not that the pair are true binaries rather than optical doubles. Distinguishing between optical and binary doubles has important ramifications for many aspects of astrophysics especially stellar formation models (Guinan, Harmanec, & Hartkopf, 2007). Indeed there has been renewed interest in wide binary systems because of their potential to distinguish between the mainstream-accepted WIMP-based hypothesis of dark matter, and Modified Newtonian Dynamics (Chanamé & Gould, 2004; Longhitano, Binggeli, & Zejda, 2010; Németh et al., 2016).

Various methods have been proposed to extract orbital information from short arcs. Two in particular are used regularly in publications.

The first is the so-called "Kovole" method which is the Kowalski method modified for short-arcs (see for example, Catović, Olević, & Hartkopf (1992), Catović & Olević (1995), and Olević & Cvetković (2004, 2005)). The second is a method based on the more well-known Thiele-Innes-Van-den-Bos method (see for example, Docobo (1985) and Docobo and Hestroffer (2012)). Neither of these two methods are satisfactory when applied to a single very short arc and we outline our method for optimising an elliptical fit to very short arcs in the next section.

We also present here Paper III of a study of the double stars identified nearly two hundred years ago at Sir Thomas Brisbane's observatory at Parramatta, Australia by Carl Rümker (see Paper I, Letchford, White, and Ernest 2017, and Paper II, 2018, and references therein). In the present study we present the orbital elements of five pairs from Rümker's double star list. Section 2 is our method for the computation of the elements of orbital motion, Section 3 discusses the validity of the claim that the stars are in a binary system, and Section 4 presents the elements and the Appendix the diagrammatic results for the five pairs.

2. Computation of Orbital Parameters for Very Short Arcs

As with all techniques of orbit fitting to the astrometric observations of double stars, we start with the fundamental step of fitting of an ellipse to the projection of the orbit onto the plane of the sky (the *apparent orbit*). Standared orbital elements follow from the satisfactory fitting of the ellipse.

The fundamental problem in finding an optimum ellipse from a small fraction of a potential ellipse is that the family of possible solutions can be very large with large variations of measured parameters and large uncertainties. We maintain however, that this difficulty can be mostly, but not completely, overcome by introducing into the calculations as many constraints on the solutions as possible.

Apart from making the initial assumption that the doubles are possibly binaries, we have identified and incorporated into our calculations the following constraints:

a) The orbit (defined by the motion of the secondary stars relative to the primary) must cross to all sides of the primary star. That is, in a projection on the sky, the N-S and E-W axes will be crossed in (only) two places for each axis, one either side of the origin (0,0).
b) Because the secondary is slow moving the positions obtained from the space-based HIPPARCOS and GAIA missions will be relatively close together. Thus the rectilinear line passing through these two positions (see Paper II, Letchford, White, & Ernest, (2018)) can be considered as approximating a tangential to the orbit. Therefore the elliptical orbit will only exist on the side of this line containing the primary. The space on the other side of the rectilinear line can be eliminated from consideration.
c) The fixed primary star must appear inside the apparent relative obit of the secondary.
d) In most double star work the ultimate aim is to obtain direct measures of the stellar masses using Kepler's third law,

$$M_1 + M_2 = \frac{a_{"}^3}{\Pi_{"}^3 P_{yrs}^2}$$

where M_1 and M_2 are the masses of the primary and secondary in solar masses, respectively; a is the semi-major axis of the real apparent orbit in arcseconds; Π is the parallax of the system (the primary star) in arcseconds; and P is the orbital period in years. Here we reverse the exercise and with the sum of the masses obtained from their spectral types, and their parallax from an online catalogue (see Paper I and the reference to ASCC 2.5 in that Paper) provide an additional constraint on P and a.
e) The calculated positions at 1991.25 and 2015.0 must be within the uncertainty ellipses of the HIPPARCOS and GAIA positions respectively, and close as possible to all other (historic) measures considered. All historic observational data are included and unweighted. No outliers were discarded.

A Monte Carlo approach was used to incorporate four random positions along the axes into the observed positional data (constraints a) and b)). An orbit was obtained using the Kowalski method along with Kepler's equation. The orbit was then tested to see if it fulfilled constraints c), d) and e). If it did, the resulting residual and orbital elements were calculated and binned and compared with subsequent possible orbits. We limited the outer boundary of the random axial points to 4 minutes from the fixed star, noting that in the 6th Orbit catalog, only two pairs have a semi-major axis greater than 180 arcseconds and a period measured in centuries. This approach was repeated until 16 orbits were recovered whose residuals (sum of the squares of distances between the observed and calculated positions) varied by no more than 0.04 square arcseconds from the lowest residual ellipse. The best orbit is the orbit that produces the lowest residual.

The elements of the best orbit are accepted and given in Table 1. They have been rounded to their significant figures on the basis of their uncertainties.

As the best orbit was not the average of the resulting orbits, but the one with the lowest residual, we estimated the uncertainty of the orbital parameters of the best orbit using the square root of its variance:

$$\sigma_{OE} = \pm\sqrt{\left(OE - OE_{mean}\right)^2} = \left|OE - OE_{mean}\right|$$

3. Orbits or Rectilinear Motion – Bound or Unbound Systems

We return to the differentiation of the orbital motion displayed by a bound binary system and the physically unbounded motion that displays straight (rectilinear) relative motion.

We assert, as above, that if the residuals from an elliptical orbit are smaller than those obtained from a rectilinear motion then the pair are more likely to be a binary rather than an optical double. In paper II in this series (Letchford, White and Ernest, 2018) the rectilinear motion of 14 Rümker pairs was presented. In that work, the rectilinear motion was computed based on the two precise astrometric positions at ICRS and epoch 1991.25 and 2015.0 respectively obtained from the HIPPARCOS and GAIA space-based missions, where the precision in position is typically 5 milli-arcseconds for HIPPARCOS and 0.4 milliarcseconds for GAIA in the pairs under consideration. Ground-based historic observations extending over ~200 years are ignored in that analysis as the associated uncertainties are estimated to be ~0.3 to ~0.5 arcseconds, two orders of magnitude greater than those of the HIPPARCOS and GAIA uncertainties.

The published rectilinear models of the Rümker pairs in paper II are therefore not compatible with computed orbits presented here. To compare like-with-like, we recalculated the rectilinear motion of the pairs in Table 1 using a Monte Carlo method, similar to that used for the computation of the orbit parameters, so that the rectilinear fit results in the lowest residual but the rectilinear motion passes within the formal uncertainties of the HIPPARCOS and GAIA positions. To keep the calculations as objective as possible, we have not weighted nor discarded any historic data. This reanalysis results in a rectilinear fit which is close to but not identical to that of Paper II.

The like-with-like comparison of the residuals resulting from the orbit computation, and the residuals resulting from the rectilinear fit, are presented in Table 1, last two columns, and the percentage probability that the pair represent a gravitationally bound pair in an elliptical orbit is given by:

$$\exp\left[\frac{orbit\ residual}{linear\ residual}\ln(0.5)\right]\times 100\%$$

which is defined such that if, for example, the orbital residual equals the rectilinear residual (percentage probability = 50%), the binary nature of the pair (or their possible rectilinear nature) remains unknown. Values greater than 50% (orbital residual < linear residual) imply the increasing validity of a bound pair, while values less than 50% (orbital residual > linear residual) imply an increasing probability of an unbound system.

4. Results

The orbital elements, their uncertainties, along with the resulting residual of orbital and (recalculated) rectilinear motion are presented in Table 1. The associated plots are in the Appendix.

5. Discussion

From Paper II, only RMK 1, 3, 4, 5, 6, 8, 10, 11, 12, 17, 20, 25, 27 and 28 had both full HIPPARCOS and GAIA DR1 data. Of those only 1, 3, 4, 6, 8, yielded potential orbits given the constraints of Sections 2 and 3. We conclude from this that either the remaining pairs are optical doubles or that the secondary is moving too slowly with respect to the primary for even our brute-force method to detect (as yet) any curvature in their astrometric motion.

For the five orbits reported here, the orbital period are all long, ranging from 2400 years up to one million years. The average period is some 330000 years, and the formal errors are, on average 60% of the period. The largest semi-major axis is 10 ± 5 arcminutes for RMK 6.

RMK 1 (WDS 00524-6930). The probability of this pair being a binary system is ~ 49%. This system is unlikely to be binary, and further accurate observations are warranted. Figures 1 – 5 in the Appendix.

Table 1: Orbital Elements. All orbital elements have been rounded to their significant figures on the basis of their uncertainties. Residuals rounded to the nearest 10th arcsecond squared.

RMK	P yrs +/-	a " +/-	i ° +/-	Ω ° +/-	T yr +/-	e +/-	ω ° +/-	Orbital Residual "^{2} +/-	Linear Residual "^{2}
1	580000	170	66.4	14.1	6100	0.74	105	14.5	14.2
	40000	11	0.5	0.9	800	0.02	4	0.00	
3	26000	14	105	55	4900	0.8	210	8.8	8.7
	9000	3	2	13	900	0.3	40	0.01	
4	2400	8.0	70	54.9	1900	0.22	200	7.2	13.0
	400	1.1	2	0.7	110	0.05	30	0.01	
6	1000000	600	85	146	10000	0.9	340	23.7	23.7
	700000	300	1	2	4000	0.3	120	0.03	
8	30000	21	73	70.4	1950	0.8	350	1.2	1.34
	9000	7	3	0.7	150	0.3	30	0.00	

RMK 3 (WDS 04177-6315). Probability of this pair being a binary system is ~ 49%. This system is unlikely to be binary, and further observations are warranted. Figures 6 - 10 in the Appendix.

RMK 4 (WDS 04242-5704). Probability of this pair being a binary system is ~ 68%. This system is likely to be binary, and confirmation observations are needed. Figures 11 – 15 in the Appendix.

RMK 6 (WDS 07204-5219) Probability of this pair being a binary system is ~ 49%. This system is unlikely to be binary, and further observations are warranted. Figures 16 – 20 in the Appendix.

RMK 8 (WDS 08153-6255) Probability of this pair being a binary system is ~ 53%. This system is likely to be binary, and further confirmation observations are needed. Figures 21 - 25 in the Appendix.

5.1 Grading of These Orbits.

The Sixth Catalog of Orbits of Visual Binary Stars†, http://ad.usno.navy.mil/wds/orb6/orb6text.html#grading, maintained by the USNO, grades each orbit; Grade 1 being for a definitive orbit and Grade 5 being for an indeterminate orbit. The elements of Grade 5 orbits "may not even be approximately correct". Grade 5 orbits usually have short arcs with little curvature. We classify our orbits of the five RMK pairs (RMK 1, 3, 4, 6 and 8) as Grade 5 orbits, and claim validity only for the orbits of RMK 4 and 8.

6. Conclusion

Our objective method of quantifying the binary status of very wide slow moving binaries suggests that most of the Rümker doubles are unbound optical doubles. RMK 1, 3, and 6 are also likely to be optical doubles but further observation is warranted as the probability of them being a binary pair is about 50%. However, RMK 4, and to a lesser extent RMK 8, are likely to be binaries as this paper has detected an elliptical trend in the astrometric observations which leads to a probability of a bound pair of 68% and 53% respectively. As above, we classify the orbits of RMK 1, 3, 4, and 6 as Grade 5 orbits and claim validity only for the orbits of RMK 4 and 8. Given the importance of binarity in stellar formation models and possibly in the detection of dark matter, this outcome fully justifies the value of finding orbits for very short arc doubles.

Acknowledgements

The following were used in this research:

- The HIPPARCOS Catalogue (The Hipparcos and Tycho Catalogues (ESA 1997)) from VizieR, http://vizier.u-strasbg.fr/viz-bin/VizieR-3?-source=I/239/h_dm_com
- The GAIA Catalogue (Gaia DR1 (Gaia Collaboration, 2016)) from VizieR, http://vizier.u-strasbg.fr/viz-bin/VizieR-3?-source=I/337/gaia
- The *Washington Double Star Catalog* maintained by the USNO.

Spectral Types were obtained from Paper I, and the corresponding masses were obtained from Landon Curt Noll's website, http://www.isthe.com/chongo/tech/astro/HR-temp-mass-table-byhrclass.html .

References

Catović, Z., & Olević, D., 1995, "Visual Binary Orbit Determination. One Possible Approach", *Bulletin Astronomique de Belgrade*, **195**, 65–70. Retrieved from http://adsabs.harvard.edu/abs/1995BABel.152...65C.

Catović, Z., Olević, D., & Hartkopf, W. I., 1992, "Visual Binary Orbit Calculation with Help of a PC: One Possible Approach", *IAU Colloq. 135: Complementary Approaches to Double and Multiple Star Research*, H. A. McAlister, ed., **32**, 217–219. Retrieved from http://adsabs.harvard.edu/abs/1992ASPC...32..217C.

Chanamé, J., & Gould, A., 2004, "Disk and Halo Wide Binaries from the Revised Luyten Catalog: Probes of Star Formation and MACHO Dark Matter", *The Astrophysical Journal*, **601**, 289–310. Retrieved from http://adsabs.harvard.edu/abs/2004ApJ...601..289C.

Docobo, J. A., 1985, "On the analytic calculation of visual double star orbits", *Celestial Mechanics*, **36**, 143–153. Retrieved from http://adsabs.harvard.edu/abs/1985CeMec..36..143D.

Docobo, J. A., & Hestroffer, D., 2012, "The use of Docobo's analytic method for calculating visual double star orbits", *Orbital Couples: Pas de Deux in the Solar System and the Milky Way*, F. Arenou, ed., Retrieved from http://adsabs.harvard.edu/abs/2012ocpd.conf..119D.

Guinan, E. F., Harmanec, P., & Hartkopf, W. I., 2007, "Introduction & Overview to Symposium 240: Binary Stars as Critical Tools and Tests in Contemporary Astrophysics", *IAU Symposium 240: Binary Stars as Critical Tools and Tests in Contemporary Astrophysics*, W. I. Hartkopf, E. F. Guinan, and P. Harmanec, eds., **240**, 5–16, Cambridge, UK, Cambridge University Press. Retrieved from http://adsabs.harvard.edu/abs/2007IAUS..240....5G.

† http://ad.usno.navy.mil/wds/orb6/orb6text.html#grading

Letchford, R. R., White, G. L., & Ernest, A. D., 2017, "The Southern Double Stars of Carl Rümker I: History, Identification, Accuracy", *JDSO*, **13**(2), 220–232, Retrieved from http://www.jdso.org/volume13/number2/Letchford_220-232.pdf.

Letchford, R. R., White, G. L., & Ernest, A. D., 2018, "The Southern Double Stars of Carl Rümker II: Their Relative Rectilinear Motion", *JDSO*, **14**(2), 208–222. Retrieved from http://www.jdso.org/volume14/number2/Letchford_208_222.pdf.

Longhitano, M., Binggeli, B., & Zejda, M., 2010, "The Widest Binary Stars: A Statistical Approach", *ASP Conference Proceedings 435: Binaries - Key to Comprehension of the Universe,* A. Prša and M. Zejda, eds., **435**, 67–70. Retrieved from http://adsabs.harvard.edu/abs/2010ASPC..435...67L.

Németh, P., Ziegerer, E., Irrgang, A., Geier, S., Fürst, F., Kupfer, T., & Heber, U., 2016, "An extremely fast halo hot subdwarf star in a wide binary system, *The Astrophysical Journal Letters*, **821**(L13), 1–7. Retrieved from http://adsabs.harvard.edu/abs/2016ApJ...821L..13N.

Olević, D., & Cvetković, Z., 2004, "Orbits of 10 interferometric binary systems calculated by using the improved Koval'skij method", *Astronomy and Astrophysics*, **415**, 259–264. Retrieved from http://adsabs.harvard.edu/abs/2004A&A...415..259O.

Olević, D., & Cvetković, Z., 2005, "Improved Koval'skij method and its new possibilities", *Publications of the Astronomical Society "Rudjer Boskovic,"* **5**, 237–240. Retrieved from http://adsabs.harvard.edu/abs/2005PASRB...5..237O.

van den Bos, W. H., 1962, "Is This Orbit Really Necessary?", *Publications of the Astronomical Society of the Pacific*, **74**(439), 297–301. Retrieved from http://adsabs.harvard.edu/abs/1962PASP...74..297V.

Worley, C. E., 1990, "Is this Orbit Really Necessary? (II)", *Errors, Bias and Uncertainties in Astronomy* (p. 419), Cambridge, UK, Cambridge University Press. Retrieved from http://adsabs.harvard.edu/abs/1990ebua.conf..419W.

Appendix

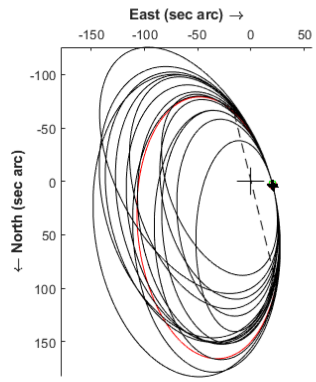

Figure 1: RMK 1 family of orbits. The best orbit, whose elements are in Table 1, is in red.

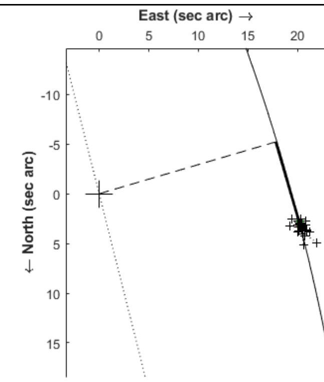

Figure 2: RMK 1 enlargement showing rectilinear line (thick line) and section of orbit (thin line).

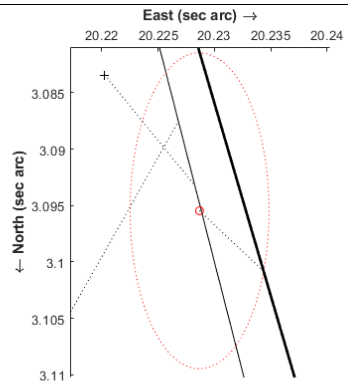

Figure 3: RMK 1 enlargement showing rectilinear line (thick line) and section of orbit (thin line) passing through the uncertainty space of the HIP position.

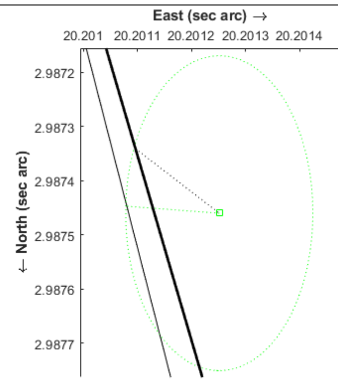

Figure 4: RMK 1 enlargement showing rectilinear line (thick line) and section of orbit (thin line) passing through the uncertainty space of the GAIA position.

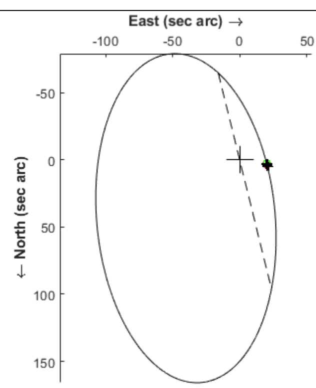

Figure 5: RMK 1 best obtained possible orbit. Orbital Elements in Table 1. Direction of motion is prograde (anti-clockwise).

The Southern Double Stars of Carl Rümker III: Quantified Probability of Boundedness ...

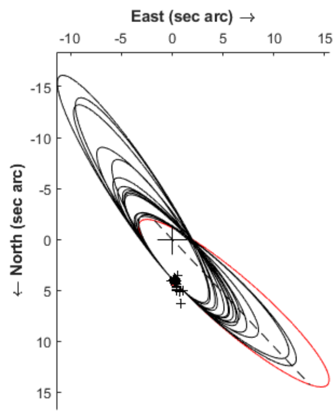

Figure 6. RMK 3 family of orbits. The best orbit, whose elements are in Table 1, is in red.

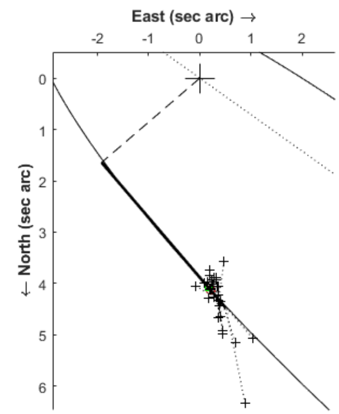

Figure 7. RMK 3 enlargement showing rectilinear line (thick line) and section of orbit (thin line).

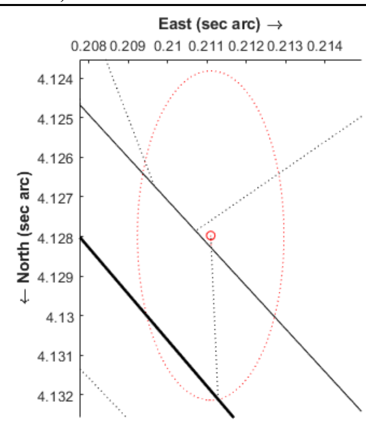

Figure 8. RMK 3 enlargement showing rectilinear line (thick line) and section of orbit (thin line) passing through the uncertainty space of the HIP position.

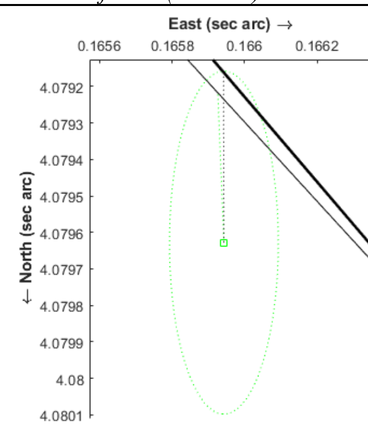

Figure 9. RMK 3 enlargement showing rectilinear line (thick line) and section of orbit (thin line) passing through the uncertainty space of the GAIA position.

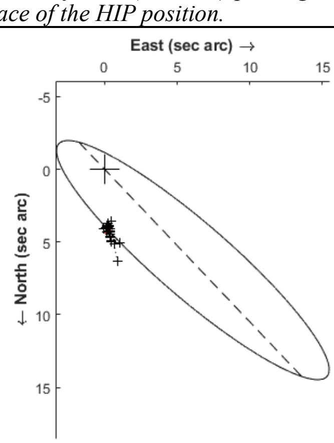

Figure 10. RMK 3 best obtained possible orbit. Orbital Elements in Table 1. Direction of motion is retrograde (clockwise).

The Southern Double Stars of Carl Rümker III: Quantified Probability of Boundedness ...

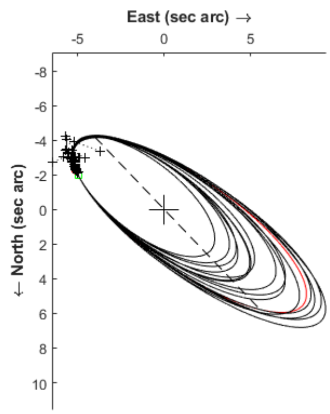

Figure 11. RMK 4 family of orbits. The best orbit, whose elements are in Table 1, is in red.

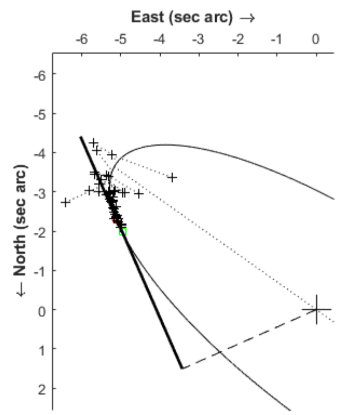

Figure 12. RMK 4 enlargement showing rectilinear line (thick line) and section of orbit (thin line).

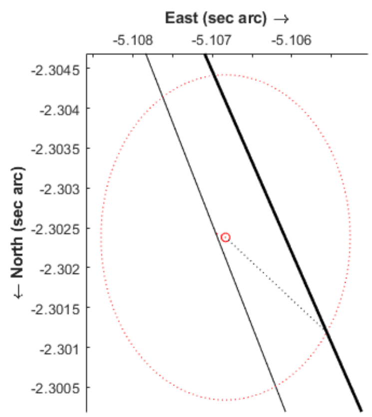

Figure 13. RMK 4 enlargement showing rectilinear line (thick line) and section of orbit (thin line) passing through the uncertainty space of the HIP position.

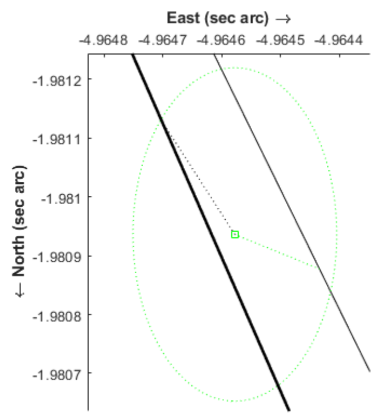

Figure 14. RMK 4 enlargement showing rectilinear line (thick line) and section of orbit (thin line) passing through the uncertainty space of the GAIA position.

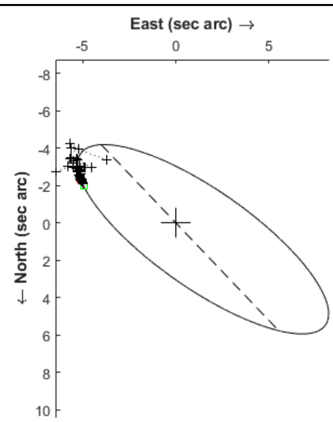

Figure 15. RMK 4 best obtained possible orbit. Orbital Elements in Table 1. Direction of motion is prograde (anticlockwise).

The Southern Double Stars of Carl Rümker III: Quantified Probability of Boundedness ...

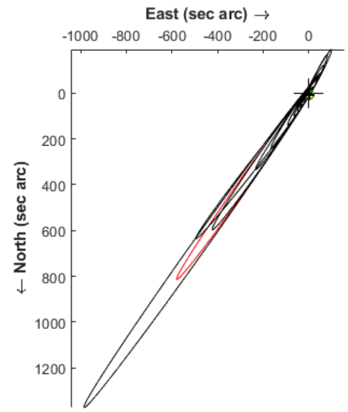

Figure 16. RMK 6 family of orbits. The best orbit, whose elements are in Table 1, is in red.

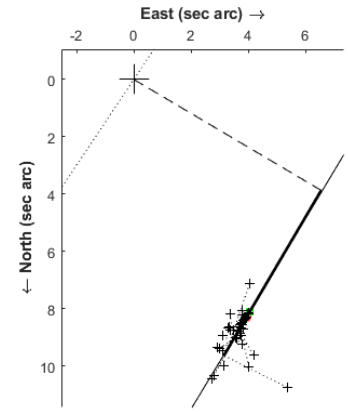

Figure 17. RMK 6 enlargement showing rectilinear line (thick line) and section of orbit (thin line).

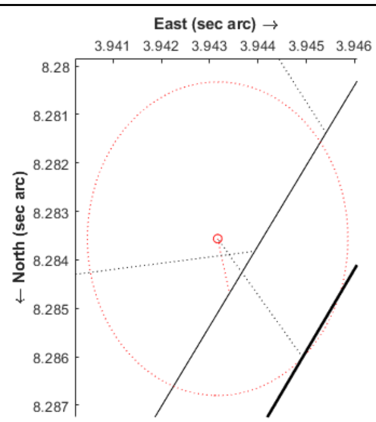

Figure 18. RMK 6 enlargement showing rectilinear line (thick line) and section of orbit (thin line) passing through the uncertainty space of the HIP position.

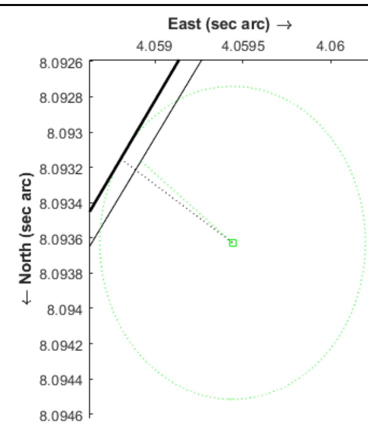

Figure 19. RMK 6 enlargement showing rectilinear line (thick line) and section of orbit (thin line) passing through the uncertainty space of the GAIA position.

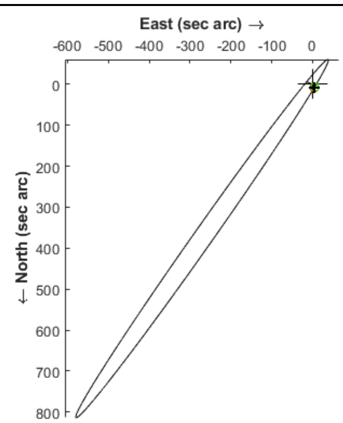

Figure 20. RMK 6 best obtained possible orbit. Orbital Elements in Table 1. Direction of motion is retrograde (clockwise).

The Southern Double Stars of Carl Rümker III: Quantified Probability of Boundedness ...

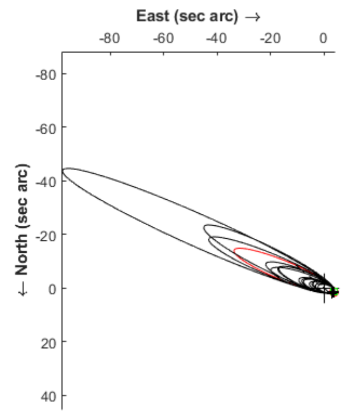

Figure 21. RMK 8 family of orbits. The best orbit, whose elements are in Table 1, is in red.

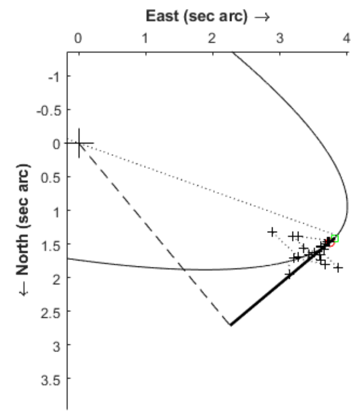

Figure 22. RMK 8 enlargement showing rectilinear line (thick line) and section of orbit (thin line).

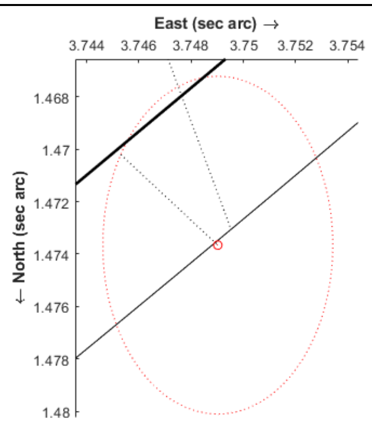

Figure 23. RMK 8 enlargement showing rectilinear line (thick line) and section of orbit (thin line) passing through the uncertainty space of the HIP position.

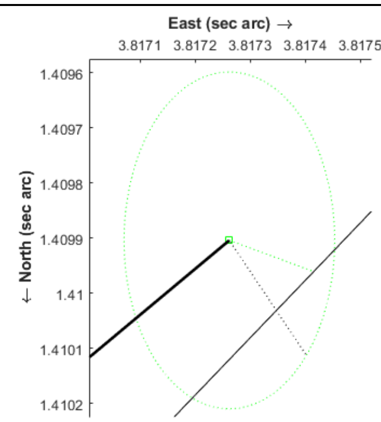

Figure 24. RMK 8 enlargement showing rectilinear line (thick line) and section of orbit (thin line) passing through the uncertainty space of the GAIA position.

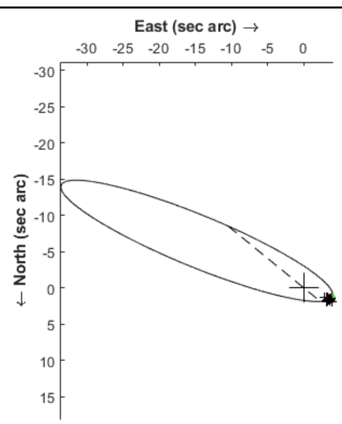

Figure 25. RMK 8 best obtained possible orbit. Orbital Elements in Table 1. Direction of motion is prograde (anti-clockwise).