The Southern Double Stars of Carl Rümker II: Their Relative Rectilinear Motion

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Abstract: A description of the relative rectilinear motion of double stars provides an important clue to the relationship of the components. The aim is to provide an objective method of obtaining Rectilinear Elements. We present a simplified method to calculate relative rectilinear motion, relying on the data obtained from the HIPPARCOS and GAIA DR1 missions, together with their uncertainties. As examples, we present the Rectilinear Elements of RMK 1, 3, 4, 5, 6, 8, 10, 11, 12, 17, 20, 25, 27, and 28.

1. Introduction

The description of the relative motion of the components of double stars in the form of rectilinear motion is the analysis of the (presumed) perceived linear motion of the secondary star relative to the primary. It is usually visualized as a straight line on a Cartesian plot where the primary star is the origin (0,0) position.

Such descriptions are an important tool in distinguishing between optical doubles and physical binaries, which in turn, have implications for stellar formation models. Relatively slow moving doubles may be either chance alignments of unrelated stars or very long period bound pairs. A comparison of the best-fit rectilinear motion and curved orbital motion should result in a clear distinction between these two types, since it is the variations from linearity that allows a sensitive identification of a Keplerian system.

We present here a portion of a continuing study of the double stars identified nearly two hundred years ago at Sir Thomas Brisbane's observatory at Parramatta, Australia (see paper 1, Letchford, White, and Ernest 2017, and references therein). In this study, and this paper, we present the rectilinear elements of pairs from the double star list of Rümker (Rümker 1832). Section 2 is our revised method for the computation of the elements of rectilear motion, and Section 3 presents the elements and the diagrammatic results for 14 of the 28 Rümker pairs.

2. Derivation of Rectilinear Elements

Rectilinear motion has been studied for many double star pairs, and a *Catalog of Rectilinear Elements* (CORE) is maintained by the USNO[†]. Utilizing the USNO method as a starting point, we see several ways that the CORE can be improved. Our improvements are designed to address:

1. The present conversion from polar to Cartesian coordinates is non-standard resulting in a plot that is East-West/North-South reversed relative to adopted orientation.
2. The quoted uncertainties for at least one of the computed parameters appear to be underestimated.
3. The weights associated with the individual observations are subjective.
4. The method used to arrive at the *Rectilinear Elements* (RE) is not stated in easily reproducible form.

We therefore propose an alternate procedure for rectilinear motion that eliminates each of the above inadequacies by (i) applying the convention in converting from polar to cartesian coordinates: $y = \rho \sin\theta$ and

† http://ad.usno.navy.mil/wds/lin1/lelements.html

$x = \rho\cos\theta$ where ρ is the separation of the primary and the secondary in arcseconds and θ is the precessed (to Equinox J2000.0) position angle, in degrees, initially given at the equinox of date (epoch), (ii) the calculation of the uncertainties following the standard definition,

$$\sigma_{f(x_i)} = \pm\sqrt{\sum\left(\frac{\partial f(x_i)}{\partial x_i}\sigma_{x_i}\right)^2}$$

and (iii) further modifications as discussed in section 2.1 below.

2.1 Adoption of Space-Based Data

The rectilinear elements in the CORE are the result of assigning weights to each observation (which, despite the best of efforts, always involves an element of subjectiveness), and then calculating the line of best fit based on historical observations. Instead we propose to avoid altogether the need for weighting by using only the positions observed by the HIPPARCOS (HIP) and GAIA (GAIA) space missions. These two positions offer uncertainties far smaller than is currently available with terrestrial measurements. HIPPARCOS positional uncertainties are approximately 5 milliarcseconds (mas) and GAIA are 0.4 mas for the RMK stars in this paper. The line of motion can then be fitted to the two-point astrometric positions in an entirely objective fashion. This utilizes the most powerful data available for astrometric work; relegating the other ground-based and historic measures to a secondary "supportive" role.

The adoption of the HIPPARCOS positions for the individual stars comes, however, with a caveat that the HIPPARCOS mission had technical difficulties handling double stars within specific seperation and magnitude limits. Details of the limitations of the HIPPARCOS mission's handling of double stars are given in Lindegren et al. (1997), and analysis of data from that mission must be treated in light of that paper.

We give here in section 2 full derivations of all elements and their uncertainties.

2.2 Input Data

To compute the CORE elements, the following data are needed:

- The Right Ascension (RA) and Declination (DE) of the primary and secondary (1 and 2, respectively) stars from HIPPARCOS, together with their uncertainties.
- The Right Ascension (RA) and Declination (DE) of the primary and secondary (1 and 2, respectively) stars from GAIA DR1 (Gaia Data Release 1) together with their uncertainties.

The above data for the Rümker doubles in this paper are presented in Table 1.

HIPPARCOS and GAIA positions are both given in the International Celestial Reference System (ICRS), with the epoch of HIPPARCOS set at J1991.25 and those of GAIA DR1 at J2015.0. Alignment to the ICRS at epoch J1991.25 is estimated to be within 0.6 mas (Hilton and Hohenkerk, 2004). This is smaller than the uncertainties of positions of our RMK stars in the HIPPARCOS catalogue, and less than those associated with any transformation required for the ground-based observations, and so the differences in the definitions of ICRS can be safely ignored.

2.3 Computation of Rectilinear Elements

Let:

ΔRA	Difference in RA between the primary and secondary
ΔDE	Difference in DE between the primary and secondary
$RA1$	RA of primary
$DE1$	DE of primary
$RA2$	RA of secondary
$DE2$	DE of secondary
θ	Position angle
ρ	Separation
xh	x cartesian coordinate of HIP relative position of secondary
yh	y cartesian coordinate of HIP relative position of secondary
xg	x cartesian coordinate of GAIA relative position of secondary
yg	y cartesian coordinate of GAIA relative position of secondary
σ	Uncertainty of quantity

And so:

$$\Delta RA^{rad} = \left(RA2^{rad} - RA1^{rad}\right)\cos\left(DE1^{rad}\right)$$

$$\Delta DE^{rad} = DE2^{rad} - DE1^{rad}$$

Therefore, the position angle is:

$$\vartheta^o = \begin{cases} \frac{180}{\pi}\arctan 2\left[\Delta RA^{rad}, \Delta DE^{rad}\right] \\ \vartheta^o + 360^o, \text{ if } \vartheta^o < 0^o \\ \vartheta^o - 360^o, \text{ if } \vartheta^o > 360^o \end{cases}$$

and the separation is

$$\rho" = \frac{360.180}{\pi}\sqrt{\left(\Delta RA^{rad}\right)^2 + \left(\Delta DE^{rad}\right)^2}$$

(Text continues on page 212)

Table 1: Data from the HIP (ICRS, epoch J1991.25) and GAIA DR1 (ICRS, epoch J2015.0) catalogues

RMK	HIP RA °	HIP DE °	HIP pmRA mas/yr	HIP pmDE mas/yr	GAIA RA °	GAIA DE °	GAIA pmRA mas/yr	GAIA pmDE mas/yr
	± mas	± mas	± mas/yr	± mas/yr	± mas	± mas	± mas/yr	± mas/yr
1 A	13.10213861	-69.50359643	3.81	-68.87	13.10221204	-69.50404326	4.040	-67.635
	1.73	1.78	1.84	2.14	0.288	0.161	0.115	0.111
1 B	13.11818630	-69.50273658	-2.47	-59.75	13.11823831	-69.50321341	3.265	-72.464
	15.63	14.16	11.60	12.41	0.376	0.247	0.325	0.312
3 A	64.41777199	-63.25549171	5.48	35.53	64.41785704	-63.25526588	5.837	34.244
	0.66	0.61	0.51	0.60	0.308	0.454	0.025	0.030
3 B	64.41790229	-63.25434505	5.02	28.29	64.41795947	-63.25413265	N/A	N/A
	4.04	4.13	1.68	2.17	0.113	0.124	N/A	N/A
4 A	66.05133135	-57.07115747	-104.32	-73.73	66.05005359	-57.07164998	-105.265	-74.750
	0.97	0.94	1.09	0.92	0.149	0.306	0.041	0.043
4 B	66.04872176	66.04872176	-99.63	-59.98	66.04751666	-57.07220024	N/A	N/A
	1.93	1.93	1.42	1.28	0.056	0.096	N/A	N/A
5 A	107.60195006	-55.58769155	-1.46	-9.42	107.60194970	-55.58776986	0.217	-12.031
	1.27	1.17	1.16	1.25	0.174	0.230	0.132	0.117
5 B	107.59949405	-55.58903491	-1.46	-9.42	107.59949880	-55.58911163	N/A	N/A
	2.13	2.18	1.16	1.25	0.058	0.108	N/A	N/A
6 A	110.08940661	-52.31188250	-35.17	147.45	110.08900990	-52.31091536	-36.881	146.689
	1.00	0.96	0.85	1.00	0.780	0.614	0.050	0.045
6 B	110.09119822	-52.30958151	-30.09	139.97	110.09085430	-52.30866713	N/A	N/A
	3.46	3.46	1.79	2.01	0.635	0.777	N/A	N/A
8 A	123.81647486	-62.91561677	-26.87	-10.95	123.81613060	-62.91569353	N/A	N/A
	0.60	0.55	0.62	0.57	0.270	0.327	N/A	N/A
8 B	123.81876212	-62.91520742	-26.87	-10.95	123.81845950	-62.91530189	N/A	N/A
	6.84	7.13	0.62	0.57	0.057	0.084	N/A	N/A
10 A	139.47917885	-69.80468837	-4.17	4.71	139.47903390	-69.80464588	-7.794	6.518
	2.16	2.22	2.55	2.08	0.254	0.233	0.063	0.069
10 B	139.48185926	-69.80195281	-12.53	8.73	139.48171690	-69.80191018	-6.978	6.315
	3.97	3.75	3.59	2.85	0.245	0.226	0.085	0.098

Table 1 concludes on next page.

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Table 1 (conclusion): Data from the HIP (ICRS, epoch J1991.25) and GAIA DR1 (ICRS, epoch J2015.0) catalogues

RMK	HIP RA °	HIP DE °	HIP pmRA mas/yr	HIP pmDE mas/yr	GAIA RA °	GAIA DE °	GAIA pmRA mas/yr	GAIA pmDE mas/yr
	± mas	± mas	± mas/yr	± mas/yr	± mas	± mas	± mas/yr	± mas/yr
11 A	146.77557340	-65.07201888	-11.55	4.97	146.77536750	-65.07197603	N/A	N/A
	0.42	0.39	0.49	0.38	1.200	1.273	N/A	N/A
11 B	146.77819338	-65.07287694	-11.55	4.97	146.77800690	-65.07283570	N/A	N/A
	9.06	7.82	0.49	0.38	0.255	0.210	N/A	N/A
12 A	148.77381591	-69.18905000	-67.46	31.29	148.77258570	-69.18886855	-66.237	27.490
	0.68	0.63	0.85	0.66	0.341	0.307	0.033	0.031
12 B	148.76989518	-69.19119668	-67.46	31.29	148.76870420	-69.19102761	N/A	N/A
	6.18	5.28	0.85	0.66	0.141	0.128	N/A	N/A
17 A	203.01630920	-63.04189266	-1.40	-5.60	203.01623910	-63.04190578	-4.946	-2.433
	4.40	3.80	5.50	4.10	0.189	0.219	0.298	0.316
17 B	203.01589640	-63.03745126	16.30	23.90	203.01584320	-63.03747152	-4.610	-3.541
	6.10	6.00	8.20	7.50	0.236	0.322	0.344	0.378
20 A	236.97123090	-65.44218988	-27.98	-41.74	236.97076230	-65.44244124	-29.371	-38.218
	1.11	1.20	1.08	1.39	0.179	0.349	0.036	0.045
20 B	236.97189982	-65.44261411	-28.00	-34.83	236.97142560	-65.44284167	N/A	N/A
	2.01	2.08	1.32	1.59	0.086	0.175	N/A	N/A
25 A	303.73387783	-56.97624609	38.22	-96.11	303.73432470	-56.97686335	36.677	-93.431
	1.75	1.46	2.43	2.00	0.151	0.194	0.073	0.062
25 B	303.73563959	-56.97448410	38.22	-96.11	303.73606750	-56.97513412	N/A	N/A
	3.19	3.37	2.43	2.00	0.068	0.070	N/A	N/A
27 A	354.86635383	-46.63789839	22.44	40.99	354.86656960	-46.63766756	22.353	34.941
	1.04	0.81	0.94	0.65	0.323	0.436	0.038	0.033
27 B	354.86478545	-46.63777300	28.98	36.14	354.86503090	-46.63752002	N/A	N/A
	3.59	3.24	2.07	1.55	0.122	0.123	N/A	N/A
28 A	356.88918210	-60.51913771	34.80	-14.40	356.88942030	-60.51912624	17.492	2.075
	7.40	9.50	9.60	13.10	0.214	0.214	0.478	0.438
28 B	356.89237990	-60.51941088	20.20	8.70	356.89260510	-60.51940647	17.510	1.857
	9.20	12.40	12.10	17.10	0.207	0.205	0.472	0.431

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and the above in Cartesian coordinates:

$$x" = \rho"\cos\left(\vartheta^{rad}\right)$$
$$y" = \rho"\sin\left(\vartheta^{rad}\right)$$

This has the effect that increasing x is in the direction of increasing North and increasing y is in the direction of increasing East.

The relative proper motions of the secondary in the x (xa) and y (ya) coordinates are:

$$xa^{"/yr} = \frac{xg" - xh"}{2015.0 - 1991.25}$$
$$ya^{"/yr} = \frac{yg" - yh"}{2015.0 - 1991.25}$$

The secondary position at $t = 0$ (not $t0$) on the x axis (xb) and y axis (yb) is:

$$xb" = xg" - xa".2015.0$$
$$yb" = yg" - ya".2015.0$$

So the time of closest approach ($t0$) and position ($x0$,$y0$) of secondary is:

$$t0^{yr} = \frac{xa.xb + ya.yb}{xa^2 + ya^2}$$
$$x0" = xa"(t0 - 2015.0) + xg"$$
$$y0" = ya"(t002015.0) + yg"$$
$$\vartheta 0 = \left\{ \begin{array}{l} \frac{180}{\pi}\arctan 2\left[y0, x0\right] \\ \vartheta^o + 360^o \quad \text{if } \vartheta^o < 0^o \\ \vartheta^o - 360^o \quad \text{if } \vartheta^o > 360^o \end{array} \right\}$$
$$\rho 0" = \sqrt{x0^2 + y0^2}$$

The seven Rectilinear elements ($x0$, xa, $y0$, ya, $t0$, $\theta 0$, $\rho 0$) thus calculated are defined in Section 3.1.

2.4 Uncertainties of the Rectilinear Elements

The uncertainties for the RA (σ_{RA}) and DE (σ_{DE}) of the HIP and GAIA positions are presented in the catalogues in milli-arcseconds (mas). Here, they are assumed to be in radians.

$$\sigma_{\Delta RA}^{rad} = \pm\sqrt{\left(\sigma_{RA2}^2 + \sigma_{RA1}^2\right)\cos^2\left(DE1\right) + \left(RA2 - RA1\right)^2 \sin^2\left(DE1\right)\sigma_{DE1}^2}$$

$$\sigma_{\Delta DE}^{rad} = \pm\sqrt{\sigma_{DE2}^2 + \sigma_{DE1}^2}$$

$$\sigma_{\vartheta}^{o} = \pm\frac{180}{\pi}\sqrt{\frac{\left(\Delta RA.\sigma_{\Delta DE}\right)^2 + \left(\Delta DE.\sigma_{\Delta RA}\right)^2}{\left(\Delta RA^2 + \Delta DE^2\right)^2}}$$

where $\frac{\partial\vartheta}{\partial\Delta RA} = \frac{\Delta DE}{\Delta DE^2 + \Delta RA^2}$ and $\frac{\partial\vartheta}{\partial\Delta DE} = -\frac{\Delta RA}{\Delta DE^2 + \Delta RA^2}$

so, $\sigma_{\vartheta}^2 = \left(\frac{\partial\vartheta}{\partial\Delta RA}\sigma_{\Delta RA}\right)^2 + \left(\frac{\partial\vartheta}{\partial\Delta DE}\sigma_{\Delta DE}\right)^2$, radians

$$\sigma_{\vartheta}^2 = \left(\frac{\partial\vartheta}{\partial\Delta RA}\sigma_{\Delta RA}\right)^2 + \left(\frac{\partial\vartheta}{\partial\Delta DE}\sigma_{\Delta DE}\right)^2, \quad \text{radians}$$

$$\sigma"_{\rho} = \pm\frac{3600.180}{\pi}\sqrt{\frac{\left(\Delta RA\,\sigma_{\Delta RA}\right)^2 + \left(\Delta DE\,\sigma_{\Delta DE}\right)^2}{\Delta RA^2 + \Delta DE^2}}$$

$$\sigma"_{x} = \pm\frac{3600.180}{\pi}\sqrt{\left[\cos\left(\vartheta^{rad}\right)\sigma_{\rho}^{rad}\right]^2 + \left[\rho^{rad}\sin\left(\vartheta^{rad}\right)\sigma_{\vartheta}^{rad}\right]^2}$$

$$\sigma"_{y} = \pm\frac{3600.180}{\pi}\sqrt{\left[\sin\left(\vartheta_{\rho}^{rad}\right)\right]^2 + \left[\rho^{rad}\cos\left(\vartheta^{rad}\right)\sigma_{\vartheta}^{rad}\right]^2}$$

$$\sigma"_{xa} = \pm\frac{3600.180}{\pi}\sqrt{\frac{\sigma_{xg}^2 + \sigma_{xh}^2}{23.75}}$$

$$\sigma"_{ya} = \pm\frac{3600.180}{\pi}\sqrt{\frac{\sigma_{yg}^2 + \sigma_{yh}^2}{23.75^2}}$$

$$\sigma"_{xb} = \pm\frac{3600.180}{\pi}\sqrt{\sigma_{xg}^2 + \left(2015.0\,\sigma_{xa}\right)^2}$$

$$\sigma"_{yb} = \pm\frac{3600.180}{\pi}\sqrt{\sigma_{yg}^2 + \left(2015.0\,\sigma_{ya}\right)^2}$$

$$\sigma"_{x0} = \pm\sqrt{\left[\left(t0 - 2015.0\right)\sigma_{xa}\right]^2 + \sigma_{xg}^2}$$

$$\sigma"_{y0} = \pm\sqrt{\left[\left(t0 - 2015.0\right)\sigma_{ya}\right]^2 + \sigma_{yg}^2}$$

$$\sigma"_{\vartheta 0} = \pm\frac{180}{\pi}\sqrt{\frac{\left(x0\,\sigma_{y0}\right)^2 + \left(y0\,\sigma_{x0}\right)^2}{\left(x0^2 + y0^2\right)^2}}$$

$$\sigma"_{\rho 0} = \pm\sqrt{\frac{\left(x0\,\sigma_{x0}\right)^2 + \left(y0\,\sigma_{y0}\right)^2}{x0^2 + y0^2}}$$

The uncertainty for t0 is much more complicated. Recall that:

$$t0^{yr} = \frac{xa\ xb + ya\ yb}{xa^2 + ya^2}$$

Then: $p = xa\ xb + ya\ yb$ and $q = xa2+ya2$, so that

$$t0 = \frac{p}{q}$$

Thus:

$$\sigma_{t0}^2 = \left(\frac{\partial t0}{\partial p}\sigma_p\right)^2 + \left(\frac{\partial t0}{\partial q}\sigma_q\right)^2$$

$$\sigma_{t0}^2 = \left(\frac{1}{q}\sigma_p\right)^2 + \left(\frac{p}{q^2}\sigma_q\right)^2$$

$$\sigma_p^2 = \left(\frac{\partial p}{\partial xa}\sigma_{xa}\right)^2 + \left(\frac{\partial p}{\partial xb}\sigma_{xb}\right)^2 + \left(\frac{\partial p}{\partial ya}\sigma_{ya}\right)^2 + \left(\frac{\partial p}{\partial yba}\sigma_{yb}\right)^2$$

$$\sigma_p^2 = \left(xb\ \sigma_{xa}\right)^2 + \left(xa\ \sigma_{xb}\right)^2 + \left(yb\ \sigma_{ya}\right)^2 + \left(ya\ \sigma_{yb}\right)^2$$

$$\sigma_q^2 = \left(\frac{\partial q}{\partial xa}\sigma_{xa}\right)^2 + \left(\frac{\partial q}{\partial xb}\sigma_{xb}\right)^2$$

$$\sigma_q^2 = \left(2xa\ \sigma_{xa}\right)^2 + \left(2ya\ \sigma_{ya}\right)^2$$

2.5 Ephemeris

With the rectilinear elements (REs) thus calculated, the equations derived can be used to compute the position angle (θ, PA in degrees) and separation (ρ in arcseconds) of the components at any epoch (t_{Eph}). Such computations can be used to establish positions for calibration pairs to be used in later observations.

$$x"_{Eph} = xa\left(t_{Eph} - 2015.0\right) + xg$$

$$y"_{Eph} = ya\left(t_{Eph} - 2015.0\right) + yg$$

$$\vartheta^o_{Eph} = \begin{cases} \frac{180}{\pi}\arctan 2\left(y_{Eph}, x_{Eph}\right) \\ \vartheta^o_{Eph} + 360^o,\ \text{ if } \vartheta^o_{Eph} < 0^o \\ \vartheta^0_{eph} - 360^o,\ \text{ if } \vartheta^0_{Eph} > 360^o \end{cases}$$

$$\rho"_{Eph} = \sqrt{x_{Eph}^2 + y_{Eph}^2}$$

2.6 Uncertainties of the Ephemeris

Note first that $\sigma_{tEph} = 0$, i.e. the uncertainty in the ephemeris date is zero.

$$\sigma"_{xEph} = \pm\sqrt{\left(\left(t_{Eph} - 2015.0\right)\sigma_{xa}\right)^2 + \sigma_{xg}^2}$$

$$\sigma"_{yEph} = \pm\sqrt{\left(\left(t_{Eph} - 2015.0\right)\sigma_{xa}\right)^2 + \sigma_{xg}^2}$$

$$\sigma^o_{Eph} = \pm\frac{180}{\pi}\sqrt{\frac{\left(x_{Eph}\ \sigma_{yEph}\right)^2 + \left(y_{Eph}\ \sigma_{xEph}\right)^2}{\left(x_{Eph}^2 + y_{Eph}^2\right)^2}}$$

$$\sigma"_{\rho Eph} = \pm\sqrt{\frac{\left(x_{Eph}\ \sigma_{xEph}\right)^2 + \left(y_{Eph}\ \sigma_{yEph}\right)^2}{x_{Eph}^2 + y_{Eph}^2}}$$

3. Application to the Rümker Double Stars

As stated in Section 1, this paper is a continuation of a series that retrospectively analyzes the double star observations from the private observatory built by Sir Thomas Brisbane in Parramatta, Australia, in 1822. The three astronomers associated with Parramatta observatory were Brisbane himself, and two employees: Carl Rümker and James Dunlop. This paper builds on our study of the double star catalog of Rümker (WDS designation RMK, see Letchford et al., 2017), and is undertaken with the aim of improving the data sets and our understanding of the quality of the associated historic data.

Again as stated (Section 2), we are working with the milli-arcsecond results from the space missions HIPPARCOS and GAIA, and present all ground-based and historic observations only as a starting point for a later study of the precision of such data.

HIPPARCOS and GAIA positions and proper motions are currently available for only 14 RMK pairs: 1, 3, 4, 5, 6, 8, 10, 11, 12, 17, 20, 25, 27, and 28 (i.e. 50% of the 28 pairs in the Rumker catalog). The Rectilinear Elements and their uncertainties for these pairs are given in Table 2. Following the USNO lead, we leave all digits from the computation rather than round off the elements and the uncertainties in the elements.

In addition, we have also adopted the ‘one line’ formatting of the CORE elements rather that utilizing the subscripted form (for example we use t0 for the date of closest approach rather than t_0) as seen in current references.

3.1 The Rectilinear Elements as defined in the CORE

- x0 - The RA position of the secondary (usually defines as the fainter) star relative to the primary (brightest) in the Cartesian frame centered on the primary, in units of arcseconds, at the time of closest approach, t0.

- xa - The RA proper motion of the secondary star relative to the primary in the Cartesian frame centered on the primary, in units of arcseconds per year.
- y0 - The Declination position of the secondary star relative to the primary in the Cartesian frame centered on the primary in units of arcseconds, at the time of closest approach, t0.
- ya - The Declination proper motion of the secondary star relative to the primary in the Cartesian frame centered on the primary, in units of arcseconds per year.
- t0 - The date of closest apparent approach of the two stars, in calendar years.
- θ0 - The Position Angle of the secondary star relative to the primary at time of closest approach, t0, in units of degrees measured from celestial North via East.
- ρ0 - The separation of the two stars in the Cartesian frame at the time of closest approach, t0, in units of arcseconds.

3.2 The Rectilinear Motion as a Test for Binary Orbit

The differentiation of an optical pair and a physically-bound binary system is a skill in its own right. For a pair to be bound, the relative motion of the stars as determined by astrometric means should show a curved orbital path (and the two stars should have the same parallax, appropriate radial velocities, and other physical and inferred properties). In contrast, the astrometric paths of the optical pairs will show no deviation from a straight line (although it is conceded that an orbit may present itself as a straight line under rare edge-on alignment of the orbit).

This analysis is, of course, made more difficult for wide, slow moving binary systems of great period. For all work of this type, there is no substitute for good quality (low uncertainty) data made over a long time baseline.

Also, well-defined relative proper motions can allow scale calibration for imaging systems and improvement in the determined proper motions of individual components.

3.3 Explanation of Figures

Plots of the Rümker doubles are given in Figures 1-28, presented in the Appendix. Historical data from the WDS have been incorporated into the figures and their position angles have been precessed from Equinox of date to J2000.0 and then converted to Cartesian coordinates. The WDS data for 1991.25 (HIP) were not precessed because they are already presented at Equinox J2000.0 and not at Equinox of date as the remaining WDS measures are presumed to be. Precessed WDS observations are prepesented in the plots by a '+'.

The HIP and GAIA positions are represented by a red circle and green square respectively. The dotted ellipses are the uncertainty ellipses for the t0 (unzoomed figure for each RMK pair). If they cannot be seen in the plots, it is because of the plot scale. Uncertainty ellipses for the HIP and GAIA were also plotted but in each case they are too small to see at the scales that are needed to represent all relevant data.

4 Notes on Individual Pairs.

RMK 1: Closing, secondary moving 4.69 ± 0.06 mas/yr a linear velocity along the line of best fit ($= \sqrt{(xa^2 + ya^2)}$). Proper motion data from both the HIP and GAIA missions are available. The red line is the rectilinear movement based on only HIP proper motions, the green line based on GAIA. The GAIA relative proper motion is similar to this paper; the HIP very different. The proper motions from the HIP data for RMK 1 is suspect, see note above re the limitation of the HIP data.

RMK 3: Closing, 2.79 ± 0.05 mas/yr. Primary is RMK 3AB and secondary is RMK 3B, between is RMK 3A or θ Ret. Proper motion available only from HIP and our own calculations.

RMK 4: Closing, 14.80 ± 0.03 mas/yr. Motion in close agreement with the relative proper motion as determined by HIPPARCOS.

RMK 5: Closing, 0.50 ± 0.07 mas/yr. Primary is itself a double (HD 55598 and CPD-55 1174B). Rümker secondary is CD-55 1708.

RMK 6: Closing, 9.38 ± 0.06 mas/yr. Primary is a spectroscopic binary. Close agreement between the determined rectilinear motion and that inferred by the HIP proper motion.

RMK 8: Widening, 3.93 ± 0.14 mas/yr.

RMK 10: Widening, 0.14 ± 0.10 mas/yr. Proper motion data from both the HIP and GAIA missions are available. The large uncertainty in t0 is due to: the extremely slow relative motion (the slowest of our sample); and the large separation (~10.4 arcseconds at J2015.0).

RMK 11: Widening: 1.26 ± 0.25 mas/yr.

RMK 12: Widening, 2.82 ± 0.12 mas/yr.

RMK 17: Closing, 1.59 ± 0.11 mas/yr. Disparate proper motions.

RMK 20: Closing, 3.62 ± 0.01 mas/yr. Similar proper motions.

(Text continues on page 217)

The Southern Double Stars of Carl Rümker II: Their Relative Rectilinear Motion

Table 2: Rectilinear Elements and their uncertainties, all ICRS

RMK	x0 "	xa "/yr	y0 "	ya "/yr	t0 yr	θ0 °	ρ0 "
	+/-	+/-	+/-	+/-	+/-	+/-	+/-
1	-4.63768	-0.00455	18.26516	-0.00115	3691.82472	104.24681	18.84474
	0.98846	0.00059	0.43580	0.00026	1027.75053	2.93110	0.48744
3	1.81763	-0.00204	-1.94640	-0.00190	3126.16037	313.04064	2.66313
	0.19615	0.00018	0.08739	0.00008	380.67083	3.34059	0.14833
4	1.51283	0.01353	-3.41838	0.00599	2273.13836	293.87220	3.73818
	0.02239	0.00009	0.01717	0.00007	30.87234	0.33146	0.01813
5	-1.60271	0.00024	0.88157	0.00044	15407.22262	151.18704	1.82916
	1.13936	0.00009	1.13873	0.00009	5901.71816	35.67360	1.13921
6	4.01399	-0.00800	6.55690	0.00490	2525.12668	58.52597	7.68798
	0.07207	0.00014	0.06016	0.00012	87.91699	0.51444	0.06363
8	2.65704	-0.00268	2.48239	0.00287	1550.42556	43.05369	3.63622
	0.12623	0.00027	0.08616	0.00019	248.17533	1.68162	0.10940
10	9.10521	0.00002	-1.42270	0.00014	-3012.00720	351.11925	9.21569
	5.90460	0.00017	3.46400	0.00010	101342.38340	22.01999	5.85827
11	-2.22133	-0.00024	-0.43684	0.00124	-1564.21086	191.12556	2.26388
	0.95782	0.00027	0.92232	0.00026	908.24950	23.37677	0.95652
12	-6.80683	-0.00188	-6.05101	0.00211	1500.33914	221.63586	9.10756
	0.09344	0.00018	0.08306	0.00016	301.12882	0.55229	0.08900
17	8.22116	-0.00108	7.66155	0.00116	9168.62396	42.98211	11.23774
	2.13949	0.00030	1.03391	0.00014	3433.77602	8.37729	1.71658
20	0.08279	0.00361	0.84265	-0.00035	2437.53812	84.38862	0.84671
	0.03475	0.00008	0.03093	0.00007	131.73156	2.34897	0.03097
25	-0.41727	-0.00497	1.32091	-0.00157	3352.66840	107.53102	1.38525
	0.17939	0.00013	0.15245	0.00011	197.22727	7.32583	0.15509
27	2.13848	0.00336	-2.32489	0.00309	2493.73377	312.60843	3.15883
	0.06756	0.00014	0.05244	0.00011	164.97803	1.10818	0.05985
28	-3.26241	-0.00107	3.60044	-0.00097	4120.86184	132.18019	4.85865
	1.35100	0.00064	0.59959	0.00028	3203.61894	12.72483	1.01011

Table 3: HIP and GAIA position data and Ephemeris, all ICRS.

RMK	1991.25	(HIP)	2015.0	(GAIA)	2020.0		2025.0		2030.0	
	θ °	ρ "	θ °	ρ "	θ °	ρ "	θ °	ρ "	θ °	ρ "
	+/-	+/-	+/-	+/-	+/-	+/-	+/-	+/-	+/-	+/-
1	81.29989	20.46414	81.58779	20.42096	81.64856	20.41193	81.70937	20.40293	81.77025	20.39395
	0.00069	0.00586	0.00001	0.00017	0.00824	0.00137	0.01643	0.00271	0.02465	0.00406
3	2.92738	4.13337	2.32927	4.08300	2.20148	4.07246	2.07302	4.06193	1.94389	4.05142
	0.00045	0.00417	0.00004	0.00047	0.00594	0.00100	0.01132	0.00183	0.01686	0.00269
4	245.73211	5.60185	248.24731	5.34520	248.80760	5.29255	249.37909	5.24043	249.96200	5.18883
	0.00038	0.00143	0.00006	0.00014	0.00543	0.00040	0.00971	0.00072	0.01434	0.00106
5	225.93633	6.95384	225.91055	6.94238	225.90511	6.93997	225.89966	6.93755	225.89422	6.93514
	0.00029	0.00199	0.00003	0.00019	0.00386	0.00047	0.00720	0.00087	0.01066	0.00129
6	25.45552	9.17420	26.63652	9.05461	26.88910	9.02992	27.14307	9.00542	27.39842	8.98109
	0.00027	0.00338	0.00008	0.00093	0.00633	0.00110	0.00929	0.00162	0.01281	0.00222
8	68.54127	4.02825	69.72832	4.06931	69.97514	4.07818	70.22090	4.08712	70.46557	4.09613
	0.00168	0.00391	0.00008	0.00017	0.01894	0.00101	0.03714	0.00198	0.05543	0.00296
10	18.68870	10.39617	18.70465	10.39768	18.70801	10.39800	18.71136	10.39832	18.71472	10.39864
	0.00020	0.00416	0.00001	0.00031	0.00316	0.00086	0.00605	0.00165	0.00900	0.00246
11	127.84868	5.03443	127.69577	5.06128	127.66379	5.06693	127.63187	5.07259	127.60003	5.07825
	0.00131	0.00567	0.00021	0.00089	0.01865	0.00163	0.03184	0.00279	0.04604	0.00404
12	212.97951	9.21250	212.56752	9.22284	212.48091	9.22507	212.39433	9.22733	212.30780	9.22961
	0.00037	0.00462	0.00002	0.00029	0.00542	0.00092	0.01051	0.00178	0.01566	0.00265
17	357.58727	16.00323	357.68222	15.97641	357.70225	15.97077	357.72229	15.96513	357.74235	15.95949
	0.00021	0.00710	0.00001	0.00039	0.00265	0.00154	0.00522	0.00301	0.00782	0.00450
20	146.76197	1.82595	145.45490	1.75013	145.16521	1.73429	144.87018	1.71850	144.56969	1.70275
	0.00084	0.00208	0.00013	0.00032	0.01556	0.00049	0.02706	0.00084	0.03965	0.00122
25	28.58665	7.22378	28.77798	7.10244	28.81910	7.07690	28.86051	7.05137	28.90223	7.02584
	0.00034	0.00336	0.00002	0.00019	0.00497	0.00067	0.00974	0.00131	0.01461	0.00195
27	276.64163	3.90289	277.95003	3.84026	278.23090	3.82733	278.51366	3.81450	278.79833	3.80176
	0.00085	0.00258	0.00012	0.00024	0.01244	0.00061	0.02213	0.00113	0.03245	0.00168
28	99.84728	5.75019	100.13696	5.73192	100.19818	5.72809	100.25949	5.72427	100.32087	5.72046
	0.00268	0.00632	0.00005	0.00015	0.03181	0.00152	0.06346	0.00303	0.09518	0.00454

(Continued from page 214)

RMK 25: Closing, 5.21 ± 0.03 mas/yr. Component A is a spectroscopic binary. X-ray source at 5.2" from component Aa.

RMK 27: Closing, 4.56 ± 0.07 mas/yr.

RMK 28: Closing, 1.44 ± 0.19 mas/yr.

5 Conclusions

Our method of describing the relative Rectilinear motion of double stars produces objective results. It eliminates a number of the problems we believe are associated with the current *Catalog of Rectilinear Elements* (CORE) maintained by the USNO (see Section 2). We present the Rectilinear Elements of RMK 1, 3, 4, 5, 6, 8, 10, 11, 12, 17, 20, 25, 27, and 28 based on the data obtained from the HIPPARCOS and GAIA missions.

Acknowledgments

This research has made use of:

- The *Aladin sky atlas* developed at CDS, Strasbourg Observatory, France.
- The *Washington Double Star Catalog* maintained by the USNO.
- The *Catalog of Rectilinear Elements* (CORE) maintained by the USNO. We wish to particularly thank Bill Hartkopf of the UNSO who gave the first author a copy of the fortran program from which data in the CORE is currently generated.
- The HIPPARCOS Catalogue (The Hipparcos and Tycho Catalogues (ESA 1997)) from VizieR†.
- The GAIA Catalogue (Gaia DR1 (Gaia Collaboration, 2016)) from VizieR††.

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Lindegren, L, F Mignard, S Söderhjelm, M Badiali, H.-H. Bernstein, Patricia Lampens, R Pannunzio, et al. 1997. "Double Star Data in the HIPPARCOS Catalogue." *Astronomy and Astrophysics* 323: L53–56. http://adsabs.harvard.edu/abs/1997A&A...323L..53L.

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† http://vizier.u-strasbg.fr/viz-bin/VizieR-3?-source=I/239/h_dm_com

††http://vizier.u-strasbg.fr/viz-bin/VizieR-3?-source=I/337/gaia

The Southern Double Stars of Carl Rümker II: Their Relative Rectilinear Motion

Appendix: Figures

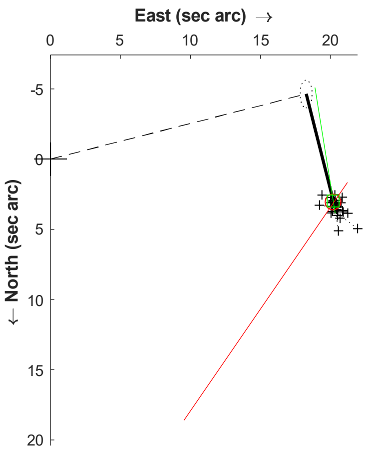

Figure 1. RMK 1

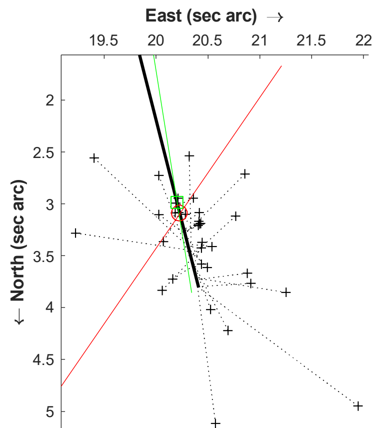

Figure 2. RMK 1 zoom

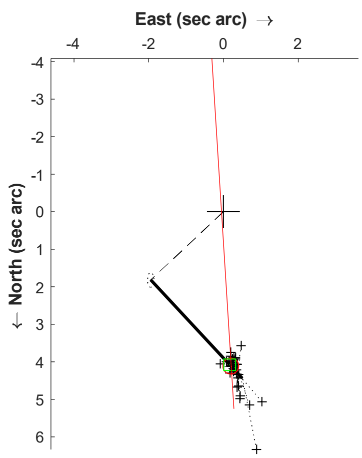

Figure 3. RMK 3

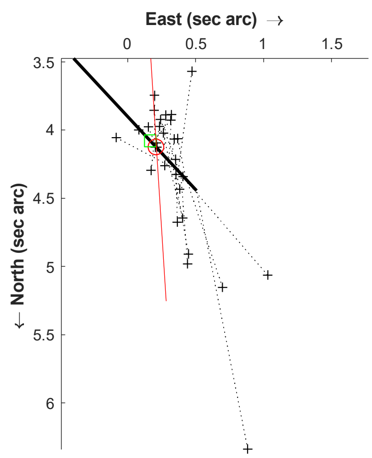

Figure 4. RMK 3 zoom

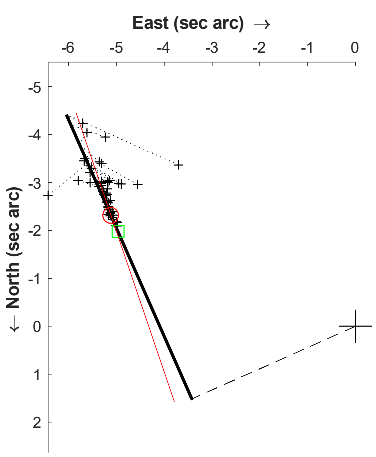

Figure 5. RMK 4

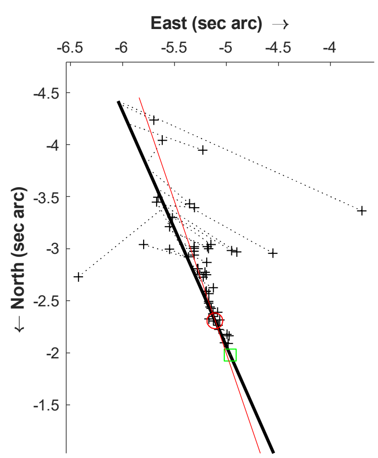

Figure 6. RMK 4 zoom

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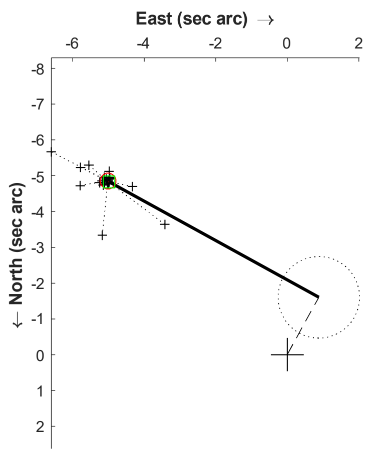

Figure 7. RMK 5

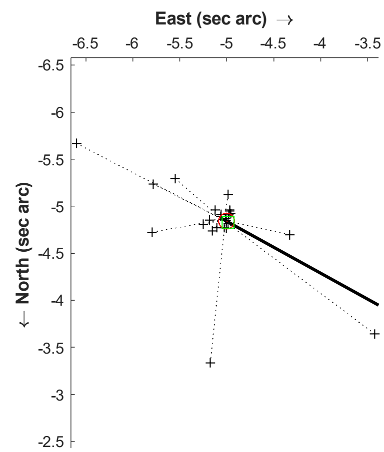

Figure 7. RMK 5 zoom

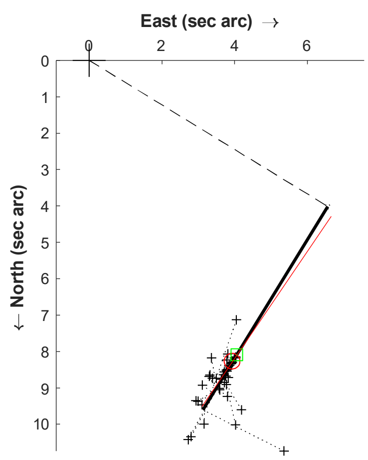

Figure 9. RMK 6

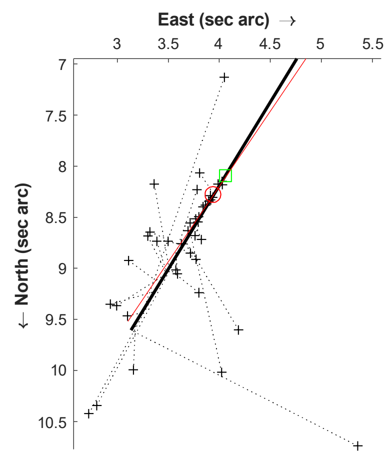

Figure 10. RMK 6 zoom

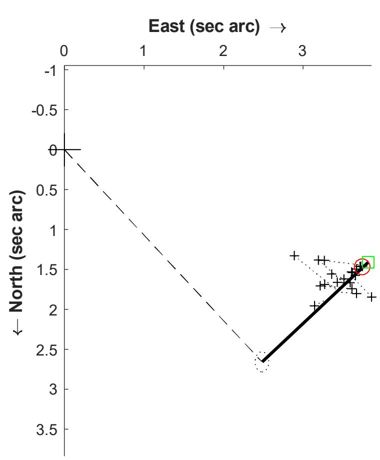

Figure 11. RMK 8

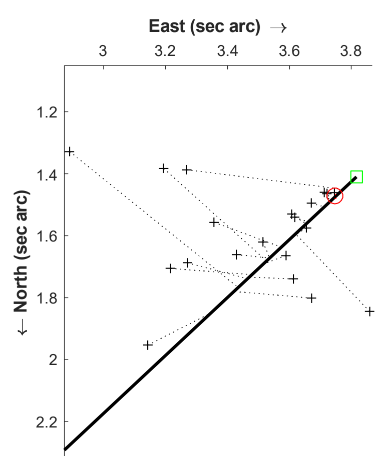

Figure 12. RMK 8 zoom

The Southern Double Stars of Carl Rümker II: Their Relative Rectilinear Motion

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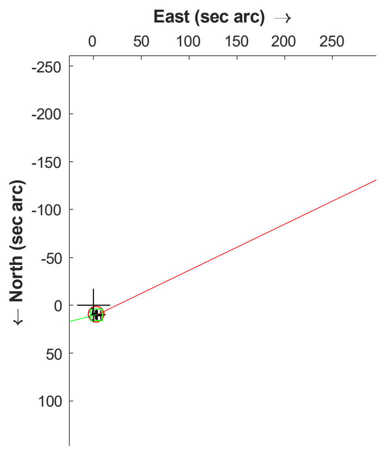

Figure 13. RMK 10

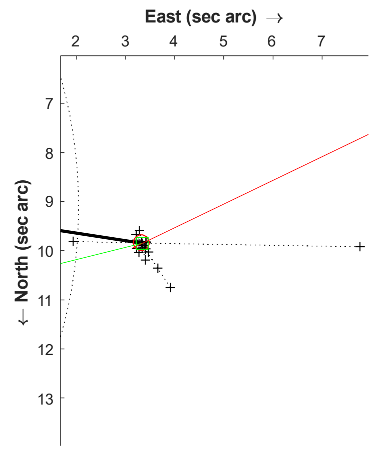

Figure 14. RMK 10 zoom

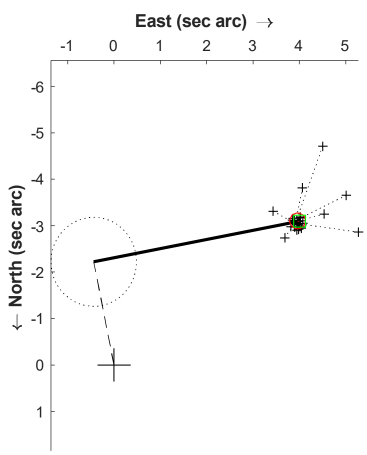

Figure 15. RMK 11

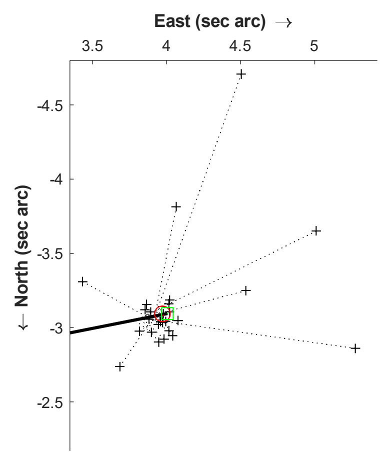

Figure 16. RMK 11 zoom

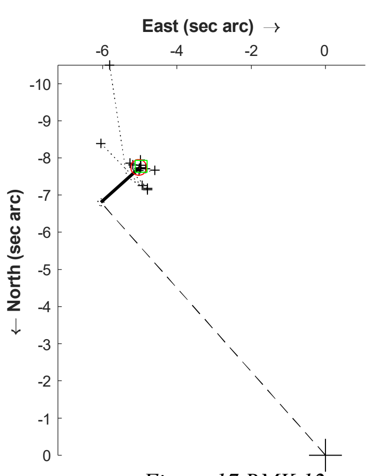

Figure 17 RMK 12

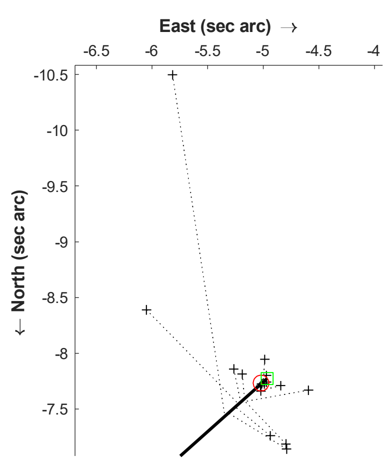

Figure 18. RMK 12 zoom

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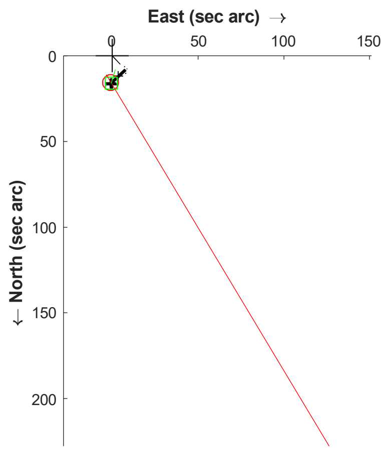

Figure 19. RMK 17

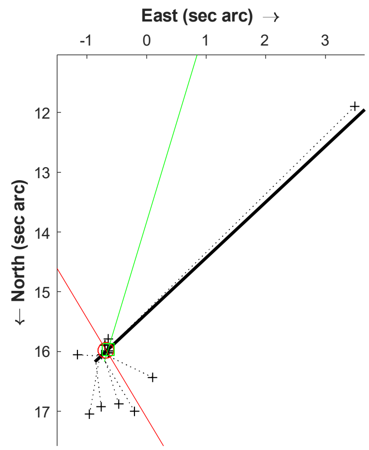

Figure 20. RMK 17 zoom

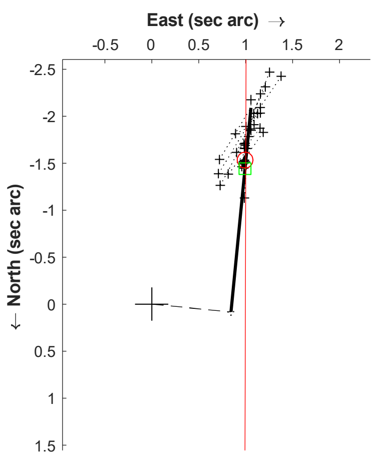

Figure 21. RMK 20

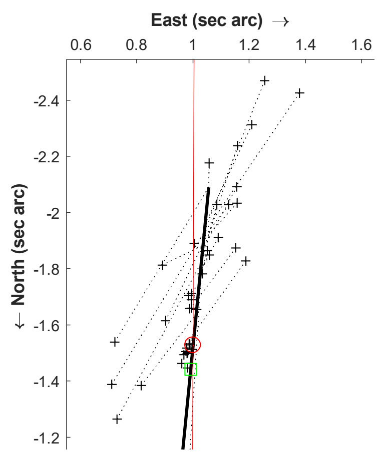

Figure 22. RMK 20 zoom

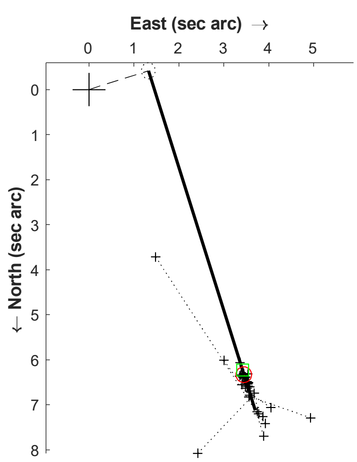

Figure 23 RMK 25

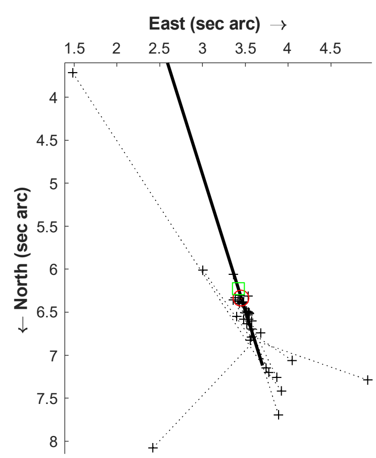

Figure 24. RMK 25 zoom

The Southern Double Stars of Carl Rümker II: Their Relative Rectilinear Motion

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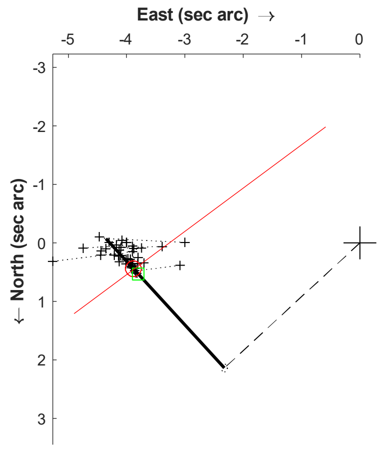

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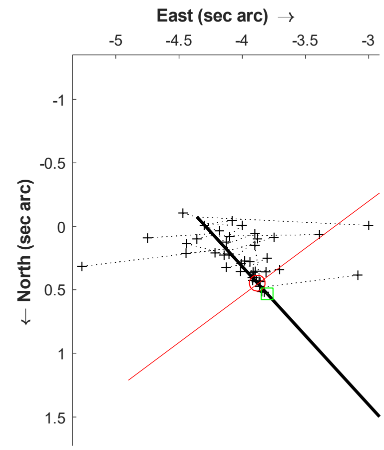

Figure 26. RMK 27 zoom

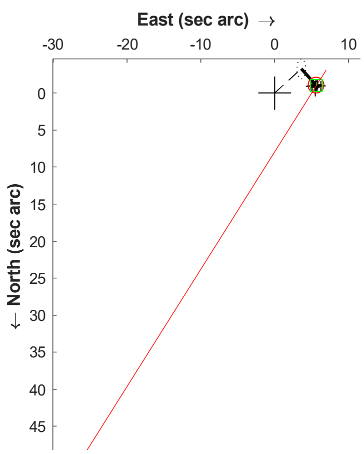

Figure 27. RMK 28

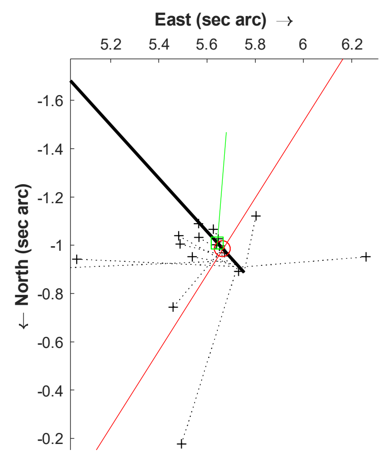

Figure 28. RMK 28 zoom