CCD Measurements and Linear Solution for WDS 02041-7115

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Abstract: The double star system WDS 02041-7115 or HJ 3483 was imaged using the Skynet Robotic Telescope Network on Jan 31 (Besselian date 2017.085). Using AstroImageJ, the secondary was measured to have a position angle of 263° and a separation of 7.6". Because the distance to the primary is known, along with the spectral classes and proper motions of both, we show that a gravitational relationship is not only unlikely, but mathematically impossible for these stars. Even if the secondary is at the same distance as the primary, its proper motion relative to the primary would exceed its escape velocity. We also present a linear solution for the pair.

Introduction

By calculating the period and orbit of a binary star system, it is possible to determine the dynamical (combined) mass of the stars if the distance to the binary is known. With additional information, such as radial velocity curves, the dynamical mass of the binary can be parsed into individual stellar masses, and, by extension, luminosities, to a high level of accuracy. This leads to a better understanding of the structure of the galaxy and the astrophysical properties of stars. Following perusal of the Washington Double Star (WDS) Catalog, the system WDS 02041-7115 was selected as one with high potential for a new solution to be discovered. The reason for this choice was that the system was easy to observe, had a long record of past observations showing significant relative movement, and did not yet have either a linear or orbital solution.

Target Selection

There were two main categories of constraints on our selection of a target: those imposed by the project goal of finding a new solution for a double star system, and those imposed by the available telescopes.

First, the system's most recent observation was constrained to be several years in the past, so that the new observation might show movement beyond what had already been published. Having a first observation before 1900, with at least 10 recorded measurements since then, further increased the probability that sufficient data existed to find a solution. The system was also constrained to have changed in position angle (PA) by at least 20 degrees between its first and most recent observation in order to ensure that significant relative movement was occurring for this system.

The telescopes used were part of the Skynet Robotic Telescope Network (https://skynet.unc.edu/). Since the Skynet telescopes are located all over the world, including many locations in the Southern Hemisphere, the location of the star was relatively unconstrained. However, these CCD telescopes cannot easily resolve double stars with a separation of less than 5 arcseconds, so that defined the minimum possible separation. Also, the double star's primary had to be less than 3 magnitudes brighter than the secondary in order for both components to be measurable in the same image.

Stelledoppie (http://stelledoppie.goaction.it/index2.php) is a website created by Gianluca Sordiglioni that facilitates searching through the WDS catalog based on constraints. It also contains some additional data beyond what is available in the WDS catalog, such as distance to the system, and designations to facilitate lookup in other catalogs. The constraints above narrowed the Stelledoppie catalog down to just 43 stars. From these, the one that looked the most promising was manually chosen. This was WDS 02041-7115. The Stelledoppie listing is shown in Figure 1.

History

This star is a Southern Hemisphere star, as shown

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02041-7115 HJ 3483

02H 04M 04.31S -71° 15' 10.7" P.A. 268 SEP 7.4 MAG 10,10.29 SP G3IV/V DIST. 164.74 PC (537.38 L.Y.)

Coord 2000	02041-7115	**Discov num**	HJ 3483	**Comp**		**Coord arcsec 2000**	02 04 04.31 -71 15 10.7
Date first	1835	**Date last**	1999	**Obs**	23		
Pa first	306	**Pa last**	268	**P.A. Now (θ)**	268°		
Sep first	6.3	**Sep last**	7.4	**Sep. Now (ρ)**	7.4"		
Mag pri	10	**Mag sec**	10.29	**delta mag (ΔM)**	0.29	**Spectral class**	G3IV/V (yellow)
Pri motion ra	+031	**Sec motion ra**	+019				
Pri motion dec	+024	**Sec motion dec**	-006				
Notes							

rPM=0.82 (> 0.8, Optical double)

OTHER CATALOGS AND DESIGNATIONS

Constellation	Hydrus	**HIP**	9651	**Tycho2**	9146-02056-1	**HD**	13021
CP	CP-71 102	**Distance**	164.74	**Distance ly**	537.38	**last precise pa**	267.5
last precise sep	7.41						

Figure 1. Stelledoppie Catalog Listing for WDS 02041-7115

in Figure 2.

John Herschel's *Results of Astronomical Observations Made During the Years 1834, 5, 6, 7, 8, at the Cape of Good Hope*, published in 1847, contains the record of the discovery of this system. In this document, rather than listing the stars by their declination, he gives the North Polar Distance (NPD), which is the angle from Celestial North. Currently, however, declination is measured as a positive or negative angle from the Celestial Equator. With this reference difference, the celestial coordinates of WDS 02041-7115 most closely match 3483, on page 175 of Herschel's document, because 162° NPD is equivalent to -72° in the current system. The four measurements that he originally made on the night of April 26th, 1835 are shown in Figure 3.

On page 165 of the document, Herschel mentions the weather conditions on the night of his discovery. There are other notes in the "remarks" column where HJ 3483 is described as "neat" even though the condi-

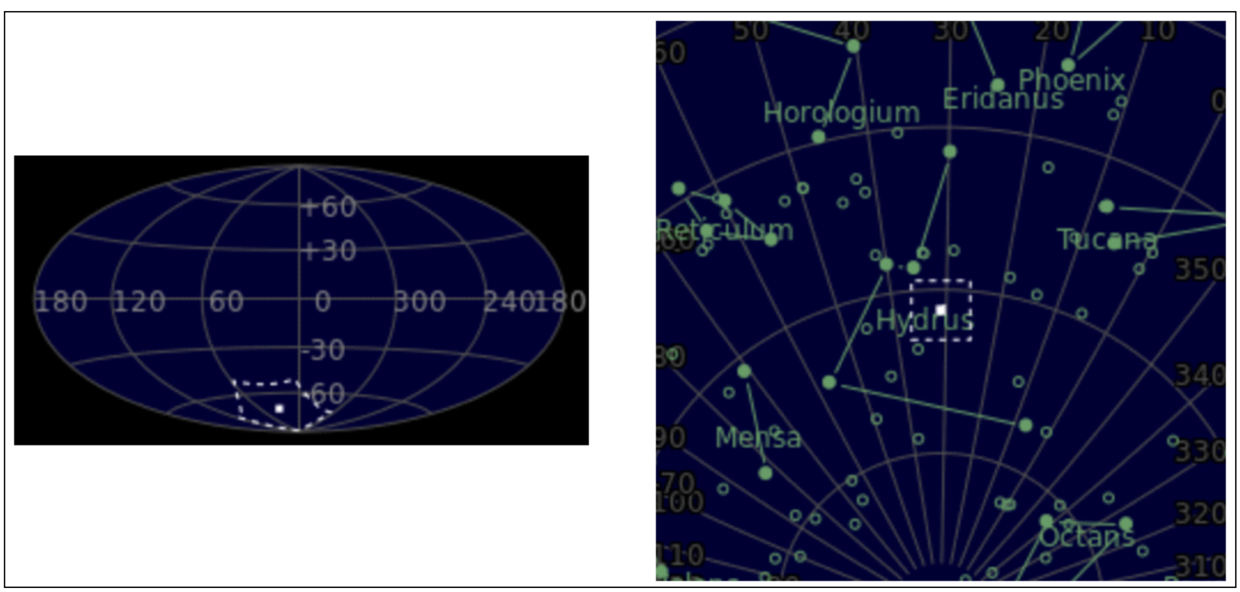

Figure 2. Location of WDS 02041-7115, as depicted at nova.astrometry.net

3483	2 1 9.7	162 4 20	303.4	6	9′ 10	....	......	513
	9.7	4 13	305.6	6	9 9′	....	......	514
	13.5	3 59	309.3	5	9′ 10	Neat star	......	482
	14.7	5 58	306.7	8	9′ 9′	P D right reduced. Some slip..................	......	745

Figure 3. John Herschel's Original Four Measurements of WDS 02041-7115

tions were "hazy." Possibly the hazy conditions allowed him to avoid the turbulence associated with clearer skies.

Since Herschel, 28 other astronomers have measured this system. All 29 past observations are listed in Table 1. The "Technique Code" column contains information about the way in which the measurement was made. In this designation Ma and Mb are micrometry techniques; T is a transit circle observation; Pa and Po are photography; C is CCD; Hh, Ht, and Hg are satellite -based observations (Hipparcos, Tycho, and Gaia, respectively); Eu is USNO CCD Astrograph Catalogue; 4th reduction (UCAC4); and E2 is the Two-Micron All-Sky Survey (2MASS). Full references, as well as a description of technique codes, can be found at the WDS website.

Observations

The PROMPT telescope array was selected as optimal for viewing this Southern Hemisphere star. Based at the Cerro Tololo Inter-American Observatory (CTIO) in Chile, the PROMPT array consists of six 16-inch telescopes, along with a 32-inch and a 24-inch telescope. The 16-inch PROMPT5 was used for this ob-

Table 1. Past Observations of 02041-7115 HJ 3483

Date	PA (deg)	Separation	Telescope Aperture (m)	# of observations	Reference Code	Technique Code
1835.32	306.2	6.3	0.5	4	HJ_1847a	Mb
1850.93	308.5	8.5	0.1	2	Gli1868	T
1882.99	296.3	7.11	0.2	1	Hrg1892	Ma
1893.93	293.8	7.357	0.3	1	WFC1998	Pa
1893.94	290.2	6.969	0.3	1	WFC1998	Pa
1916.94	287.3	7.20	0.4	3	Daw1918a	Ma
1930.63	283.8	7.05	0.7	4	B__1932b	Ma
1947.90	279.3	5.893	0.1	1	WFC1966b	Pa
1955.89	278.8	6.768	0.1	1	WFC1971	Pa
1955.932	277.97	7.194	0.3	1	Sms1965	Pa
1956.808	277.50	7.172	0.7	1	Kpr1985	Po
1956.813	277.63	7.152	0.7	1	Kpr1985	Po
1957.717	277.28	7.167	0.7	1	Kpr1985	Po
1959.943	277.36	7.240	0.3	1	Sms1965	Pa
1960.762	277.29	7.124	0.3	1	Sms1965	Pa
1977.874	272.1	7.22	0.6	1	Hln1978c	Mb
1991.25	269.4	7.377	0.3	1	HIP1997a	Hh
1991.69	269.9	7.443	0.3	1	TYC2000b	Ht
1992.6251	269.15	7.385	0.9	3	Lmp2001b	C
1992.652	268.7	7.15	0.3	5	War2000a	Ma
1998.6	267.9	7.441	0.2	4	UC_2013b	Eu
1999.89	267.5	7.41	1.3	1	TMA2003	E2
2015.0	266.8	7.198	0.7	1	Dam2017a	Hg

servation. The PROMPT5 and its neighboring telescopes are connected to the Skynet Robotic Telescope Network (SRTN), which allows remote access to more than 14 observatories and telescopes around the globe.

Initially, 8 images were taken with a "clear" filter on the evening of January 28, 2017 with varying exposure times ranging from 0.1s to 20s. The best of the returned images had a 10s exposure time, so 5 additional 10s exposures were requested. These 5 images were taken and returned on the evening of January 31, 2017 (Besselian date 2017.085).

Measurement Results

The returned images were uploaded to nova.astrometry.net for plate solving. The astrometry.net results listed the image orientation as 1.66 degrees East of North. Corroborating this, the compass rose in AstroImageJ (AIJ) showed the y axis canted very slightly East of North, as shown in Figure 4. Since AstroImageJ uses astrometry.net to plate solve images and writes the World Coordinate System headers from astrometry.net into the image .fits header, the canted compass axis implies that the AIJ (x,y) pixel coordinates apply to a coordinate system that is rotated 1.66 degrees relative to one in which Celestial North is "up".

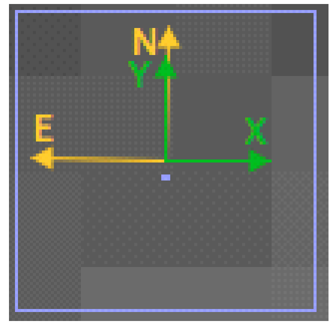

Figure 4. The AstroImageJ Compass Rose for 10-second Exposure Images of WDS 02041-7115

Although the separation of the stars is invariant to axis orientation, the position angle (PA) is not. Therefore, prior to calculation of PA, the AIJ x- and y- pixel coordinates of the star centroids were adjusted to a coordinate system in which the vertical axis aligns with Celestial North.

A sample AIJ image reduction is shown in Figure 5. The five 10-second exposure images were each measured by three different researchers using a six-

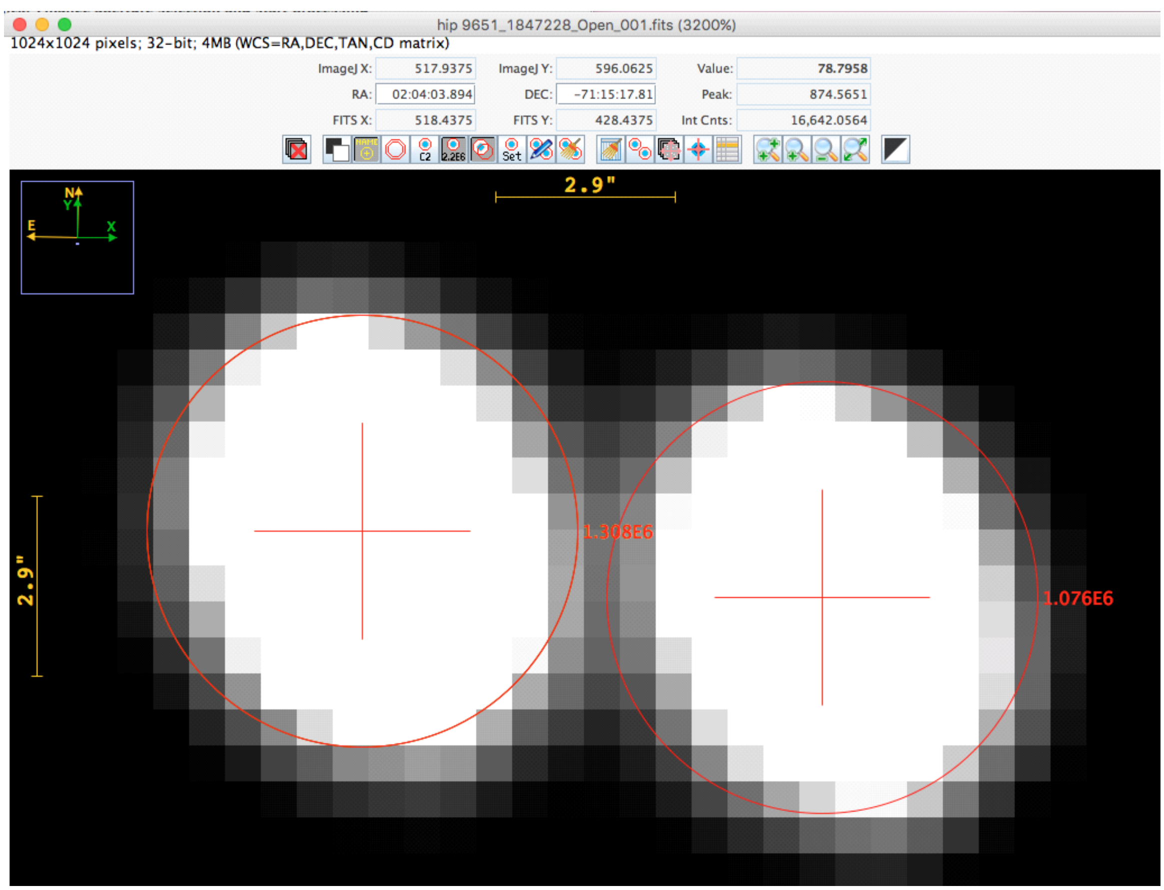

Figure 5. Image of 02041-7115 HJ 3483 with Centroids Located by AstroImageJ

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I/340/ucac5 UCAC5 Catalogue (Zacharias+ 2017) 2017yCat.1340....0Z ReadMe+ftp
Post annotation Fifth U.S. Naval Observatory CCD Astrograph Catalog (original column names in green) *(107758513 rows)*

start AladinLite plot the output query using TAP/SQL

Full	_r arcmin	_RAJ2000 "h:m:s"	_DEJ2000 "d:m:s"	RAJ2000 deg	DEJ2000 deg	SrcIDgaia	RAgaia deg	e_ mas	DEgaia deg	e_ mas	Org	Nu	EPucac yr	pmRA mas/yr	e_ (...)	pmDE mas/yr	e_ (...)	Gmag mag
1	0.0013	02 04 04.301	-71 15 10.64	031.0178906	-71.2529669	4692871105164087296	31.0182447	0.2	-71.2528408	0.2	1	4	1998.599	25.0	1.7	27.7	1.7	9.759
2	0.1254	02 04 02.755	-71 15 10.96	031.0114574	-71.2530428	4692871173883563904	31.0116983	0.2	-71.2530611	0.1	2	4	1998.599	17.0	1.0	-4.0	1.0	9.959
3	0.3824	02 04 04.397	-71 14 47.75	031.0182986	-71.2466000	4692871178178577280	31.0185778	0.2	-71.2465767	0.1	2	2	1998.599	19.7	3.2	5.2	3.1	15.519
4	0.5305	02 03 57.709	-71 15 11.63	030.9904603	-71.2532289	4692871143818839808	30.9904117	0.1	-71.2532517	0.1	3	4	1998.599	-3.4	1.3	-5.0	1.3	14.027
5	1.2501	02 04 09.321	-71 13 59.70	031.0388439	-71.2332489	4692871551840910976	31.0387661	0.2	-71.2332522	0.1	2	2	1998.599	-5.5	3.8	-0.7	3.6	15.852
6	1.5137	02 04 02.253	-71 16 40.96	031.0093408	-71.2780489	4692870868940942592	31.0098783	0.2	-71.2779844	0.1	2	2	1998.598	37.9	2.6	14.1	2.8	15.138

Figure 6. The VizieR Result Page from UCAC5, Searched on Precise Coordinates 020404.31-711510.7

pixel aperture for the centroid finder. Our measured separation of 7.6" and PA of 263° represent the mean of those fifteen measurements. The standard error of the mean for separation and position angle were 0.019" and 0.34 degrees, respectively.

Since these measurements were made, the AIJ software has been updated to compensate for the image orientation automatically, outputting the position angle relative to Celestial North. The updated software consistently finds the same star centroids between researchers. The images were reduced in the updated version of the software as a check of both the manual method and the software update. To the precision reported, the values for position angle and separation were unchanged.

Analysis

The proper motion of the stars was determined by using their WDS precise position (020404.31-711510.7) to search the USNO CCD Astrograph Catalog, 5th reduction (UCAC5). The search results are shown in Figure 6. The stars were identified by their magnitudes as the first two among the six search results.

Thus, the primary star has a proper motion of 25.0 milliarcseconds per year (mas/yr) right ascension and 27.7 mas/yr declination, and the secondary star has a proper motion of 17.0 mas/yr right ascension and -4.0 mas/yr declination.

The relative proper motion ratio of a binary system can be calculated using the proper motion vector of the primary and secondary components, with the formula:

$$PM_{ratio} = \frac{\left\| PM_{secondary} - PM_{primary} \right\|}{\left\| PM_{max} \right\|}$$

where PM_{max} is the larger proper motion vector. If this ratio is greater than or equal to 0.80, the stars are considered to have "differing proper motion" (DPM), implying that they are probably not a physical pair.

For this system, the secondary's proper motion relative to the primary is -8 mas/yr right ascension and -31.7 mas/yr declination. Dividing the magnitude of this vector by the magnitude of the primary proper motion vector, which is the larger of the two, gives a ratio of 0.876. This is greater than the threshold of 0.80 for differing proper motion, meaning that the stars are unlikely to be a physical pair.

We will show more rigorously that if the stars were gravitationally bound, then the relative speed of the secondary would likely exceed the escape velocity of the system.

To compute both speeds, we must first know the distance to the system. The SIMBAD astronomical database includes the Van Leeuwen re-reduction of the Hipparcos catalog, published in 2007. In this catalog, WDS 02041-7115 is identified as HIP 9651, and is listed as having a parallax of 6.07 ±1.72mas. Therefore the distance to the primary is between 128.4 parsecs and 229.9 parsecs. Because the stars have a separation of 7.6", this corresponds to a minimum physical separation of (by the small–angle approximation) 128.4*7.6 = 978.5 AU, which could be higher if the orbital plane of the stars is at a high inclination. The WDS catalog reports that the primary's spectral class is G3IV/V, meaning that it has a temperature of 5700 K and is roughly on the main sequence, and its mass is approximately the same as that of the sun, or possibly as large as 1.5 solar masses. The secondary's class is not known, but, because it is very similar in brightness to the primary, a reasonable assumption can be made that it is around the same mass as the primary, provided that they are the same distance from Earth.

With this information, it is possible to estimate the maximum possible escape velocity v_e of the secondary star. Assuming that both the secondary and the primary are solar-mass stars with a separation of no less than

978.5 AU, the total gravitational potential energy of the system is $-2GM^2/r$ where M is the mass of each star, G is the gravitational constant, and r is the distance between the two stars. In order for the stars to be exactly at escape velocity, the total kinetic energy of both stars must be equal to $2GM^2/r$. So we have $Mv_e^2 = 2GM^2/r$, giving:

$$v_e = \sqrt{\frac{2GM}{r}}$$

This gives us an escape velocity of between 0.28 AU/year and 0.35 AU/year, using the lower and upper estimates of the stars' mass, respectively.

If the stars are at their minimum distance of 128.47 parsecs away, then the relative proper motion of the stars (-8 mas/yr right ascension and -31.7 mas/yr declination) corresponds to a relative tangential velocity of 4.2 AU/year. If the standard errors of the stars' proper motions work together to minimize this relative velocity, such that the primary has proper motion (25.0 - 1.7) mas/yr right ascension and (27.7 - 1.7) mas/yr declination and the secondary has proper motion (17.0 + 1.0) mas/yr right ascension and (-4.0 + 1.0) mas/yr declination, then the relative tangential velocity would be 3.1 AU/year. This is the minimum possible relative velocity the stars could have if the standard error on the distance measurement and the standard errors on the stars' proper motions conspire to minimize their tangential velocity, and there is no additional radial component. Since this minimum relative velocity is still an order of magnitude greater than the highest possible escape velocity, these stars are highly unlikely to be a physically orbiting pair, i.e. a gravitationally bound binary. This, along with the fact that the stars have ≥ 0.80 different proper motion, strongly supports the existence of a linear solution.

We now find the best–fit linear solution. The R^2 value of a linear solution can be calculated as:

$$R^2 = 1 - \frac{SS_{res}}{SS_{tot}}$$

where SS_{res} is the sum of the squares of the weighted residuals of the past observations relative to the solution, and SS_{tot} is the sum of the squares of the weighted residuals relative to the average of the past observations of the system. The best-fit linear solution should maximize this value.

A linear solution can be expressed most easily by converting the past observations into rectangular coordinates X and Y, where X is the RA component of the stars' separation in arcseconds, and Y is the declination component of the stars' separation in arcseconds. Any linear solution can be specified using four parameters: X at time of discovery, Y at time of discovery, ΔX motion per year, and ΔY motion per year. These four variables comprise a four-dimensional search space that must contain the best-fit linear solution. By starting off with an educated-guess solution of X_disc = 6.0, Y_disc = 5.0, ΔX = 0.008, ΔY = -0.03, which visually appears to closely fit the data, and performing a gradient-descent search algorithm from that start point, we find a best-fit linear solution with parameters X_disc = 6.1586, Y_disc = 4.3236, ΔX = 0.0075, ΔY = -0.0280, shown graphically in Figure 7. The R^2 value of this solution is well within the range of relative certainty with a value of 0.9863.

Conclusion

The current measurements of the position angle and separation of 02041-7115 HJ 3483 are 263° and 7.6", respectively. The proper motion of these stars strongly suggests that they are not gravitationally bound, because the secondary is moving relative to the primary with a speed that exceeds its likely escape velocity. A linear trend in the observations made over the course of the past 180 years corroborates the classification of this system as an optical double with a linear solution (dX, dY) = (7.51 x 10^{-3} as/year, 2.80 x 10^{-2} as/year), and a closest approach of ($X_{\text{closest approach}}$, $Y_{\text{closest approach}}$) = (6.83", 1.83") in 1924.35.

Acknowledgements

This research has made use of the Washington Double Star Catalog maintained at the U.S. Naval Observatory, the SkyNet Robotic Telescope Network, the Stelledoppie catalogue maintained by Gianluca Sordiglioni, the SIMBAD database, operated at CDS, Strasbourg, France, and AstroImageJ software written by Karen Collins and John Kielkopf at the University of Louisville, updated for double star astrometry by Karen Collins.

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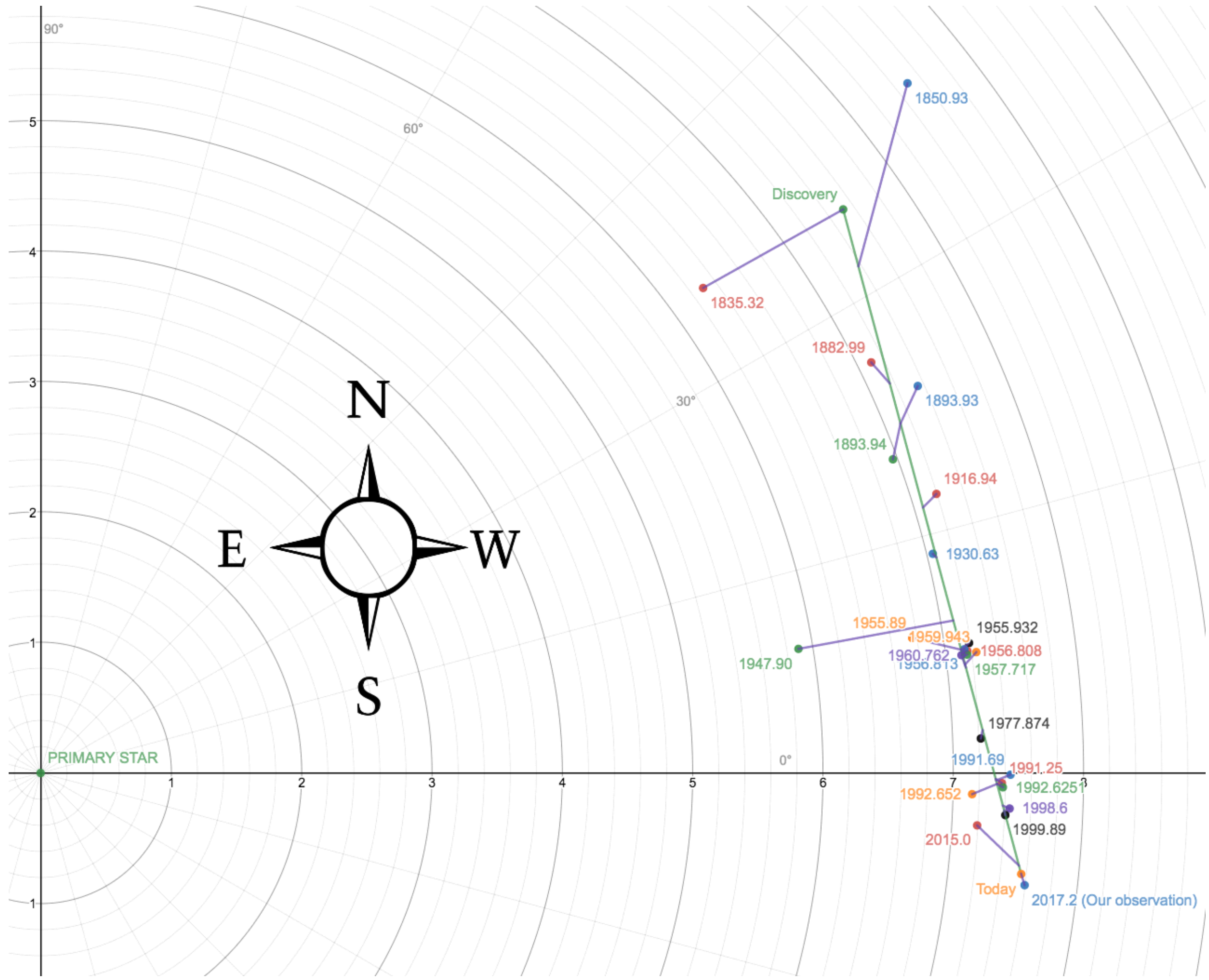

Figure 7: Measurement of 02041-7115 HJ 3483 together with the past observations and best fit line

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