Dynamic Studies of Struve Double Stars: STF4 and STF 236AB Appear Gravitationally Bound

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Abstract: Dynamics of two Struve double stars, WDS 00099+0827 (STF 4) and WDS 02556+2652 (STF 326 AB) are analyzed using astrometric criteria to determine their natures as gravitationally bound or unbound systems. If gravitationally bound, then observed relative velocity will be within limits according to the orbital energy conservation equation. Full implementation of this criterion was possible because the relative radial velocities as well as proper motions have been estimated. Other physical parameters were taken from literature or estimated using published protocols. Monte Carlo analysis indicates that both pairs have a high probability of being gravitationally bound and thus are long-period binaries.

Introduction

The Washington Double Star Catalog (Mason et al., 2001), hereafter WDS, contains many systems with common proper motions but no detailed analysis of their nature. While many are unbound common origin systems, others may be gravitationally bound systems with long periods and only a small part of the orbit measured. Herein we analyze two systems, WDS 00099+0827 (STF 4) which comprises two components first measured by Struve in 1829 and WDS 02556+2652 AB (STF 326 AB), a triple system first measured by Struve in 1831 with a fainter companion with similar proper motions (LDS 883 AC) discovered by Luyten in 1936 (Struve 1837; Luyten 1969).

Several astronomers have proposed criteria for separating unbound from gravitationally bound systems based on separation and relative motion. Van de Kamp (1961) proposed a solution by calculating the conditions for a parabolic trajectory based on the vis viva equation; above a critical value, the trajectory must be hyperbolic, while below the critical value, it may be elliptical. Sinachopoulos & Mouzourakis (1992) sampled a number of wide doubles where both components were at similar distances and calculated the relative tangential and projected velocity of the secondary with respect to the primary, and from these results they calculated an orbital velocity for a circular orbit. They considered optical pairs to be those whose tangential velocity coupled with the error associated with this velocity exceeded the maximum orbital velocity, while those that did not may be bound. The limitation of both studies lay in the fact that the separation between pairs was a projected separation and did not take into account the radial velocity needed to calculate the 3-D relative velocity. Thus while a selected pair may be gravitationally bound, the answer is not definitive. Close et al. (1990) discriminated between optical and bound pairs by comparing the relative orbital velocity with the escape velocity, the minimum speed of an object needed to escape from the gravity of the companion star. Since most know wide systems have an escape velocity less...
than 5 km s⁻¹, they considered all systems with a tangential velocity of > 5 km s⁻¹ ± 5σ as not gravitationally bound. Close et al. (1990) proposed that if the total velocity of the secondary was less than the escape velocity, then the candidate pair may be gravitationally bound within a calculated probability threshold:

\[ \Delta V_{\text{tot}} < V_{\text{esc}} \]  

(1)

Rica (2011) explored the relationship between these various criteria and demonstrated how they were related based on the total mechanical energy of a system. He derived the *vis viva* equation by considering the total orbital energy using relative quantities. This allows calculation of the maximum (contra minimum) orbital velocity of the binary:

\[ v^2 = G\left(M_1 + M_2\right)\left(\frac{2}{r} - \frac{1}{a}\right) \]  

(2)

where \( G \) is the gravitational constant (\( G = 0.0043 \left(\text{pc}/\text{M}_\odot\right) \left(\text{km/s}^2\right) \)), \( M_1 \) and \( M_2 \) are the masses of primary and secondary relative to solar mass, \( r \) is the distance between the primary and secondary in parsecs, and \( a \) is the expected semi-major axis as calculated by Fischer and Marcy (1992). Masses for main sequence dwarfs were directly calculated from equations given in Henry & McCarthy (1993), given known magnitudes and distances.

The escape velocity can be derived directly from the conservation of energy equation (Rica 2011) and yields:

\[ V_{\text{esc}} = \sqrt{\frac{2GM_{\text{tot}}}{r}} \]  

(3)

where \( G \) is the gravitational constant, \( M_{\text{tot}} \) is the total mass of the system in solar units, and \( r \) is the distance between the primary and secondary in parsecs.

Using equations (2) and (3) with errors from catalogues (\( M_1, M_2, r \)) or empirically determined, Rica (2011, 2013) was able to characterize the probable relationship to a number of doubles.

Knowledge of radial velocities, distance, and relative motions permits the investigator to calculate two additional parameters, the tangential velocity (\( v_{\tan} \)) and the total 3-D velocity:

\[ v_{\tan} = 4.74 \Delta\mu \]  

(4)

\[ V_{\text{tot}} = \sqrt{\Delta v_x^2 + \Delta v_y^2 + \Delta v_z^2} \]  

(5)

where \( v_{\tan} \) is the tangential relative velocity (relative projected velocity) of the secondary to the primary in km/s, \( \Delta\mu \) is the relative proper motion of the secondary to the primary in arcsec/yr, \( \Delta v_x, \Delta v_y, \Delta v_z \) are the total relative velocities of the system in km/s, and \( \Delta v_r \) is the relative radial velocity. Determination of \( \Delta v_{\text{tot}} \) permits full implementation of the Close et al. (1990) criterion shown in formula (1) above.

Herein, we report a dynamical study of two double stars (WDS 00099+0827 = STF 4 and WDS 02556+2652 AB = STF326 AB), first cataloged by F. Struve, with common proper motions and known radial velocities. Studies of other Struve doubles are ongoing.

**Methods**

WDS historical measures for each pair were kindly supplied by Dr. Brian Mason of the U. S. Naval Observatory (Mason 2005). We import the theta and rho values into an Excel spreadsheet prepared by Francisco Rica that performs a weighted analysis resulting in estimation of the following parameters:

- Separation of component stars in \( x \) and \( y \): \( x(AU) \) \( [E-W] \), \( y(AU) \) \( [N-S] \)
- Variation of theta and rho: \( \Delta\mu/\Delta \mu \)  (mas/yr) and \( \Delta\rho/\Delta \rho \) (deg/yr)
- Variation of \( x \) and \( y \) coordinates in time: \( \Delta x/\Delta t \) (mas/yr) and \( \Delta y/\Delta t \) (mas/yr)
- Relative velocities along \( x \), \( y \), and \( z \) (km/s): \( V_x \) \( [E-W] \), \( V_y \) \( [N-S] \), \( V_z \)
- Projected total relative velocity: \( V_{xy} \)  (km/s)
- Total apparent motion of B relative to A: \( \Delta\mu \) (mas/yr)
- Total relative velocity: \( V_{\text{tot}} \) (km/s)
- Maximum orbital velocity: \( V_{\text{orb max}} \) (km/s)

Errors associated with certain values (e.g. \( V_{\text{tot}} \)) are not directly calculated in the spreadsheet were obtained through Monte Carlo simulation with 10,000 iterations. Mass is calculated from equations given in Henry & McCarthy (1993) from magnitude and distance estimates gleaned from catalogues (for sources see Table 1). Dynamics are calculated from Rica (2011), and a Monte Carlo simulation is performed with 25,000 iterations to calculate the probability that the pair is gravitationally bound.

**Catalog Sources and Astrophysical Data**

Basic astrophysical data are shown in Table 1 and include WDS number and discoverer code, position, spectral types, mass, parallax and distance, proper motions in right ascension and declination, radial velocity, first and last observations (Epoch Range), and change in theta and rho over the history of observations (\( \Delta\theta, \Delta \rho \)).
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Results

Table 2 presents the relative motions and velocities obtained. Table 3 presents parameters calculated in the Monte Carlo simulations and the probabilities that the pairs meet three criteria: (1) van de Kamp's parabolic condition, (2) the probability that the total velocity ($V_{total}$) is less than the maximum orbital velocity ($V_{orb\_max}$), and (3) the probability that the total velocity is less than the escape velocity ($V_{esc}$). Figures 1-4 illustrate the relative motions of the two pairs.

Discussion and Conclusions

Calculated mass for STF 4 A was close to estimates in Allende Prieto and Lambert (1999), 0.99 ± 0.12 M$_{\odot}$ and Casagrande et al. (2011), Padova maximum likelihood mass of 0.97 M$_{\odot}$. Mass estimates for STF326 A in the literature range from 1.0 M$_{\odot}$ to 0.74 M$_{\odot}$ (Valenti & Fischer 2005; Howard et al., 2010; Shaya and Olling 2011; Tokovinin 2008 in decreasing order) while our estimates for STF326 B was higher than that estimated by Tokovinin (2008; 0.58 M$_{\odot}$).

Both pairs have a high probability that their total velocities are less than the escape velocities and both meet the parabolic condition. Of the two, WDS 00099+0827 (STF 4) has the highest probability of being bound as its total velocity is less than half of either the estimated maximum orbital velocity or the estimated escape velocity. The case for WDS 02556+2652 AB (STF 326 AB) is less clear cut: its estimated total velocity and estimated maximum orbital velocities are within errors. Thus it is estimated to be at the limit of its allowed maximum orbital velocity. However, based on the observation that the total velocity is lower than the estimated escape velocity, we conclude that it is also gravitationally bound. STF326 AB has both linear (Hartkopf and Mason 2011) and orbital (Hopmann 1967) solutions in the WDS, a situation that might not be uncommon for “short arc” binaries. An orbital solution calculation is being carried out by Francisco Rica using instant position and velocity (in addition to parallax and stellar mass). This work will be published in a future publication.

Although we did not analyze WDS 02556+2652 AC (LDS 883 AC), we have no reason to think that it does not form part of a common origin triple system. LDS883 C is similar in proper motion to STF326 AB (Zacharias et al. 2013; C is Gliese 118.2C). Orlov et al. (1995) report all three stars as having similar parallax values and as components of one of five moving clusters in the solar neighborhood.

The two systems analyzed were taken from a number of pairs discussed by Wiley (2012) that were charac-

(Continued on page 8)
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Table 2. Relative Motions and Velocities

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<th>WDS</th>
<th>Disc</th>
<th>Epoch</th>
<th>θ (deg)</th>
<th>ρ (arcsec)</th>
<th>x (AU), [E-W]</th>
<th>y (AU), [N-S]</th>
<th>dθ/dt (mas/yr)</th>
<th>dq/dt (deg/yr)</th>
<th>dx/dt (mas/yr), [E-W]</th>
<th>dy/dt (mas/yr), [N-S]</th>
<th>Vx (km/s), [E-W]</th>
<th>Vy (km/s), [N-S]</th>
<th>Vz (km/s)</th>
<th>Vxy (km/s)</th>
<th>Vtot(km/s)</th>
<th>Vesc_max (km/s)</th>
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<td>θ (deg)</td>
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<td>ρ (arcsec)</td>
<td>5.24</td>
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<td>y (AU), [N-S]</td>
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<td>dy/dt (mas/yr), [N-S]</td>
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<td>24.3±0.4</td>
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<td>Vx (km/s), [E-W]</td>
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<td>Vy (km/s), [N-S]</td>
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<td>Vz (km/s)</td>
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<td>Vxy (km/s)</td>
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<td>Vtot(km/s)</td>
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<td>Vesc_max (km/s)</td>
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<td>4.5±0.2</td>
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Table 3: Dynamical Results from Monte Carlo Analysis

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<th>WDS</th>
<th>Disc</th>
<th>Δμ [mas/yr]</th>
<th>Vtotal [km/s]</th>
<th>Vorb_max [km/s]</th>
<th>Vesc [km/s]</th>
<th>p(Vorb_max &gt; Vtotal )</th>
<th>p(Vesc &gt; Vtotal )</th>
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<tr>
<td>WDS</td>
<td>00099+0827</td>
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<td>Disc</td>
<td>STF 4</td>
<td>STF 326AB</td>
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<td>Δμ [mas/yr]</td>
<td>1.3±0.4</td>
<td>29.5±0.5</td>
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<td>Vtotal [km/s]</td>
<td>1.1±0.6</td>
<td>3.3±0.1</td>
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<tr>
<td>Vorb_max [km/s]</td>
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<td>3.1±0.6</td>
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<td>Vesc [km/s]</td>
<td>3.0±0.2</td>
<td>4.5±0.2</td>
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<tr>
<td>p(Vorb_max &gt; Vtotal )</td>
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<td>0.383</td>
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<td>p(Vesc &gt; Vtotal )</td>
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Figure 1. WDS 00099+0827 (STF 4). Theta versus Epoch over history of observations

Figure 2. WDS 00099+0827 (STF 4). Rho versus Epoch over history of observations
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Figure 3. WDS 02556+2652 AB (STF 326 AB). Theta versus Epoch over history of observations.

Figure 4. WDS 02556+2652 AB (STF 326 AB). Rho versus Epoch over history of observations.
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(Continued from page 4)

terized as possible long-period binaries. They were selected from that list because we could access radial velocities for both components. We note that lack of radial velocities is a major impediment to analysis of many common proper motion pairs in the WDS that show some indications of interesting relative motion over their histories of observation. We also note that some pairs originally thought to have radial velocities for both components, in fact, do not; a cautionary tale that requires such data to be carefully evaluated.

Acknowledgements

This research has made use of the Washington Double Star Catalog maintained at the U.S. Naval Observatory and we gratefully thank Brian D. Mason and William I. Hartkopf for honoring our many data requests. This research also made use of the many catalogs (VizieR) and tools (Simbad, Aladin) maintained by the Centre de Données astronomiques de Strasbourg and we gratefully acknowledge the hard work of the many authors contributing to this resource.

References


Anderson E., Francis C., 2012, AstL, 38, 331. (V/137)


Mason, B. D. 2005. JDSO, 1, 57.


Orlov, V. V., Yatsevich, A. V., Panchenko, I. E., Rastorguev, A. S., 1995, AZh, 72, 495.


Rica, F. M., 2013, JDSO, 9, 221.


Struve, F.G.W., 1837, Mensurae Micrometricae, Petropoli.


