Analysis of Errors in the Measurement of Double Stars Using Imaging and the Reduc Software

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Abstract: This paper reports quantification and explanation of some sources of error in using the Reduc software to measure the PA and separation of double star images. A comparison of measures of the images of two pairs, one close and one wider, made by the two authors independently has shown that the two individuals produce measurements that are numerically nearly identical and are not significantly different statistically. Analysis of the repeated measures of 38 pairs has shown that the standard deviation of PA increases with decrease in separation due mainly to the geometry of the measurement. The standard deviation of separation increases with separation due mainly to propagation of the error in determining the separation calibration constant. All these errors can be controlled to any desired precision by conducting sufficient repeats.

Introduction

The Astronomical Association of Queensland is undertaking a program of measuring neglected double and multiple stars visible from Queensland at an approximate latitude of 27ºS. "Neglected" was arbitrarily defined as 15 or more years since the last measure as recorded in the Washington Double Star catalog (WDS). Results to date have been reported in the Webb Society Double Star Section Circulars 17 and 18 (Napier-Munn and Jenkinson; 2009, 2010), and further reports are in preparation.

The method involves acquiring an image of the chosen pair and measuring position angle and separation using Florent Losse's Reduc software (see Losse). In our 2009 paper we evaluated the errors associated with the estimation of the separation constant (specific to the camera and optical train) and the PA constant (depending on the orientation of the camera with respect to the sky). In this paper we take advantage of some recent measurements to comment on the magnitude of personal errors by comparing the measurements of the two authors on the same images. We also investigate the dependencies of the measurement uncertainties in PA and separation on the separation itself.

Method

All images were obtained using a Meade DSI CCD camera coupled to an equatorially-mounted 150mm f8 refractor. In some cases of close separations a Barlow lens was used (either 2.36x or 5.43x). Separations and position angles were measured using the software program Reduc (Losse), which was specifically designed to measure double stars using appropriate images of the target pairs together with images of calibration pairs of known separation and PA; Argyle's list of calibration pairs was used for this purpose (Argyle, 2004).

In order to obtain statistically viable results, the DSI software is used to stack a minimum of 10 individual good quality images as they are acquired, to generate one image for measuring. About 10 such images are obtained per pair per night, plus 3 trailed images with the tracking switched off in order to calibrate the E-W axis in the images. The Reduc software is then used to generate a single average measure for the 10 images. This process is repeated on about 7 separate nights, generating mean separations and position angles together with standard deviations from which a confidence interval for the measurement can be calculated based on the 7 repeat values. There is therefore a considerable amount of replication built into the final PA and separation values. A full description of the method was given in Napier-Munn and Jenkinson (2009).

Florent Losse's excellent software has a built-in automated procedure which greatly decreases the time needed to make measurements, and this works well for easy pairs, that is those with wide separations in barren

Table 1. Measures of HRG 53 Car

Date	PA °		Separation "	
	Jenkinson	Napier-Munn	Jenkinson	Napier-Munn
11-Feb-10	215.39	212.20	3.265	3.173
14-Feb-10	210.60	212.95	3.244	3.106
19-Feb-10	216.99	218.38	3.287	3.193
20-Feb-10	219.53	218.83	3.201	3.200
21-Feb-10	218.30	217.95	3.127	3.247
26-Mar-10	218.54	216.36	3.253	3.216
Mean	**216.56**	**216.11**	**3.230**	**3.189**
Std. Dev.	3.251	2.873	0.058	0.048
Coeff. of var. %	1.50	1.33	1.79	1.50
95% conf. int. ±	3.412	3.015	0.061	0.050

WDSC entry: RA 10 52.4 Dec -58 45 Mags 8.4 & 10.2 PA 218° Sep 3.6" (1991) Imaged with 5.43x Barlow. Measured using Reduc's QuadPx facility. One night's measures rejected because of poor seeing and degraded images (not shown in Table 1)

fields. However, quite often manual measurements are required which are subject to some degree of personal error in identifying the position of each star, or more specifically the center of each star's image. In this paper, therefore, we turn our attention to the effects of personal error on the measurement uncertainties by comparing the values obtained by the two authors on the same images. The test pairs for this comparison were HRG 53 in Carina and HJ 4583 in Centaurus. HRG 53 is a close pair (around 3" separation) and HJ 4583 a wider pair (around 23" separation). All measurements were made manually, that is, not using Reduc's automated procedure.

Observations of these two pairs suggested some interesting relationships between measurement uncertainties and the separation itself, and these were further investigated using data from earlier measurements of 36 pairs, plus the two mentioned above, 38 in all.

Results

Results for the two test pairs are shown in Tables 1 and 2, including the current WDS entry (as of January 2014). Barlows were used in both cases, for which separate separation calibrations were required.

Table 2. Measures of HJ 4583 Cen

Date	PA °		Separation "	
	Jenkinson	Napier-Munn	Jenkinson	Napier-Munn
21-Apr-12	173.23	173.26	21.437	21.443
11-May-12	172.76	172.75	21.402	21.669
12-May-12	173.59	173.54	21.573	21.530
18-May-12	172.35	172.40	21.298	21.651
15-Jun-12	171.95	171.89	21.807	21.726
30-Jun-12	171.61	171.71	22.148	22.104
06-Jul-12	171.88	171.91	21.924	21.913
Mean	**172.48**	**172.49**	**21.656**	**21.719**
Std. Dev.	0.740	0.716	0.312	0.225
Coeff. of var. %	0.43	0.42	1.44	1.04
95% conf. int. ±	0.685	0.751	0.289	0.237

WDSC entry: RA 13 25.2 Dec -64 29 Mags 5.3 & 11.0 PA 180° Sep 21.8" (1997) Imaged with 2.36x Barlow.

Table 3 – Comparison of means of measures by the two observers

Statistic	HRG 53 (n = 6)*		HJ 4583 (n = 7)*	
	PA °	Separation "	PA °	Separation "
Mean difference between observers	0.45	0.04	0.01	0.06
SD of difference between observers	2.09	0.09	0.06	0.17
95% confidence interval of difference ±	2.19	0.10	0.05	0.16
Paired t-test 2-sided P-value	0.62	0.33	0.57	0.36

** n = number of measures by each observer*

In the case of the close pair (HRG 53) Reduc's QuadPx facility was used to magnify the image before measurement; this resamples the displayed image and enlarges it by a factor of two while keeping the distribution of the light constant by unit of area.

The data for the subsequent study of the dependence of measurement uncertainty on separation are taken from Napier-Munn and Jenkinson (2009, 2010) and from the authors' unpublished data (report in preparation), together with the two measures presented above, 38 doubles in all.

Discussion

Comparison of measurers

Tables 1 and 2 show very close agreement between the two observers, their mean results being nearly identical for both PA and separation. The paired t-test was used to test the hypothesis that the PA and separation measures of the two observers were not significantly different. Table 3 summarizes the results.

By convention a P-value of 0.05 or less would be considered as evidence in favor of the alternative hypothesis that the measures of the two observers were different. In all four comparisons (PA and separation for each double) P is much greater than 0.05 and we may therefore conclude that there is no evidence that the measures of the two observers are statistically different. This is confirmed by the confidence intervals on each difference which all include zero.

Also of interest is the precision (repeatability) of each observer, which can be compared by comparing the ratio of the variances (i.e., the square of the standard deviations in Tables 1 and 2) of the two observers using an F-test. Again this shows no statistical difference between the observers, with 2-sided P-values in the range 0.45 – 0.94.

Dependency of measurement uncertainty on separation

Inspection of Tables 1 and 2 shows that the standard deviations of the PA measurements for the close double HRG 53 (mean sd 3.06°) were much larger than those for the wider pair HJ 4583 (mean sd 0.73°). Conversely the standard deviations of the separation measurements for the close double (mean sd 0.053") were much smaller than those of the wider pair (mean sd 0.269"). F-tests on the variances confirm that these are statistically significant differences. This suggests that there are systematic variations in measurement uncertainty as a function of the separation of the pair.

Figure 1 shows the relationship between the standard deviation of the PA measurement and the measured separation for the 38 pairs measured by the authors. Although there is a good deal of scatter there is a clear relationship which is linear on log axes. The fitted power function is shown in the figure; regression statistics show that it is statistically significant. This confirms that the uncertainty in the PA measurement increases as the separation decreases, and the rate of increase is high at small separations.

This is to be expected if we assume that the uncertainty in measuring the position of each star center is constant across the image and that the uncertainty in the PA exists in only one dimension, perpendicular to the line joining the two centers. If we consider the range of possible positions of the secondary as a proxy for uncertainty, then it is easy to show that the range of uncertainties in angle must be inversely related to the distance, as in Figure 1. The principle is illustrated in Figure 2, which shows how the angle describing the uncertainty in PA (of the secondary relative to the primary) varies with separation. In this simple diagram $\tan\theta = r/d$ where r is half the range of uncertainty, d is the separation, and θ is the half angle corresponding to the maximum range of uncertainty (we

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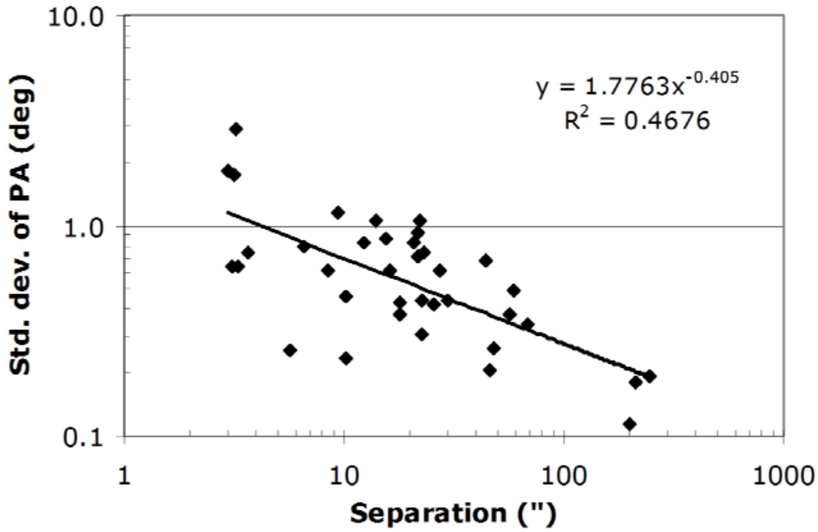

Figure 1. Standard deviation of PA measurement (°) vs measured separation (") for 38 pairs.

are dealing in half angles because for clarity the mirror image of the diagram is not shown). Thus we would expect an inverse relationship between separation and PA uncertainty, which is indeed what we see in Figure 1. (Of course the uncertainties illustrated in Figure 2 apply to the primary as much as to the secondary but the net effect will be the same).

The dependency of the separation standard deviation upon the mean separation is also worth noting, shown in Figure 3. The data have been plotted on a log x-axis, and a power law model fitted shown by the solid line.

As in Figure 1, there is considerable scatter due to factors not captured in the two-factor plot, including the use or otherwise of the QuadPx facility in Reduc and the prevailing seeing which controls the precision with which the center of each star can be determined. However, the regression statistics show that the fitted model is statistically significant and that the increase of separation standard deviation with standard deviation is real. It is likely that this is at least partly due to the propagation of the error inherent in the calibration factor, *E*, which is obtained by measuring pairs of known separation. In our case we used three Barlow configurations: no Barlow, a x2.36 Barlow and a x5.43 Barlow. *E* was calculated for each configuration together with the variance of E based on many repeat measurements of several calibration pairs.

The Reduc software calculates the separation, d, as the number of pixels between the star centers in the image, n, times the calibration factor, *E*. As there is likely to be little or no error in counting the number of

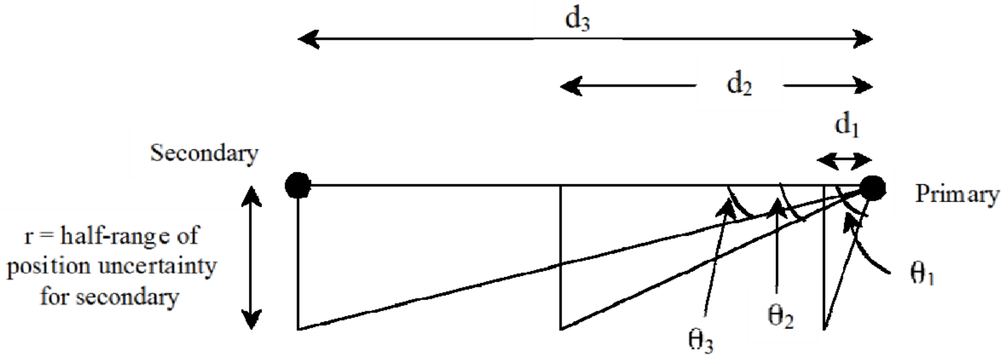

Figure 2. Relationship between the separation (d) and the angle defining the maximum range of uncertainty in determining the position of the secondary component (θ)

A New Visual Double Star in Gemini

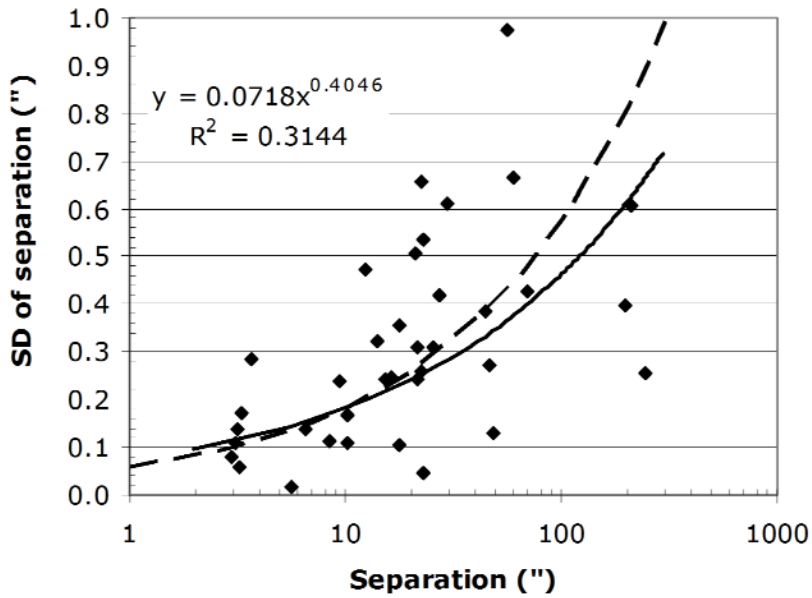

Figure 3. Standard deviation of separation measurement (") vs measured separation for 38 pairs. Continuous line is fitted function; dotted line is theoretical prediction (eqn 2).

pixels (other than the error in locating the star centres which could be significant especially with degraded images in poor seeing), the uncertainty in d is likely to be due mainly to the variance in *E* summed over the number of pixels between the centers. Thus,

$$\sigma_d^2 = n\sigma_E^2 \quad (1)$$

where σ_d^2 = separation variance, σ_E^2 = calibration constant variance, and n = number of pixels between the star centres. As $n = d/E$, then

$$\sigma_d = \sqrt{\frac{d\sigma_E^2}{E}} \quad (2)$$

This value of the predicted separation standard deviation, σ_d, from equation 2 is plotted against the separation, d, as the dotted line in Figure 3, using the average prediction of the three Barlow conditions (each having different values of E and σ_E). The agreement is good. Although this should not be over-interpreted, it does suggest that at least a part of the dependency of the standard deviation of separation on the separation itself is due to the propagation of the error in the calibration constants.

There was no apparent correlation between the standard deviation of PA or separation and the magnitudes of either component or their difference.

Conclusions

An error analysis has been conducted of repeated measures of some southern doubles using the Reduc software to determine PA and separation from stacked images. This adds to earlier analysis by the authors of the errors inherent in determining the PA and separation constants in Reduc (Napier-Munn and Jenkinson, 2009).

It has been shown that independent measures of the same images for two pairs, one close and one more distant, by the two authors give results which are to all intents and purposes identical. We therefore conclude that application of Reduc by competent users should include a negligible component of personal equation, even in the application of the manual option in Reduc.

Analysis of the repeated measures of 38 pairs shows clearly that the uncertainty in the estimate of PA increases as the separation decreases. This is attributed to the simple geometry of the measurement process, whereby a constant uncertainty in the position of the stars' centres has much more effect at small separations than at large separations and cannot be overcome in principle other than by averaging many measures, which is the authors' strategy.

Conversely there is evidence that the uncertainty in the estimate of separation increases with separation.

This is due mainly to propagation of the variance in the separation calibration constants over the number of pixels between the star centers.

For both PA and separation arbitrary precision can be achieved by repeating the measurements enough times according to the usual formula:

where n = required sample size (number of repeats), z

$$n = \left(\frac{z\sigma}{m} \right)^2 \quad (2)$$

= standard normal deviate for desired confidence level (= 1.96 for 95% confidence), σ = known standard deviation of measurement, and m = acceptable margin of error (±).

Particular attention needs to be given to the measurement of the PA of close doubles. The procedure adopted by the authors delivers confidence intervals which are within the precision required for the effective determination of orbits.

As Reduc is essentially an astrometric program that determines PA and separation by counting pixes between identified image centers, the general conclusions of this paper are likely to apply to any similar measurement method.

References

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