Another Statistical Tool for Evaluating Binary Stars

Richard Harshaw

Brilliant Sky Observatory
Cave Creek, Arizona

Abstract: Down through the years, astronomers have proposed many ways to estimate the number of binary and optical pairs in a given section of sky. In this paper, I propose a simple test to determine whether a given pair of stars is binary or optical based on the proper motions of the two stars. It will be shown that there is a very high correlation between binary status and common proper motion and optical status and different proper motions.

Early Methods

Since the earliest days of binary star astronomy, astronomers have struggled to determine whether a pair they see in their telescopes is truly binary (that is, gravitationally bound to each other) or optical (a chance alignment of two stars that are very far apart in reality). Optical pairs may exhibit a linear change in relative position due to differences in proper motion, while binaries may show a non-linear / curved change due to orbital motion. (Of course, an optical pair whose components both have very small proper motions, if they are very distant, would not exhibit any significant motion, linear or otherwise, over a human lifetime or centuries or even millenia.) An early observer of double stars, John Michell, wrote (in 1767, see ref. 1) that the chance alignment theory of double stars was not adequate to account for all the double stars that astronomers could see in their telescopes. He strongly believed that many of the double stars he and his fellow astronomers observed were actually gravitationally bound. But the instrumentation of that time did not allow him (or anyone else) to verify his hypothesis.

The great English astronomer, William Herschel, arguably one of the greatest astronomers of all time, studied double stars to see if their (assumed) different distances would show different parallaxes and thus prove they were chance alignments. Of course, instrumentation of that day did not have the accuracy to reveal even the slightest parallax. This led Herschel to ponder the question of double stars actually being gravitationally bound. But in his research, he found more double stars than he felt should be expected. A few years after his initial survey, he returned to the doubles he had discovered and found that many of them displayed a change in their measurements since his first observation. He concluded that most of the double stars astronomers could view in their telescopes were in fact gravitationally bound or true binaries.

In 1852, Wilhelm Friederich Struve [2] published a formula based on his empirical studies of double star distribution and measurements. This formula predicted how many optical pairs there would be in a piece of sky of area "A" where the total number of stars in "A" was known.

Other astronomers proposed statistical methods to identify possible binaries (Aitken [3], Couteau [4], et al).

In 1986, Jean-Louis Halbwachs wrote an article titled "Common Proper Motion Stars in the AGK3" [5]. Halbwachs set down three statistical criteria to identify potential binary stars given their proper motions and parallax.

All of these statistical approaches have merit. But in this paper I'm going to approach the problem from the other end — I am going to examine the physical properties of known binaries and suspected optical pairs, using the proper motions of both components of the system. I will show that by examining the proper motions of a given pair, we can obtain reasonable certainty that the pair is, in fact, a binary (or a linear).

Methodology

I downloaded the most recent Washington Double Star Catalog and imported the records into an Excel

spreadsheet. I then used Excel's filtering function to select pairs that had eight or more measurements. This returned 5,347 records. I copied those records to a new spreadsheet and began a long process of sending in batch requests to the Washington Double Star Catalog. It took me approximately a year to get the data from Brian Mason and William Hartkopf at the US Naval Observatory (due to the fact that I requested the data in small batch requests) and about 10 months to analyze the records. I ran the measurement histories through an Excel spreadsheet that I created to correct the data for precession and then plot the measurements on a graph. I found that 397 of the pairs I requested had orbital solutions already; 187 had linear solutions; 935 showed what appeared to be a part of an ellipse on the plot for the measurements; another 1,118 showed signs of linear motion; and 2,124 pairs showed no discernible pattern to their motions but did have common proper motion. (The plot of the data resembled the pattern of buckshot made by a shotgun at close range.) Thus only about 600 pairs in my survey could not be classified under either category because the proper motion of only one component is known.

I then turned my attention to the pairs with known orbital or linear solutions. All of the pairs in this group had proper motions of both components on record.

The Statistical Sieve I Used

I was not surprised to see that almost all of the orbital solutions had common proper motions. Likewise almost all of the linear pairs had quite different proper motions. In only a few cases did an orbital solution involve two stars with very different proper motions, and only a few cases of linear solutions had identical proper motions. (This is especially attributable to the possibility that there were typographical errors in the original PM data.) Since the correlation to orbital and linear solutions was so strongly tied to the proper motions, I decided to create a sieve for this project based on a computation involving the proper motion vectors of each star.

It is customary to list the proper motion of a star with two numbers. The first number gives the annual displacement in right ascension in milli-arc seconds (MAS) per year with a positive number indicating motion to the east and a negative number motion to the west. Similarly, the declination number is positive when the star is moving northward and negative when it is moving southward. For example, the stars SAO 132119 and GSC 4753:1581 are a little over two minutes apart in the sky. They could be seen as a wide double star in even a modest telescope. However, the proper motions of these two stars are quite different. The SAO star has proper motion of [-15 +50] while the GSC star has [+175 +134]. Thus, the SAO star is moving to the north northwest while the GSC star is moving to the northeast. Clearly, these two are not traveling through space together.

Since I had the proper motions for both stars for all the pairs in my sifted list, it was a simple step to subtract the proper motion vectors.

Using my SAO/GSC pair as an example, the difference would be [-15 +50] + [-175 -134] = [-190 -84].

But the difference in vectors is only part of the story. I also wanted to know how the difference compared to the sum of the two proper motion vectors. So I divided the difference in vectors by the sum of each vector's length to get a number (ranging from 0.00 to 0.99…). This let me "normalize" the calculations since a small difference with small proper motions would still give a rather large number (say, 0.68) while that same small difference with large proper motions would give a small number (say, 0.05).

Using my SAO/GSC pair, the difference between the PM vectors (-190, -84) gives rise to a scalar value of $(190^2 + 84^2)^{1/2}$ or 207.74. The sum of the two PM vectors would be found by taking $(15^2 + 50^2)^{1/2}$ or 52.20, plus $(175^2 + 134^2)^{1/2}$ or 220.41, for a total of 272.6 Dividing the difference in the vectors by the sum of the vectors results in 207.74 / 272.6 or 0.8. I call this last result (0.8) the "rating" and its value will range from 0.00 for orbital pairs up to 0.99 for optical ones.

On the following pages, I present two tables: Table 1 contains the orbital pairs that I studied and Table 2 contains the linear pairs. In Table 1, note that with only a few exceptions, the ratings cluster around zero. In Table 2, notice how the ratings cluster near 100%. There are a few notable exceptions that I will discuss in

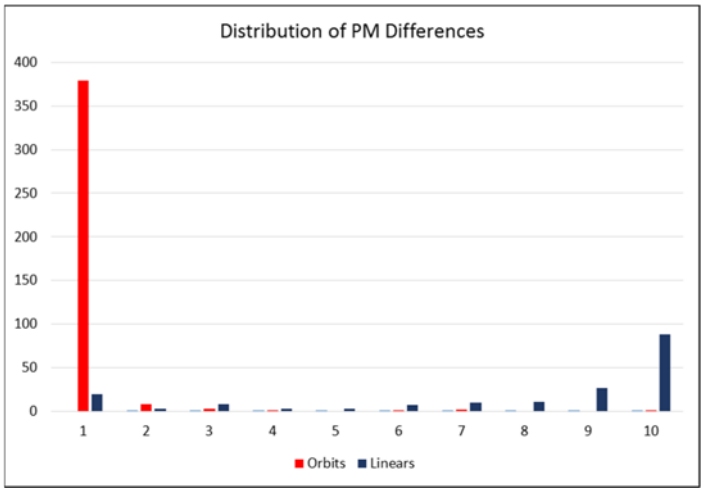

Figure 1: Distribution of proper motion differencese .

(Continued on page 48)

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Table 1: Orbital Solutions

WDS No.	PM A RA	PM A Dec	PM B RA	PM B Dec	Diff Vector Mag[1]	Mag, Sum of PM	Rating (%)
00003-4417	62	-107	62	-107	0.0	247.3	0.0
00014+3937	-30	-48	-30	-48	0.0	113.2	0.0
00048+3810	29	-60	29	-60	0.0	133.3	0.0
00057+4549 AB	879	-154	879	-154	0.0	1784.8	0.0
00063+5826	271	30	271	30	0.0	545.3	0.0
00093+7943	106	-33	106	-33	0.0	222.0	0.0
00098-3347	-62	-150	-62	-150	0.0	324.6	0.0
00162+7657	18	1	18	1	0.0	36.1	0.0
00167+3629	93	53	93	53	0.0	214.1	0.0
00184+4401 AB	2860	390	2884	410	31.2	5799.5	0.5
00206+1219	-27	7	-27	7	0.0	55.8	0.0
00210+6740	27	78	27	78	0.0	165.1	0.0
00214+6700 AB	25	0	25	0	0.0	50.0	0.0
00271-0753	-37	18	-37	18	0.0	82.3	0.0
00283+6344	63	3	63	3	0.0	126.1	0.0
00315-6257 CD	95	-49	95	-49	0.0	213.8	0.0
00321+6715 AB	172	-24	172	-32	8.0	348.6	2.3
00345-0433 AB	85	5	85	5	0.0	170.3	0.0
00360+2959 AB	191	-403	177	-388	20.5	872.4	2.4
00373-2446	1396	-14	1400	-10	5.7	2796.1	0.2
00424+0410 AB	27	-38	27	-38	0.0	93.2	0.0
00491+5749 AB	1141	-572	1105	-493	86.8	2486.3	3.5
00504+5038 AB	-36	-7	-36	-7	0.0	73.3	0.0
00521+1036	43	-33	43	-33	0.0	108.4	0.0
00521-1314	77	-93	77	-93	0.0	241.5	0.0
00546+1911 AB	15	-12	15	-12	0.0	38.4	0.0
00550+2338 AB	136	-30	136	-30	0.0	278.5	0.0
00583+2124	-13	-1	-13	-1	0.0	26.1	0.0
01030+4723	81	-12	81	-12	0.0	163.8	0.0
01055+2107	-13	-34	-13	-34	0.0	72.8	0.0
01095+4715 AB	0	-11	0	-11	0.0	22.0	0.0
01106+5101 Aa, Ab	1	21	1	21	0.0	42.	0.0
01148+6056	64	-44	64	-44	0.0	155.3	0.0
01158-6853 AB	405	108	383	87	30.4	811.9	3.7
01200-1549	53	-107	53	-107	0.0	238.8	0.0
01213+1132 AB	60	1	60	1	0.0	120.0	0.0
01337-1213	-11	-9	-11	-9	0.0	28.4	0.0
01361-2954	105	38	105	38	0.0	223.3	0.0
01388-1758	339	56	330	56	9.0	678.3	1.3
01424-0645	-32	-46	-32	-46	0.0	112.1	0.0

1. The magnitude of the difference vector in MAS

Table continues on next page.

Table 1 (cont.) : Orbital Solutions

WDS No.	PM A RA	PM A Dec	PM B RA	PM B Dec	Diff Vector Mag[1]	Mag, Sum of PM	Rating (%)
01437+0934	-45	-42	-45	-42	0.0	123.1	0.0
01456-2503 AB	162	-57	160	-72	15.1	347.2	4.4
01467+3310 AB	-26	-21	-26	-21	0.0	66.8	0.0
01499+8053	-17	1	-17	1	0.0	34.1	0.0
01551+2847 AB	28	-35	28	-35	0.0	89.6	0.0
01559+0151	164	189	164	189	0.0	500.5	0.0
02020+0246 AB	42	-11	42	-11	0.0	86.8	0.0
02022+3643 A, BC	147	-54	139	-56	8.2	306.5	2.7
02037+2556 AB	134	10	134	10	0.0	268.7	0.0
02140+4729	-77	-53	-77	-53	0.0	187.0	0.0
02158-1814	-52	-135	-52	-135	0.0	289.3	0.0
02159+0638	-111	-56	-111	-56	0.0	248.7	0.0
02174+6121 AB	51	-87	51	-87	0.0	201.7	0.0
02202+2949	-11	-14	-11	-14	0.0	35.6	0.0
02232-2952	-84	-104	-84	-104	0.0	267.4	0.0
02257+6133	17	-13	17	-13	0.0	42.8	0.0
02291+6724 AB	-18	12	-18	12	0.0	43.3	0.0
02407+2637	83	1	83	1	0.0	166.0	0.0
02460-0457	50	2	50	2	0.0	100.1	0.0
02475+1922 AB	115	-159	129	-169	17.2	408.8	4.2
02572-2458 AB	32	-35	32	-35	0.0	94.8	0.0
02589+2137	44	-19	44	-19	0.0	95.9	0.0
03124-4425 AB	92	-6	92	-6	0.0	184.4	0.0
03127+7133	15	16	15	16	0.0	43.9	0.0
03140+0044	76	-14	76	-14	0.0	154.6	0.0
03162+5810	446	-340	436	-305	36.4	1092.9	3.3
03175+6540 AB	-9	5	-9	5	0.0	20.6	0.0
03177+3838	112	-44	112	-44	0.0	240.7	0.0
03184-0056 AB	254	-62	254	-62	0.0	522.9	0.0
03217+0845	-27	-98	-27	-98	0.0	203.3	0.0
03350+6002	-25	28	-25	28	0.0	75.1	0.0
03362+4220	143	-146	143	-146	0.0	408.7	0.0
03463+2411 AB	18	-41	18	-41	0.0	89.6	0.0
03489+6445	21	-60	21	-60	0.0	127.1	0.0
03590+0947	95	-50	95	-50	0.0	214.7	0.0
04021-3429 AB	374	-12	374	-12	0.0	748.4	0.0
04041+3931	142	-98	142	-98	0.0	345.1	0.0
04064+4325	-28	-195	-28	-195	0.0	394.0	0.0
04076+3804 AB	174	-231	174	-231	0.0	578.4	0.0
04100+8042	-15	1	-15	1	0.0	30.1	0.0
04159+3142 AB	8	-51	8	-51	0.0	103.2	0.0

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Table continues on next page.

Table 1 (cont.) : Orbital Solutions

WDS No.	PM A RA	PM A Dec	PM B RA	PM B Dec	Diff Vector Mag[1]	Mag, Sum of PM	Rating (%)
04170+1941	1	-101	1	-101	0.0	202.0	0.0
04179+5847	-17	-5	-17	-5	0.0	35.4	0.0
04182+2248	-5	-27	-5	-27	0.0	54.9	0.0
04215-2544 AB	48	-53	48	-53	0.0	143.0	0.0
04227+1503 AB	114	-26	114	-26	0.0	233.9	0.0
04233+1123	25	-38	25	-38	0.0	91.0	0.0
04269-2405	-9	-19	-9	-19	0.0	42.0	0.0
04290+1610	105	-25	105	-25	0.0	215.9	0.0
04301+1538	111	-28	111	-28	0.0	229.0	0.0
04366+1946	-8	-2	-8	-2	0.0	16.5	0.0
04367+1930	35	-39	35	-39	0.0	104.8	0.0
04382-1418	-76	-167	-76	-167	0.0	367.0	0.0
04422+3731	47	2	47	2	0.0	94.1	0.0
04518+1339 AB	90	-17	90	-17	0.0	183.2	0.0
04590-1623 AB	-132	136	-132	136	0.0	379.1	0.0
05005+0506	35	-38	35	-38	0.0	103.3	0.0
05017+2050	-67	-90	-67	-90	0.0	224.4	0.0
05047+7404	9	-17	9	-17	0.0	38.5	0.0
05055+1948	-10	-14	-10	-14	0.0	34.4	0.0
05079+0830	22	-61	22	-61	0.0	129.7	0.0
05098+2802 BC	103	16	26	-18	84.2	135.9	62.0
05135+0158 AB	7	2	7	2	0.0	14.6	0.0
05239-0052 BC	-5	-9	-5	-9	0.0	20.6	0.0
05247+6323	-121	-51	-121	-51	0.0	262.6	0.0
05308+0557	5	-42	5	-42	0.0	84.6	0.0
05320-0018 Aa, Ab	2	1	2	1	0.0	4.5	0.0
05364+2200	-35	-77	-41	-87	11.7	180.8	6.5
05371+2655 AB	7	-24	7	-24	0.0	50.0	0.0
05386+3030 AB	-21	-12	-21	-12	0.0	48.4	0.0
05407-0157	4	3	4	3	0.0	10.0	0.0
05474+2939	-15	-122	-15	-122	0.0	245.8	0.0
05589+1248	-12	3	-12	3	0.0	24.7	0.0
06003-3102 AC	-47	41	-44	102	61.1	173.5	35.2
06024+0939 AB	12	-34	12	-34	0.0	72.1	0.0
06048-4828	-106	-27	-106	-27	0.0	218.8	0.0
06298-5014 AB	-65	-52	-65	-52	0.0	166.5	0.0
06298-5014 CD	-68	-52	-68	-52	0.0	171.2	0.0
06344+1445	14	4	14	4	0.0	29.1	0.0
06364+2717	7	-98	7	-98	0.0	196.5	0.0
06462+5927 AB	-26	-17	-26	-17	0.0	62.1	0.0

1. The magnitude of the difference vector in MAS

Table continues on next page.

Table 1 (cont.) : Orbital Solutions

WDS No.	PM A RA	PM A Dec	PM B RA	PM B Dec	Diff Vector Mag[1]	Mag, Sum of PM	Rating (%)
06474+1812	5	-39	5	-39	0.0	78.6	0.0
06478+0020	-1	3	-1	3	0.0	6.3	0.0
06546+1311 AB	71	-80	70	-78	2.2	211.8	1.1
06555+3010	224	-239	224	-239	0.0	655.1	0.0
07128+2713 AB	14	-101	14	-101	0.0	203.9	0.0
07176+0918	-16	-114	-16	-114	0.0	230.2	0.0
07294-1500 AB	-188	-237	-188	-237	0.0	605.0	0.0
07303+4959	-11	-47	-11	-47	0.0	96.5	0.0
07346+3153 AB	-206	-148	-206	-148	0.0	507.3	0.0
07366-1429 AB	2	2	-32	-24	42.8	42.8	99.9
07417+0942	-11	-30	-11	-30	0.0	63.9	0.0
07461+2107	33	-7	33	-7	0.0	67.5	0.0
07518-1354	-60	-339	-60	-339	0.0	688.5	0.0
07546-0248	-22	-5	-22	-5	0.0	45.1	0.0
08024+0409	65	-105	65	-105	0.0	247.0	0.0
08041+3302	-13	7	-13	7	0.0	29.5	0.0
08044+1217 AB	101	-138	101	-138	0.0	342.0	0.0
08061-0047	87	-64	87	-64	0.0	216.0	0.0
08095+3213 AB	33	-8	33	-8	0.0	67.9	0.0
08122+1739 AB	80	-129	80	-129	0.0	303.6	0.0
08122+1739 AB, C	80	-129	86	-91	38.5	277.0	13.8
08122+1739 BC	80	-129	86	-91	38.5	277.0	13.9
08213-0136	-9	-28	-9	-28	0.0	58.8	0.0
08267+2432 BC	-37	-80	-37	-80	0.0	176.3	0.0
08331-2436 AB	-4	-31	-4	-31	0.0	62.5	0.0
08391-2240 AB	-261	430	-261	430	0.0	1006.0	0.0
08468+0625 AB, C	-194	-54	-262	14	96.2	463.7	20.7
08531+5457	38	42	38	42	0.0	113.3	0.0
08549+2612 AB	54	-441	54	-441	0.0	888.6	0.0
08554+7048 AB	-133	-36	-136	-41	5.8	279.8	2.1
08568-1726	-47	-2	-52	-28	26.5	106.1	25.0
09013+1516	-124	-314	-111	-320	14.3	676.3	2.1
09104+6708 AB	7	-95	4	-30	65.1	125.5	51.8
09144+5241 AB	-156	-57	-156	-66	9.0	335.5	2.7
09186+2049	-4	-50	-4	-50	0.0	100.3	0.0
09210+3811	-20	-23	-20	-23	0.0	61.0	0.0
09273+0614	-178	-151	-178	-151	0.0	466.8	0.0
09278-0604	-227	-77	-227	-77	0.0	479.4	0.0
09285+0903	50	-5	50	-5	0.0	100.5	0.0
09307-4028	-191	73	-191	73	0.0	408.9	0.0

1. The magnitude of the difference vector in MAS

Table continues on next page.

Table 1 (cont.) : Orbital Solutions

WDS No.	PM A RA	PM A Dec	PM B RA	PM B Dec	Diff Vector Mag[1]	Mag, Sum of PM	Rating (%)
09512+3629 AB	-98	-20	-98	-20	0.0	200.0	0.0
09525-0806	-57	-42	-57	-42	0.0	141.6	0.0
09551-2632	-149	-42	-149	-42	0.0	309.6	0.0
10043-2823	-41	-23	-41	-23	0.0	94.0	0.0
10062-4722	-1	-63	-1	-63	0.0	126.0	0.0
10131+2725	-50	-123	-50	-123	0.0	265.5	0.0
10163+1744	-7	1	-7	1	0.0	14.1	0.0
10192+2034	-164	-20	-164	-20	0.0	330.4	0.0
10200+1950 AB	311	-153	306	-161	9.4	692.4	1.4
10227+1521	-251	-101	-262	-86	18.6	546.3	3.4
10250+2437	12	-194	12	-194	0.0	388.7	0.0
10269+1713	-42	-67	-42	-67	0.0	158.2	0.0
10275+0334	-131	-22	-131	-22	0.0	265.7	0.0
10292+1009	-3	-30	-3	-30	0.0	60.3	0.0
10361-2641	-8	-86	-8	-86	0.0	172.7	0.0
10397+0851	-105	6	-105	6	0.0	210.3	0.0
10480+4107	-21	-3	-21	-3	0.0	42.4	0.0
10525-1838	-71	27	-71	27	0.0	151.9	0.0
10596+2527	-184	-50	-178	-82	32.6	386.7	8.4
11035+5432	-126	77	-126	77	0.0	295.3	0.0
11047-0413 BC	-178	-104	-178	-104	0.0	412.3	0.0
11125-1830	-21	-27	-21	-27	0.0	68.4	0.0
11137+2008 AB	-388	-125	-388	-125	0.0	815.3	0.0
11152+7329 AC	-402	110	-404	112	2.8	836.0	0.3
11182+3132 AB	-454	-591	-454	-591	0.0	1490.5	0.0
11190+1416	54	-161	54	-161	0.0	339.6	0.0
11210-5429	-40	-3	-40	-3	0.0	80.2	0.0
11239+1032 AB	153	-65	153	-65	0.0	332.5	0.0
11247-6139	-491	99	-562	77	74.3	1068.1	7.0
11272-1539	2	-197	2	-197	0.0	394.0	0.0
11308+4117	92	-98	92	-98	0.0	268.8	0.0
11363+2747 AB	27	13	27	13	0.0	59.9	0.0
11368-1221	7	-46	7	-46	0.0	93.1	0.0
11387+4507 AB	-594	15	-577	2	21.4	1171.2	1.8
11390+4109 AB	-81	-38	-81	-38	0.0	178.9	0.0
11486+1417	-102	4	-102	4	0.0	204.2	0.0
12018-3439	-191	17	-191	17	0.0	383.5	0.0
12036-3901	-383	-39	-383	-39	0.0	770.0	0.0
12158-2321	-9	-39	-9	-39	0.0	80.1	0.0
12160+0538	-320	-66	-320	-66	0.0	653.5	0.0

1. The magnitude of the difference vector in MAS

Table continues on next page.

Table 1 (cont.) : Orbital Solutions

WDS No.	PM A RA	PM A Dec	PM B RA	PM B Dec	Diff Vector Mag[1]	Mag, Sum of PM	Rating (%)
12244+2535 AB	-13	-13	-13	-13	0.0	36.8	0.0
12266-6306 AB	-35	-15	-43	-8	10.6	81.8	13.0
12272+2701	81	-248	81	-248	0.0	521.8	0.0
12301-1324 AB	-251	-47	-238	-32	19.8	495.5	4.0
12306+0943	51	-58	51	-58	0.0	154.5	0.0
12396-3717	-262	-268	-262	-268	0.0	749.6	0.0
12415-4858 AB	-194	-12	-194	-12	0.0	388.7	0.0
12422+2622	2	80	2	80	0.0	160.1	0.0
12533+2115 AB	-48	-27	-48	-27	0.0	110.1	0.0
12567-4741	-60	-3	-60	-3	0.0	120.1	0.0
13038-2035	134	1	134	1	0.0	268.0	0.0
13100+1732 AB	-430	138	-430	138	0.0	903.2	0.0
13169+1701 AB	624	-259	631	-261	7.3	1358.5	0.5
13235+2914	-468	245	-468	245	0.0	1056.5	0.0
13237-0043	-53	-23	-53	-23	0.0	115.6	0.0
13258+4430 AB	22	-8	22	-8	0.0	46.8	0.0
13284+1543	-57	13	-57	13	0.0	116.9	0.0
13328+1649	270	-226	270	-226	0.0	704.2	0.0
13336+2944	-52	66	-52	66	0.0	168.0	0.0
13343-0019 AB	-219	21	-219	21	0.0	440.0	0.0
13372-6142 AB	154	-111	154	-111	0.0	379.7	0.0
13375+3618 AB	-104	28	-85	2	32.2	192.7	16.7
13461+0507	-103	-35	-103	-35	0.0	217.6	0.0
13491+2659	-427	-90	-436	-111	22.8	886.3	2.6
13520-3137	-38	-45	-38	-45	0.0	117.8	0.0
13535-3540 AB	-86	-21	-86	-21	0.0	177.1	0.0
13550-0804	-175	-34	-166	-50	18.4	351.6	5.2
13577+5200	236	-7	236	-7	0.0	472.2	0.0
14020+5713 AB	-8	-11	-8	-11	0.0	27.2	0.0
14131+5520	-345	-12	-345	-12	0.0	690.4	0.0
14135+5147	61	-11	65	-4	8.1	127.1	6.3
14153+0308	-198	50	-198	50	0.0	408.4	0.0
14160-0704	108	-19	108	-19	0.0	219.3	0.0
14195-1343	-174	-47	-190	-45	16.1	375.5	4.3
14203+4830	-73	-14	-73	-14	0.0	148.7	0.0
14234+0827 BC	-75	-10	-75	-10	0.0	151.3	0.0
14369+4813	-17	46	-17	46	0.0	98.1	0.0
14411+1344 AB	66	-21	66	-21	0.0	138.5	0.0
14428+0635 AB	-164	64	-164	64	0.0	352.1	0.0
14455+4223 AB	-67	54	-67	54	0.0	172.1	0.0

1. The magnitude of the difference vector in MAS

Table continues on next page.

Another Statistical Tool for Evaluating Binary Stars

Table 1 (cont.) : Orbital Solutions

WDS No.	PM A RA	PM A Dec	PM B RA	PM B Dec	Diff Vector Mag[1]	Mag, Sum of PM	Rating (%)
14463+0939 AB	71	-266	71	-266	0.0	550.6	0.0
14464-0723 AB	-150	103	-150	103	0.0	363.9	0.0
14489+0557	-8	-89	-8	-89	0.0	178.7	0.0
14514+1906 AB	159	-136	90	-147	69.9	381.6	18.3
14515+4456	-13	-39	-13	-39	0.0	82.2	0.0
14534+1542	-20	30	-20	30	0.0	72.1	0.0
14542-6625	-280	-178	-280	-178	0.0	663.6	0.0
14575-2125 AB	104	-173	99	-166	8.6	395.1	2.2
14575-2125 BC	99	-167	106	-167	7.0	391.9	1.8
14587-2739	-44	-11	-44	-11	0.0	90.7	0.0
15038+4739	-444	10	-374	39	75.8	820.1	9.2
15160-0454 AB	23	-29	23	-29	0.0	74.0	0.0
15183+2650 AB	88	77	88	77	0.0	233.9	0.0
15245+3723 Ba, Bb	-151	87	-151	87	0.0	348.5	0.0
15348+1032 AB	-72	13	-73	3	10.0	146.2	6.9
15351-4110	-28	-42	-28	-42	0.0	101.0	0.0
15360+3948 AB	-455	51	-455	51	0.0	915.7	0.0
15382+3615 AB	-62	43	-60	35	8.2	144.9	5.7
15382+3615 CD	-62	43	-62	43	0.0	150.9	0.0
15396+7959	-33	39	-33	39	0.0	102.2	0.0
15413+5959	-220	165	-220	165	0.0	550.0	0.0
15427+2618	-107	48	-107	48	0.0	234.5	0.0
16035-5747 AB	-157	-95	-157	-95	0.0	367.0	0.0
16044-1122 AB	-72	-35	-72	-35	0.0	160.1	0.0
16044-1122 AC	-72	-35	-75	-28	7.6	160.1	4.8
16085-1006	36	-110	36	-110	0.0	231.5	0.0
16147+3352 AB	-265	-84	-289	-85	24.0	579.2	4.1
16289+1825 AB	-340	383	-340	383	0.0	1024.3	0.0
16309+0159 AB	-21	-88	-21	-88	0.0	180.9	0.0
16413+3006	-43	92	-43	92	0.0	203.1	0.0
16439+4329	-75	-51	-75	-51	0.0	181.4	0.0
16492+4559 A, BC	22	-56	23	-51	5.1	116.1	4.4
16511+0924 AB	16	-122	16	-122	0.0	246.1	0.0
16518+2840 AB	-6	36	-6	36	0.0	73.0	0.0
16564+6502 AB	-33	34	-33	34	0.0	94.8	0.0
17053+5428 AB	-68	89	-68	89	0.0	224.0	0.0
17066+0039	-21	-10	-21	-10	0.0	46.5	0.0
17082-0105	-14	-41	-14	-41	0.0	86.6	0.0
17141+5608	-17	15	-17	15	0.0	45.3	0.0
17146+1423 AB	-17	47	-7	33	17.2	83.7	20.6

1. The magnitude of the difference vector in MAS

Table continues on next page.

Table 1 (cont.) : Orbital Solutions

WDS No.	PM A RA	PM A Dec	PM B RA	PM B Dec	Diff Vector Mag[1]	Mag, Sum of PM	Rating (%)
17157-0949 AB	12	-21	12	-21	0.0	48.4	0.0
17166-0027	-31	-73	-31	-73	0.0	158.6	0.0
17191-4638 AB	105	14	97	14	8.0	203.9	3.9
17248+3044	-2	5	-2	5	0.0	10.8	0.0
17293+2924 AB	-189	-278	-189	-278	0.0	672.3	0.0
17304-0104	-127	-180	-127	-180	0.0	440.6	0.0
17350+6153 AB	265	-521	277	-526	13.0	1179.0	1.1
17386+5546	4	-12	4	-12	0.0	25.3	0.0
17457+1743	-25	74	-25	74	0.0	156.2	0.0
17471+1742	12	-13	12	-13	0.0	35.4	0.0
17506+0714	12	19	12	19	0.0	44.9	0.0
17530+8354	5	8	5	8	0.0	18.9	0.0
17533+2459	-70	-50	-70	-50	0.0	172.0	0.0
17571+0004	26	-9	26	-9	0.0	55.0	0.0
18025+4414 AB	-40	-67	-40	-67	0.0	156.1	0.0
18031-0811 AB	9	-48	9	-48	0.0	97.7	0.0
18055+0230 AB	28	-109	44	-125	22.6	245.1	9.2
18058+2127 AB	-29	-40	-29	-40	0.0	98.8	0.0
18063+3824	-17	-10	-17	-10	0.0	39.4	0.0
18068-4325	14	-105	14	-105	0.0	211.9	0.0
18068-4325	14	-105	14	-105	0.0	211.9	0.0
18096+0400 AB	46	-7	46	-7	0.0	93.1	0.0
18097+5024	7	7	7	7	0.0	19.8	0.0
18146+0011	20	-35	20	-35	0.0	80.6	0.0
18250+2724 AB	3	-12	3	-12	0.0	24.7	0.0
18250-0135	5	-6	5	-6	0.0	15.6	0.0
18355+2336	5	3	5	3	0.0	11.7	0.0
18359+1659 AB	49	-71	50	-67	4.1	169.9	2.4
18360+1144	12	-11	12	-11	0.0	32.6	0.0
18428+5938 AB	-133	181	-141	184	8.5	456.4	1.9
18437+3141	-43	-41	-43	-41	0.0	118.8	0.0
18443+3940 AB	11	61	2	47	16.6	109.0	15.3
18443+3940 CD	8	54	8	54	0.0	109.2	0.0
18537-0533	-24	-33	-24	-33	0.0	81.6	0.0
18575+5814	4	37	39	2	49.5	76.3	64.9
19027-0043 BC	37	-9	37	-9	0.0	76.2	0.0
19043-2132	69	-41	69	-41	0.0	160.5	0.0
19055+3352	17	-31	17	-31	0.0	70.7	0.0
19062+3026 AB	64	-5	64	-5	0.0	128.4	0.0
19064-3704	88	-284	88	-284	0.0	594.6	0.0

1. The magnitude of the difference vector in MAS

Table continues on next page.

Table 1 (cont.) : Orbital Solutions

WDS No.	PM A RA	PM A Dec	PM B RA	PM B Dec	Diff Vector Mag[1]	Mag, Sum of PM	Rating (%)
19121+4951 AB	-210	622	-184	629	26.9	1311.9	2.1
19143+1904	32	-27	32	-27	0.0	83.7	0.0
19266+2719	97	88	97	88	0.0	261.9	0.0
19296-1239	104	-36	104	-36	0.0	220.1	0.0
19316+1747	-7	-120	5	-116	12.6	236.3	5.4
19394+2215	3	13	3	13	0.0	26.7	0.0
19406+6240	26	111	26	111	0.0	228.0	0.0
19418+5032 AB	-148	-159	-132	-163	16.5	427.0	3.9
19464+3344 FG	19	-446	19	-446	0.0	892.8	0.0
19487+3519	74	63	74	63	0.0	194.4	0.0
19520-1021 AB	-14	-49	-14	-49	0.0	101.9	0.0
19580+0456	26	3	26	3	0.0	52.3	0.0
20012-3835	18	-70	18	-70	0.0	144.6	0.0
20014+1045 AB	83	25	82	54	29.0	184.9	15.7
20102+4357	9	89	9	89	0.0	178.9	0.0
20176+2622	-8	-78	-8	-78	0.0	156.8	0.0
20182+2912	-6	-87	-6	-87	0.0	174.4	0.0
20198+4522	18	-29	18	-29	0.0	68.3	0.0
20289-1749 AB	-3	-17	-3	-17	0.0	34.5	0.0
20312+1116 BC	-6	-10	-6	-10	0.0	23.3	0.0
20393-1457 AB	2	-21	2	-21	0.0	42.2	0.0
20396+4035 AB	0	-4	0	-4	0.0	8.0	0.0
20407+4321	147	70	145	68	2.8	323.0	0.9
20450+1244 AB	-11	-4	-11	-4	0.0	23.4	0.0
20462+1554 AB	95	67	100	75	9.4	241.2	3.9
20467+1607	-31	-198	-1	-204	30.6	404.4	7.6
20494+1124 AB	196	-132	196	-132	0.0	472.6	0.0
20519+0544	75	-4	75	-4	0.0	150.2	0.0
20550+2805 AB	109	-81	109	-81	0.0	271.6	0.0
20591+0418 AB	-102	-152	-102	-152	0.0	366.1	0.0
21031+0132 AB	-107	-53	-107	-53	0.0	238.8	0.0
21069+3845 AB	416	326	412	313	13.6	1045.9	1.3
21148+3803 AB	159	431	159	431	0.0	918.8	0.0
21208+3227 AB	53	-45	53	-45	0.0	139.1	0.0
21223+5734 AB	74	26	77	28	3.6	160.4	2.2
21289+1105	71	14	71	14	0.0	144.7	0.0
21395-0003 AB	228	15	228	15	0.0	457.0	0.0
21410+2920	73	-14	73	-14	0.0	148.7	0.0
21426+4103 AB	26	-1	26	-1	0.0	52.0	0.0
21441+2845 AB	277	-251	277	-251	0.0	747.6	0.0

1. The magnitude of the difference vector in MAS

Table concludes on next page.

Table 1 (conc.) : Orbital Solutions

WDS No.	PM A RA	PM A Dec	PM B RA	PM B Dec	Diff Vector Mag[1]	Mag, Sum of PM	Rating (%)
21555+1053 AB	-85	-120	-85	-120	0.0	294.1	0.0
21582+8252 AB	-133	-47	-137	-40	8.1	283.8	2.8
22038+6438 AB	208	89	202	86	6.7	445.8	1.5
22086+5917 BC	10	1	10	1	0.0	20.1	0.0
22100+2308	-19	-8	-19	-8	0.0	41.2	0.0
22202+2931	78	18	78	18	0.0	160.1	0.0
22266-1645 AB	251	-21	251	-21	0.0	503.8	0.0
22280+5742 AB	-802	-386	-713	-321	110.2	1672.0	6.6
22300+0426	-31	-146	-31	-146	0.0	298.5	0.0
22302+2228	22	-22	22	-22	0.0	62.2	0.0
22330+6955	117	66	117	66	0.0	268.7	0.0
22400+0113	66	60	66	60	0.0	178.4	0.0
22408-0333	-3	-48	-3	-48	0.0	96.2	0.0
22409+1433 AB	274	137	274	137	0.0	612.7	0.0
22419+2126	64	-69	64	-69	0.0	188.2	0.0
22431+4710 BC	-1	-2	-1	-2	0.0	4.5	0.0
22455+1112 AB	14	-173	14	-163	10.0	337.2	3.0
22478-0414 AB	-200	-313	-200	-313	0.0	742.9	0.0
22514+2623 AB	-4	9	-4	9	0.0	19.7	0.0
22537+4445 AB	-3	-17	-3	-17	0.0	34.5	0.0
22557+1547	122	25	122	25	0.0	249.1	0.0
22592+1144	32	-41	32	-41	0.0	104.0	0.0
23026+4245 AB	59	-6	59	-6	0.0	118.6	0.0
23079+7523 AB	6	-31	6	-31	0.0	63.2	0.0
23103+3229 AB	15	-4	15	-4	0.0	31.0	0.0
23176-0131 AB	255	-93	255	-93	0.0	542.9	0.0
23186+6807 AB	53	24	46	13	13.0	106.0	12.3
23280+3333	-22	-17	-22	-17	0.0	55.6	0.0
23340+3120	49	-17	49	-17	0.0	103.7	0.0
23375+4426 AB	14	-13	14	-13	0.0	38.2	0.0
23431+1150	102	-2	102	-2	0.0	204.0	0.0
23440+2922	71	-39	71	-39	0.0	162.0	0.0
23487+6453 AB	21	-14	21	-14	0.0	50.5	0.0
23506-5142	71	-100	71	-100	0.0	245.3	0.0
23516+4205 AB	-1	-8	-1	-8	0.0	16.1	0.0
23595+3343 AB	-47	-97	-63	-92	16.8	219.3	7.6

1. The magnitude of the difference vector in MAS

Another Statistical Tool for Evaluating Binary Stars

Table 2: Linear Solutions

WDS No.	PM A RA	PM A Dec	PM B RA	PM B Dec	Diff Vector Mag[1]	Mag, Sum of PM	Rating (%)
00028+0208	62	-94	54	-92	8.2	219.3	3.8
00047+3416 AB, C	-17	-27	123	-60	143.8	168.8	85.2
00057+4549 AD	879	-154	3	0	889.4	895.4	99.3
00059+1805 AC	-146	-146	-2	-17	193.3	223.6	86.5
00066+2901 AB	381	-178	5	-13	410.6	434.5	94.5
00159+5233 AB	56	-45	30	-7	46.0	102.6	44.9
00175+0019 AB	20	107	-32	-9	127.1	142.1	89.5
00184+4401 AC	2860	390	-33	-19	2921.8	2924.5	99.9
00187+2545 AB	-3	0	-17	-38	40.5	44.6	90.7
00201+4232	29	31	-15	-3	55.6	57.7	96.3
00224+1329	62	21	-8	-24	83.2	90.8	91.7
00272+4959 AB	5	-4	53	-4	48.0	59.6	80.6
00277-1625 AB	208	33	-76	-142	333.6	371.7	89.8
00282-6555 AC	-11	-6	87	-5	98.0	99.7	98.3
00296-2311	9	-8	-3	5	17.7	17.9	99.0
00305+2208	65	10	-11	-17	80.7	86.0	93.8
00308+1602	-29	-3	42	-26	74.6	78.6	95.0
00360+2959 AC	191	-403	-3	-60	394.1	506.0	77.9
00384+4059	22	-15	-9	-3	33.2	36.1	92.0
00387+4657 AB	-49	3	16	8	65.2	67.0	97.3
00396+8445 AB	50	21	77	-24	52.5	134.9	38.9
00408-0714	9	-102	1	-92	12.8	194.4	6.6
00434-0054 AB	-116	-224	5	6	259.9	260.1	99.9
00444+7713	-9	20	25	-14	48.1	50.6	95.1
00458-4155	307	-76	288	-111	39.8	624.9	6.4
00464+3057	-20	-64	-41	-28	41.7	116.7	35.7
00491+5749 AE	114	-57	-3	-1	129.7	130.6	99.3
00496-5410	68	-1	15	-11	53.9	86.6	62.3
00498+7027	371	203	24	-3	403.5	447.1	90.3
00504+5038 AB, C	-36	-7	3	-3	39.2	40.9	95.8
00568+3830 AD	152	33	-13	12	166.3	173.2	96.0
00594+0047 AB	-26	-109	3	-8	105.1	120.6	87.1
00595+8341	18	-1	102	10	84.7	120.5	70.3
01044-0518 AB	-10	47	14	-29	79.7	80.3	99.3
01048-0528 AB	6	1	58	-1	52.0	64.1	81.2
01083+5455 AB	342	-160	-5	7	385.1	386.2	99.7
01207+4620 AB	-36	8	-9	-6	30.4	47.7	63.8
01211+6439	59	-25	-10	-13	70.0	80.5	87.0
01259+6808 AC	74	26	-2	8	78.1	86.7	90.1
01270-0009 AB	298	-358	-3	-6	463.1	472.5	98.0

Table continues on next page.

Table 2 (cont.): Linear Solutions

WDS No.	PM A RA	PM A Dec	PM B RA	PM B Dec	Diff Vector Mag[1]	Mag, Sum of PM	Rating (%)
01399+1515 AB	-2	-9	220	-63	228.5	238.1	96.0
01417-1119	43	-417	43	-417	0.0	838.4	0.0
01510+2107 AB	63	-28	92	79	110.9	190.2	58.3
02135-2546	70	-27	-15	-15	85.8	96.2	89.2
02442+4914 AB	511	-90	336	-84	175.1	865.2	20.2
02498-2015	20	11	38	-29	43.9	70.6	62.1
02556+2652 AB	274	-185	270	-168	17.5	648.6	2.7
02578+4431 AB	48	6	109	-18	65.6	158.8	41.3
03053+4254	60	-27	-3	1	68.9	69.0	100.0
03122+3713	-38	-39	-38	-39	0.0	108.9	0.0
03440+3822 AB	-1	-4	23	16	31.2	32.1	97.2
03463+2411 AB-C	18	-41	17	-8	33.0	63.6	51.9
04130-2832	90	44	90	44	0.0	200.4	0.0
04153-0739 AD	-226	-342	34	-42	397.0	464.0	85.6
04376-0228 AB	43	-64	1	20	93.9	97.1	96.7
04385+2656 AB	41	-56	41	-88	32.0	166.5	19.2
05100-0704	-60	-240	-3	-4	242.8	252.4	96.2
05119-0907	-72	-559	7	-8	556.6	574.2	96.9
05154+3241 AB	-24	11	25	-1	50.4	51.4	98.1
05167+4600 AF	76	-425	4	-3	428.1	436.7	98.0
05191+4006 AD	679	-665	0	-1	949.7	951.4	99.8
05191+4006 AE	679	-665	-2	1	952.5	952.6	100.0
05226+7914 AB	-79	161	55	-155	343.2	343.8	99.8
05341+6940	-10	78	12	4	77.2	91.3	84.6
05418+1933 AB	17	4	-5	-4	23.4	23.9	98.1
05561+1356 AB	374	-481	-52	11	650.8	662.4	98.2
05561+1356 AD	374	-481	19	-25	577.9	640.7	90.2
06422+5038 AC	-2	1	28	-145	149.1	149.9	99.4
06443+4037	-25	-157	193	38	292.5	355.7	82.2
07166-2319	-1	4	-31	42	48.4	56.3	86.0
07294-1500 AD	-188	-237	20	-12	306.4	325.8	94.0
07366-1429 AD	2	2	-7	3	9.1	10.4	86.7
08047+1204 AB	4	-1	-2	-34	33.5	38.2	87.8
08258+3104 AB	45	-80	5	-5	85.0	98.9	86.0
08358+0637 AE	-127	-129	23	-51	169.1	237.0	71.3
08452+4140 AB	-284	-652	-11	-9	698.6	725.4	96.3
09144+5241 AD	-156	-57	4	0	169.8	170.1	99.9
09157-0114 AB	-117	-73	-24	-16	109.1	166.8	65.4
09449-8031	47	54	-5	2	73.5	77.0	95.5
09522+0313	-426	17	-12	6	414.1	439.8	94.2

Table continues on next page.

Table 2 (cont.): Linear Solutions

WDS No.	PM A RA	PM A Dec	PM B RA	PM B Dec	Diff Vector Mag[1]	Mag, Sum of PM	Rating (%)
09573-2902	-22	10	1	-40	55.1	64.2	85.8
10015+6843	-44	-24	-46	-24	2	102.0	2.0
10029+6847	-27	-19	-31	-22	5	71.0	7.0
10041-7604	35	-3	-39	-3	74	74.3	99.7
10049+5529 AB	-5	0	55	-35	69.5	70.2	99.0
10200+1950 AC	311	-153	-502	-43	820.4	850.4	96.5
10200+1950 CD	-502	-43	-11	-25	491.3	531.2	92.5
11111+3027 AC	591	-198	-63	-16	678.9	688.3	98.6
11137+2008 AB, C	-388	-125	-21	1	388.0	428.7	90.5
11152+7329 AB	-402	110	1	-2	418.3	419.0	99.8
11279+0251 AB	17	-10	-90	17	110.4	111.3	99.1
11387+4507 AC	-594	15	0	4	594.1	598.2	99.3
11387+4507 AE	-594	15	-31	7	563.1	6256.0	89.9
12023+7222	24	3	-17	13	42.2	45.6	92.6
12095-1151 AB	296	-168	296	-168	0.0	680.7	0.0
12095-1151 AC	296	-168	1	-69	311.2	409.4	76.0
12095-1151 BC	296	-168	1	-69	311.2	409.4	76.0
12115+5325	-164	-125	-170	-136	12.5	423.9	3.0
12116+3605 AB	-46	7	55	-39	111.0	114.0	97.4
12151-0715 AB	-250	-53	-241	-69	18.4	506.2	3.6
12281+4448	-180	-7	-178	4	11.2	358.2	3.1
12351+0727 AC	155	-91	-41	8	219.6	221.5	99.1
12454+1422	-50	4	82	-45	140.8	143.7	98.0
12555+1130	-22	-8	-86	20	69.9	111.7	62.5
13064+7618	-58	15	-33	10	25.5	94.4	27.0
13120+3205	15	-6	15	-6	0.0	32.3	0.0
13518-3300	-70	-22	-36	-30	34.9	120.2	29.0
14497+4843	-78	97	-78	97	0.0	248.9	0.0
14584+7108 AC	-5	-13	-48	9	48.3	62.8	77.0
15090-2144	-43	7	17	-41	76.8	88.0	87.4
15169-0817 AB	-102	-237	-100	-225	12.2	504.2	2.4
15174+4348 AB	2	-26	-6	5	32.0	33.9	94.5
15261+1810	14	-8	14	-8	0.0	32.2	0.0
15598+1723 AB	8	2	-27	-66	76.5	79.6	96.1
16060+1319 AB	18	-19	-15	-3	36.7	41.5	88.4
16081+1703 AB	-34	-6	-26	-32	27.2	75.8	35.9
16147+3352 BD	-289	-85	4	-17	300.8	318.7	94.4
16256-2327 AB	35	-32	-14	-27	49.3	77.8	63.3
16278+2054 AC	34	-119	-36	36	170.1	174.7	97.4
17048+2805 AB	7	7	-97	-97	147.1	147.1	100.0

Table continues on next page.

Table 2 (cont.): Linear Solutions

WDS No.	PM A RA	PM A Dec	PM B RA	PM B Dec	Diff Vector Mag[1]	Mag, Sum of PM	Rating (%)
17121+2114 AB	22	4	-1	-8	25.9	30.4	85.3
17153-2636 AD	-57	-115	0	3	131.0	131.4	99.8
17153-2636 BD	-53	-114	0	3	128.4	128.7	99.8
17262+2927 AB	0	3	19	3	19.0	22.2	85.4
17293+2924 AB, C	-189	-278	-9	-10	322.8	349.6	92.3
17427-2222	-7	-66	5	-10	57.3	77.6	73.9
18029+5626 AC	-17	38	6	12	34.7	55.0	63.1
18055+0230 AC	28	-109	-2	-1	112.1	114.8	97.7
18055+0230 AD	28	-109	0	0	112.5	112.5	100.0
18055+0230 AS	28	-109	1	0	112.3	113.5	98.9
18222-1505 AB	23	28	7	14	21.3	51.9	41.0
18312+1311	1	-21	21	-1	28.3	42.0	67.3
18369+3846 AB	201	288	4	-6	353.9	358.4	98.7
18485+1045 AB	127	-437	5	6	459.5	462.9	99.3
18485+1045 AC	127	-437	-4	-10	446.6	465.9	95.9
18485+1045 AD	127	-437	4	4	457.8	460.7	99.4
18512+5923 AB	79	25	-16	20	95.1	108.5	87.7
18591+1338 AB	17	-126	9	-12	114.3	142.1	80.4
19037+1658	-20	-74	7	-10	69.5	88.9	78.2
19074+3230 AC	124	30	122	111	81.0	292.5	27.7
19074+3230 CE	122	111	0	-1	165.6	165.9	99.8
19121+4951 AD	-210	622	-29	-34	680.5	701.2	97.1
19201+2639 AB	13	-52	-19	11	70.7	75.6	93.5
19246+2131 AB	15	34	8	-72	106.2	109.6	96.9
19266+2530	16	-10	8	1	13.6	26.9	50.5
19314+3643 AC	7	-15	4	3	18.2	21.6	84.7
19368-1027	-271	-299	-300	-263	46.2	802.5	5.8
19524+2551 AC	4	11	27	24	26.4	47.8	55.2
20099+2055 AC	59	97	5	-5	115.4	120.6	95.7
20099+2055 BC	78	90	5	-5	119.8	126.2	95.0
20144-0603 AB	37	-1	48	16	20.2	87.6	23.1
20329+1357 AB, CD	19	-17	-77	148	190.9	192.3	99.3
20368+1444 AC	13	13	-19	-47	68.0	69.1	98.4
20368+1444 BC	12	12	-19	-47	66.6	67.7	98.5
20387+3838 AB	202	-195	24	-8	258.2	306.1	84.4
20494+1124 AB, C	196	-132	116	-105	84.4	392.8	21.5
20517-4054	16	-11	22	-11	6.0	44.0	13.6
20520+4346	7	-2	28	6	22.5	35.9	62.6
21124-1500	78	-40	50	-22	33.3	142.3	23.4
21144+2905 AB	-4	-17	32	2	40.7	49.5	82.2

Table concludes on next page.

Table 2 (conc.): Linear Solutions

WDS No.	PM A RA	PM A Dec	PM B RA	PM B Dec	Diff Vector Mag[1]	Mag, Sum of PM	Rating (%)
21148+3803 AD	159	431	-1	-4	463.5	463.5	100.0
21330+2043 AB	-5	-43	-5	-25	18.0	68.8	26.2
21441+2845 AD	277	-251	-1	-55	340.1	428.8	79.3
21495+0324 AB	-4	-15	25	-57	51.0	77.8	65.6
21520+5548 AB	20	1	11	23	23.8	45.5	52.2
21555+5232 AC	-10	-4	81	11	92.2	92.5	99.7
21555+5232 CD	81	11	2	-1	79.9	84.0	95.1
22045+1551 AE	-25	-69	57	-64	82.2	159.1	51.6
22057+2954	-8	-15	-42	-31	37.6	69.2	54.3
22143+1711 AB	-81	-93	-83	-91	2.8	246.5	1.1
22237+2051 AC	335	-17	14	-13	321.0	354.5	90.5
22280+5742 AC	-802	-386	1	-1	890.5	891.5	99.9
22280+5742 AI	-802	-386	-8	-8	879.4	901.4	97.6
22326+0725	-17	-2	35	16	55.0	55.6	99.0
22396-1237 AB	228	-152	228	-165	13.0	555.5	2.3
22477-1403 AB	31	-10	-31	-18	62.5	68.4	91.4
22478-0414 BC	-200	-313	7	-5	371.1	380.0	97.6
22490+6834 AB	114	61	117	73	12.4	267.2	4.6
23077+0636 AC	55	1	-2	-13	58.7	68.2	86.1
23133+2205 AB	4	-50	9	-38	13.0	89.2	14.6
23141-0855 AC	556	-40	81	-39	475.0	647.3	73.4
23141-0855 BC	561	-34	81	-39	480.0	651.9	73.6
23212+3526 AB	-1	-2	24	-8	25.7	27.5	93.4
23228+2034 AC	313	-12	5	-7	308.0	321.8	95.7
23564-0930 AB	-266	-66	-287	-60	21.8	567.3	3.9

(Continued from page 33)

a moment.

I then programmed Excel to generate a histogram of all of the data, which is shown in Figure 1.

As you can see in Figure 1, orbital solutions show very small "ratings" (near 0.0) while optical pairs showed high values (near 100.0)

Orbits are graded as to their accuracy or reliability (with 1 being a strong solution and 5 classed as indeterminate). Linear solutions are not graded. This partly explains the much wider spread of the linear solution data.

But there's another force at work, too. Linear solutions can be very difficult to get right. In some cases (most notably those with values of 90.0 or higher) the plot of measurements shows a perfectly straight line over many decades. But other linear solutions, when you plot the measurements, show a much shorter plot and sometimes the points don't all lie along a nice straight line. This could be partly due to the fact that an extremely long-period binary star with an orbit nearly edge on to our line of sight would certainly look linear even over several hundred years of observations. I suspect that many of the cases where we see linear motion in the 0.0 to 40.0 range could be just such examples. We may also be dealing with measurement error, especially for close pairs where the error is a larger percentage of the value itself. This could play a significant role for these cases.

The Anomalies

WDS 05098+2802 BC (Figure 2) has a grade 4 orbit computed by Wulff Heintz in 1976. It scored 62% because the two stars have widely varying proper motions — +103, +16 versus +26, -18. The proper motion data suggest strongly that this is a linear pair. Figure 2 is a plot of the orbit Heintz computed.

As you can see, the micrometric data points (in green) are wildly scattered, while the interferometric points (in blue) are nicely placed on the orbital curve.

Another Statistical Tool for Evaluating Binary Stars

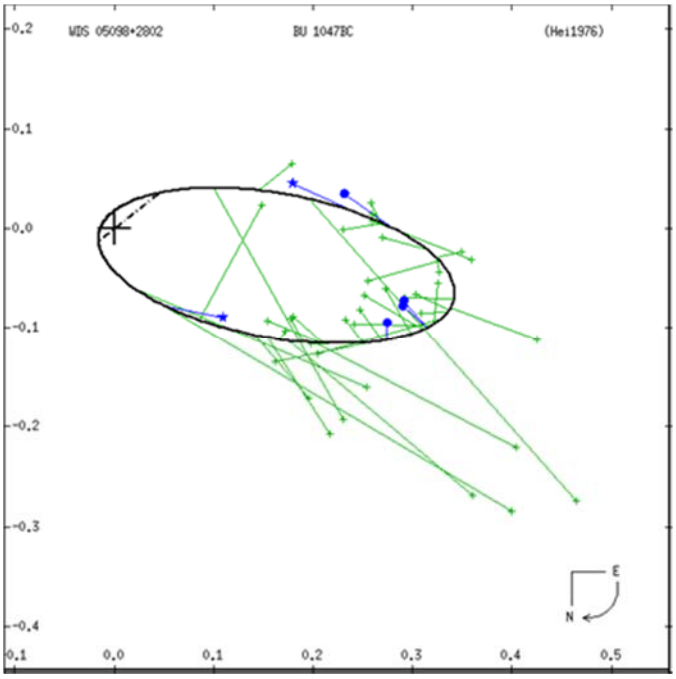

Figure 2. Grade 4 orbit of WDS 05098+2802 BC computed by Wulff Heintz in 1976

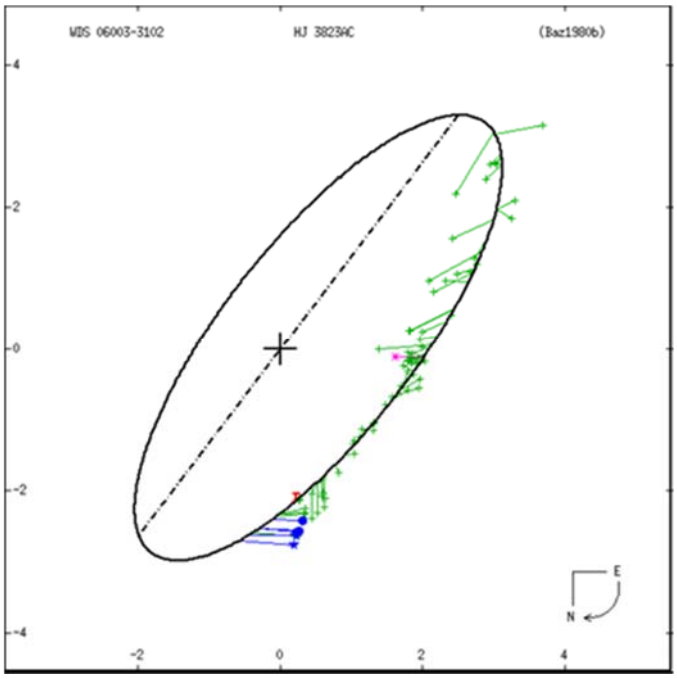

Figure 3. Grade 5 orbit of WDS 06003-3102 as computed by Baize.

Also note that the interferometric data seems to suggest a slightly different orbit than the one computed by Heintz. The micrometric points show so much scatter because this system is at the limits of micrometric measurement. In a pair this close, where an orbit has been solved, differences in proper motion are of no major concern. There could also be an error (typoraphic or even wrong data) in one or both of the proper motions.

WDS 06003-3102 AC scored 35% and has been assigned a grade of 5. The proper motions of the two stars are [-47 +41] and [-44 +102]. The RA numbers are very close together but the DEC numbers make the likelihood of this pair being binary seem low. A solution was derived in 1980 by Paul Baize. A note in the 6th Orbit Catalog reads, "This one is the closest to the line and the incomplete orbital coverage of the wider system may be the culprit." Indeed! The plot of the orbit is shown in Figure 3.

To be fair to Baize, the inteferometric data were obtained long after Baize derived his orbital solution.

Three other pairs in the orbit list show anomalies. None of them are published in the sixth orbit catalog yet so grades are not available. The first pair is WDS 07366-1429 AB. With an incredible rating of 99.94%, and proper motions of [+2 +2] and [-32 -24], every indication is that this pair is actually optical and not physical.

WDS 09104+6708 AB scored 52% and had proper

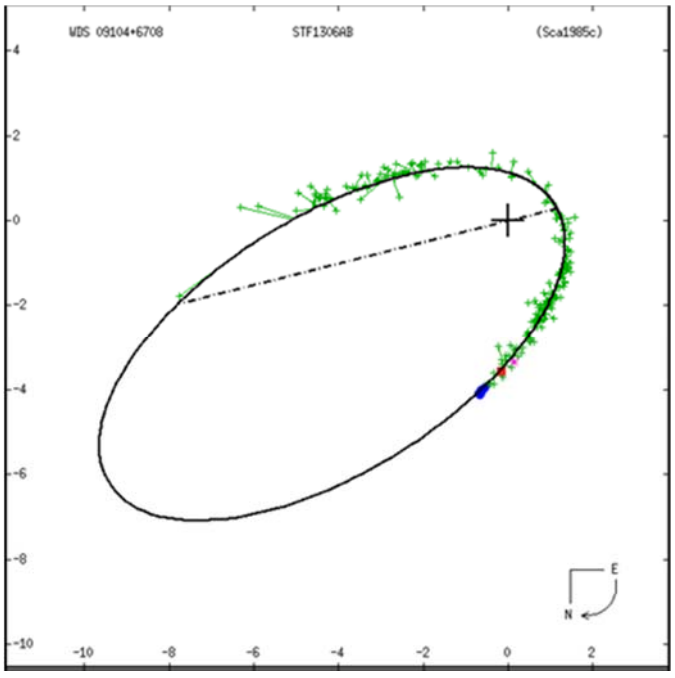

Figure 4. Orbit of WDS 09104+6708 as computed by Scardia.

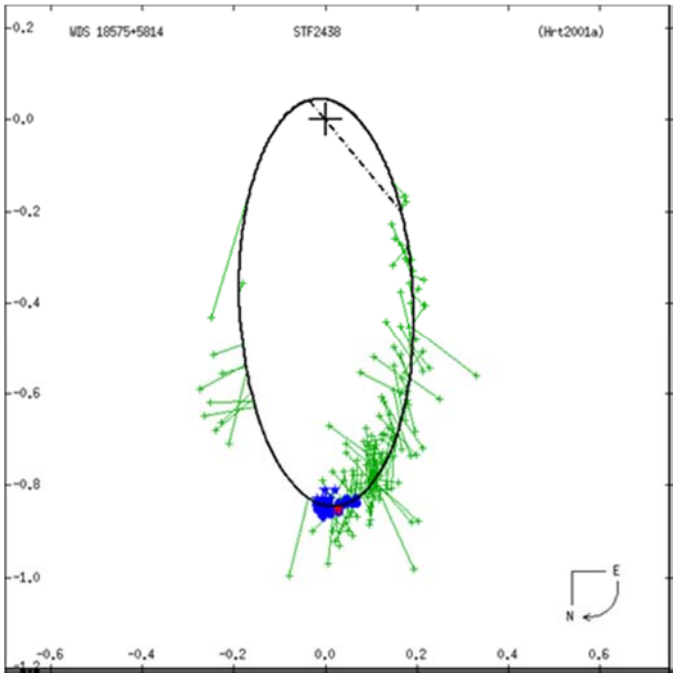

Figure 5. Orbit of WDS 18575+5814.

motions of +7, -95 and +4, -30. The orbit was computed by Scardia in 1985, and the points do indeed lie on a solid ellipse (Figure 4.)

And WDS 18575+5814 scored 65% with proper motions of +4, +37 and +39, +2. The orbital solution looks solid (Figure 5).

I have referred the possibly linear anomaly to William Hartkopf at the US Naval Observatory for possible review of the solutions on record. He replied that the proper motions for the "anomalies" should be rechecked since better data are now available than what was on file at the time of the solutions. In fact, it could be generalized that if the proper motions of a pair that shows obvious orbital motion are very different, then one or both of the proper motions is suspect.

For the linear anomalies, I extracted those systems from Table 2 and they are given in Table 3.

In particular, there are special oddities in this list. Three of these systems also have orbital solutions! They are WDS 03122+3713, WDS 04130-2832, and WDS 02442+4914.

Conclusions and Recommendations

Whereas earlier statistical criteria were useful to help predict the binary or linear nature of a given system, in this era of greater and greater precision in proper motions, it is becoming increasingly easy to compare the proper motions of both stars in a system. Even now, the US Naval Observatory is working on a project to import proper motion data for all the systems in the WDS from the new UCAC4 catalog. When this project is complete, double star astronomers will have available to them a new and powerful tool to help them know what kind of solution to seek for a pair. As the two cases of clear orbits with high rating scores show, no statistical tool is 100% reliable, but the rating system does help us know what sort of solution to begin working on.

Given the high correlation between the rating score and orbital pairs with equal or very nearly equal proper motions, it is probably safe to say that any time we see a pair with common proper motion, we are probably dealing with a true binary. It would then be necessary to precess the measurements and plot them to look for any signs of elliptical motion. A secondary approach would be to try to obtain parallax data and radial velocity data for both stars in the system. If the parallax is nearly the same and radial velocities are close to each other, the odds of a common proper motion system being binary are much higher.

At the opposite end of the spectrum, when two stars of a system have widely differing proper motions, it is probably best to try to seek a linear solution for that pair (after precessing the measurements and creating a plot).

Acknowledgments

This research has made use of the Washington Double Star Catalog maintained at the U.S. Naval Observatory.

Special thanks to William I. Hartkopf of the US Naval Observatory for filling my insatiable quest for data and for pre-publication review of this manuscript.

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Table 3. Linear Anomolies Listed in Order of Increasing Rating

WDS No.	PM A RA	PM A Dec	PM B RA	PM B Dec	Diff Vector Mag[1]	Mag, Sum of PM	Rating (%)
01417-1119	43	-417	43	-417	0.0	838.4	0.0
03122+3713	-38	-39	-38	-39	0.0	108.9	0.0
04130-2832	90	44	90	44	0.0	200.4	0.0
12095-1151 AB	296	-168	296	-168	0.0	680.7	0.0
13120+3205	15	-6	15	-6	0.0	32.3	0.0
14497+4843	-78	97	-78	97	0.0	248.9	0.0
15261+1810	14	-8	14	-8	0.0	32.2	0.0
22143+1711 AB	-81	-93	-83	-91	2.8	246.5	1.1
10015+6843	-44	-24	-46	-24	2	102.0	2.0
22396-1237 AB	228	-152	228	-165	13.0	555.5	2.3
15169-0817 AB	-102	-237	-100	-225	12.2	504.2	2.4
02556+2652 AB	274	-185	270	-168	17.5	648.6	2.7
12115+5325	-164	-125	-170	-136	12.5	423.9	3.0
12281+4448	-180	-7	-178	4	11.2	358.2	3.1
12151-0715 AB	-250	-53	-241	-69	18.4	506.2	3.6
00028+0208	62	-94	54	-92	8.2	219.3	3.8
23564-0930 AB	-266	-66	-287	-60	21.8	567.3	3.9
22490+6834 AB	114	61	117	73	12.4	267.2	4.6
19368-1027	-271	-299	-300	-263	46.2	802.5	5.8
00458-4155	307	-76	288	-111	39.8	624.9	6.4
00408-0714	9	-102	1	-92	12.8	194.4	6.6
10029+6847	-27	-19	-31	-22	5	71.0	7.0
20517-4054	16	-11	22	-11	6.0	44.0	13.6
23133+2205 AB	4	-50	9	-38	13.0	89.2	14.6
04385+2656 AB	41	-56	41	-88	32.0	166.5	19.2
02442+4914 AB	511	-90	336	-84	175.1	865.3	20.2
20494+1124 AB, C	196	-132	116	-105	84.4	392.8	21.5
20144-0603 AB	37	-1	48	16	20.2	87.6	23.1
21124-1500	78	-40	50	-22	33.3	142.3	23.4
21330+2043 AB	-5	-43	-5	-25	18.0	68.8	26.2
13064+7618	-58	15	-33	10	25.5	94.4	27.0
19074+3230 AC	124	30	122	111	81.0	292.5	27.7
13518-3300	-70	-22	-36	-30	34.9	120.2	29.0